Dynamic Behavior of Interest Rates in China[1]

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Abstract: This paper examines intuitively the dynamic behavior of two highly relevant kinds of interest rate in China. The first one is the government rate, which is decided and published by the central bank and can be simulated by pure jump process. Estimation of the jump intension is given out, and by different robustness test, it keeps stable. The jump size has met the condition to make interest rate within reasonable bounds and shows some meaning of economic cycle behavior. The second one is the market rate, which is estimated by spline approximation based on the transaction data of government bonds. Several models, including Vasicek model, Vasicek-GARCH (1,1) model, CIR model, and CIR-GARCH(1,1), are empirically tested and the best performance is done by the simple Vasicek model. Furthermore, the estimate bias problem due to the near unit root process is tested and evidenced by both traditional methods and GPH test. Impact of government rate on market rate is finally checked and analyzed.

Key Words: Dynamic Behavior, Jump Process, Mean Reversion, Unit Root Process，Interest Rate

Introduction

Interest rate has been always one focus of financial research. Some hypothesis and dynamic models have been suggested, many empirical tests have been done on these hypothesis and models, and some models have been applied in the pricing of interest derivatives. Since Vasicek (1977) and Cox, Ingersoll and Ross (CIR (1985a,b)), hundreds of other complicated models have been suggested and analyzed. In pure diffusion models, not only the interest rate level is considered, some other factors are also analyzed and put into the model (Longstaff and Schwartz (1992), Brenner, Harjes and Kroner (1996)). Nonlinear generalization of CIR model is proposed by Constantinides (1992). Regime shift problem of interest rate is also analyzed by Bansal and Zhou (2001) and Sanders and Unal (1988). More especially, jump process has become necessary to the analysis of interest rates and can improve greatly the explanation power of models (Lin and Yeh (2001), Das(2002))[2].

The empirical tests of the dynamic models develop with the theoretical analysis. Different dynamic models of interest rates are tested and compared by Durham (2002), Bali (1999), Chan el.(1992), Brown and Dybvig (1986). Ball and Torous (1996) suggested that the estimation of the mean reversion process will be biased if the data is nearly unit root process. Some research has found evidence of unit root for the interest rate (Mankiw and Miron (1986), Rose (1988)), While

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[2] A good survey of term structure analysis can be found in Dai and Singleton (2002).

others rejected the hypothesis (Lai (1997), Wang and Zhang (1997), Pesando (1979)).

In China, some basic research has also been done on the interest rate. Xie and Xu (2002) empirically tested the Vasicek model and CIR model by one-month inter-bank interest rate. Fan and Fang (2002) empirically analyzed the pricing of convertible bonds in China. But they did not test the robustness of models. Zheng and Lin (2002a,b,c) estimated the term structure, liquidity premium and credit risk premium in China's bond market. But they were all static approximation and did not test the dynamic behavior of interest rate in China.

There are two highly relevant kinds of interest rate in China: government rate and market rate. The government rate is decided and published by the central bank and is subject to change suddenly. It can be denoted as pure jump process. The market rate changes every day and can be estimated by the price of government bonds. Its dynamics shows some mean reversion of Vasicek model, but due to the problem of I(1) process, this estimate is biased.

This paper is aimed to make an intuitive estimation of the dynamic behavior of interest rate in China, including the government rate and the market rate. Since for the pure jump process, the likelihood function does not exist, we can only roughly estimate the parameters by moment estimation. Some evidence of cycle behavior was found, but is subject to robust test. For the market rate, according to the estimation results of Zheng and Lin (2002a), we tested some general dynamic models, such as Vasicek model, CIR model with and without ARCH effect, and found some evidence of simple Vasicek model. But due to the biased estimation problem of near unit root process proposed by Ball and Torous (1996), the stationary test of the market rate is necessary to do the robust test of Vasicek model. In order to avoid the problem of under rejection problem of traditional unit root test, this paper use the GPH test by spectrum analysis. The test showed strong evidence of I(1) process of market rate, so the parameter estimation of Vasicek model, is biased upward. To avoid the possible impact of government rate jump on market, we readjusted the market rate and found similar results.

This paper is organized as follows. Section 1 suggests the pure jump process model of government rate. In section 2, jump intension parameter, l is estimated and the robustness test is given out. In section 3, the analysis on the jump size estimate of government is done. In section 4, using the static estimation results of term structure in China by Zheng and Lin (2002a), some general dynamic models on the market rate are tested and the preference of Vasicek model is found. Section 5 tested the estimate bias due to near unit root process and the bias evidence is found strongly. In Section 6, the impact of government rate jump on dynamics of market rate is checked and analyzed. Section 7 is a simple conclusion.

1. Pure Jump Process of Government Rates

A general one-factor mean reversion diffusion model of interest rate is

$$dr(t) = k(u(t) - r_t)dt + s(t)dW_t$$ [3]

Where k means the speed of reverting to the mean interest rate, $u(t)$ is the time t mean interest rate, and $s(t)$ is volatility, $W(t)$ is Brownian motion. Considering the diffusion-jump process,

[3] Multi-factors, ARCH effect and regime shift problems can also be considered.

$$dr(t) = k(u(t) - r_t)dt + s(t)dW_t + JdP,$$

Where J is a stochastic variable following some distribution, in general, normal distribution, $N(a, q^2)$, dP is Poisson process with parameter l .

Under the diffusion-jump process, the likelihood function exists and can be written as in discrete time series:

$$f(u(t), k, s(t), a, q, l) = \sum_{n=0}^{\infty} \exp(-l) \frac{l^n}{n!} \frac{1}{\sqrt{2p(s(t)^2 + nq^2)}} \exp\left(-\frac{(\Delta r_t - ku(t) + kr_t - na)^2}{2(s(t)^2 + nq^2)}\right)$$

So the parameters can be estimated by maximum likelihood method[4].

But in China, the government rate is decided by central bank and then keeps unchanged for some period. It is totally different from the diffusion process or diffusion-jump process, under which interest rate changes all the time. Fortunately, it is similar to pure jump process.

$$dr_t = JdP$$

Where J follows normal distribution $N(a, q^2)$, dP is the Poisson process with parameter l .

Proposition 1. Under the pure jump process, the likelihood function does not exist, so the maximum likelihood method can not be used to estimate the parameters.

Proof: if $dP = n \neq 0$, then in discrete time series, $\Delta r_t \to N(na, nq^2)$ with probability $\exp(-l)\frac{l^n}{n!}$, which has its part in the likelihood function as

$$\exp(-l) \frac{l^n}{n!} \frac{1}{\sqrt{2p nq^2}} \exp\left(-\frac{(\Delta r_t - na)^2}{2nq^2}\right).$$

But when $dP = 0$, $p(\Delta r_t = 0) = 1$, $p(\Delta r_t \neq 0) = 0$, is not continuous and the continuous density function does not exist.

Proposition 2. For pure jump process with discrete data of relatively short period, moment's method can be used to estimate the parameters intuitively.

[4] Since

$$\int f(u(t), k, s(t), a, q, l) d\Delta r_t = \sum_{n=0}^{\infty} \exp(-l) \frac{l^n}{n!} \int \frac{1}{\sqrt{2p(s(t)^2 + nq^2)}} \exp\left(-\frac{(\Delta r_t - ku(t) + kr_t - na)^2}{2(s(t)^2 + nq^2)}\right) d\Delta r_t$$

$$= \sum_{n=0}^{\infty} \exp(-l) \frac{l^n}{n!} = 1$$

Proof: Since jump process is the limit of binominal process when time horizon increases to a long period, in a relatively short period, we can approximate the jump process $dr_t = JdP$ by binominal process:

$$dr_t = 0, \text{ with probability } 1 - l \text{ ; } dr_t \to N(a, q^2) \text{ with probability } l \text{ .}$$

then $E(dr_t) = l\,a,\ s^2(dr_t) = l\,q^2$. Furthermore, in discrete time series,

$\hat{E}(dr_t) = \dfrac{\sum_{i=1}^{N} \Delta r_i}{N} = \dfrac{\sum_{j=1}^{n} \Delta r_j}{n} \times \dfrac{n}{N} = \hat{a} \times \dfrac{n}{N}$, where N, n denotes the number of time series data and the jump times respectively. Then the intuitive estimation of l is $\hat{l} = \dfrac{n}{N}$.

2. Estimation of l and robustness test

In China, the interest rate is regulated, decided and published by the central bank without any daily change. Since there are absolute changes in the policies before and after 1980, we neglect the government rate date before 1980 and utilize the data after 1980 to estimate the jump process parameters. We use the monthly data of one year government saving rate from April 1980 to November 2002, totally 271 data. The series of interest rate difference has 270 data respectively.

In the total 270 data, 15 data are different from zero and can denote the jump times. Then a rough estimate of l is $\dfrac{15}{270} \approx 0.056$.

But this estimation is so rough that it needs other test to show its robustness. Since the parameter of l is independent from normal distribution, we can estimate l independently.

Proposition 3. If the process N(t) follows Poisson distribution, the series of jump time $T_n, n = 1,2,3,4...$ ***follows the* gamma *distribution with parameters*** l ***and n.***

Proof:

$$P(T_n \le t) = P(N(t) \ge n) = \sum_{i=n}^{\infty} e^{-lt} \frac{(lt)^i}{i!},$$

$$\frac{\partial P}{\partial t} = \sum_{i=n}^{\infty} [(-l)e^{-lt} \frac{(lt)^i}{i!} + e^{-lt} \frac{l^i t^{i-1}}{(i-1)!}] \quad ,$$

$$= l e^{-lt} \sum_{i=n}^{\infty} (\frac{(lt)^{i-1}}{(i-1)!} - \frac{(lt)^i}{i!}) = l e^{-lt} \frac{(lt)^{n-1}}{(n-1)!}$$

This is the gamma probability density function for $P(T_n = t)$. Furthermore,

$$f(T_n = t) = l e^{-lt} \frac{(lt)^{n-1}}{(n-1)!},$$

$$f(T_1 = t_1, T_2 = t_2, \ldots, T_n = t_n) = \prod_{i=1}^{n} l e^{-l t_i} \frac{(l t_i)^{i-1}}{(i-1)!},$$

$$\log f(T_1, \ldots, T_n) = \sum_{i=1}^{n} (\log l - l t_i + (i-1)\log l + (i-1)\log t_i - \log(i-1)!)$$

$$= \frac{n(n+1)}{2} \log l - l \sum t_i + \sum ((n-1)\log t_i + \log(n-1)!)$$

$$\frac{\partial f}{\partial l} = \frac{n(n+1)}{2l} - \sum t_i = 0$$

$$\hat{l} = \frac{n(n+1)}{2\sum t_i}$$

Proposition 4. If the N (t) follows Poisson distribution with parameter l , then the length of time interval between two jumps $M_1, M_{2,} \ldots, M_n$, $M_i = T_i - T_{t-1}$, follows the exponential distribution with mean $1/l$.

Proof:

$$P(M \le t) = 1 - P(M > t) = 1 - P(N(t) = 0) = 1 - e^{-l t},$$

$$\frac{\partial P}{\partial t} = -l e^{-l t}$$

The estimation of l by direct Poisson distribution, moment estimation, gamma distribution and exponential distribution is listed in table 1.

Table 1. Estimation of l

	Poisson	Moments	Gamma	Exponential
$\hat{l}$	0.056	0.056	0.048	0.057
5% confidential interval	[0.031,0.091]			[0.037,0.10]

From table 1, we can see that the estimation result by Poisson distribution, Moments methods, Exponential distribution are nearly the same, and the estimation result of Gamma distribution is also in the 5% confidential interval of the other estimations. Different estimations of parameter l are similar and stable.

3. Jump Size of Government Rate

Since l is estimated and keep stable to much extent, we can then further discuss the jump size (a, q^2).

Proposition 5: for the government rate to be within reasonable bounds a should be zero.

Proof: if $a \neq 0$, then $E(dr_t) = l a \neq 0$ .When $a \succ 0$, it is upward trend, and the government rate will increase to an unreasonable high level. When $a \prec 0$, it is downward trend, and the government rate will decrease to an unreasonable low level[5].

The estimation of (a, q^2) is listed in table 2. We can see that a is not significantly different from zero, which is consistent with the reasonable bounds condition. Furthermore, it has some meaning of economic cycle behavior. When the economic falls into deflation and the government wants to speed up the economic development, the government rate will fall down, and the change of interest rate will be minus. When the economy is too hot and the government wants to slow down the speed of economic development, the government rate will increase and the change of interest rate will be plus. The minus and plus change of interest rate with economic cycle behavior counteracted each other and made the mean of jump to be near zero.

Table 2. Estimation of jump size (a, q)

	$\hat{a}$	$\hat{q}$
Estimation	-0.25%	1.70%
Std. error	0.44%	---
T value	0.56	---

It should be noted that the estimation is an unconditional estimation and does not consider the conditional information. So if we use this model to forecast and simulate the government rate, the minus interest rate may give out in some circumstances. Conditional information, including history of interest change, economic position, etc, should also be considered if we want to forecast the change of government rate.

4. Dynamic Behavior of Market Rates in China

From this section, we turn to the market rate in China. The market rate is decided by the equilibrium of demand and supply and can reflect the market situation. Since in China, people have no way of savings in banks or lending from banks by market rate. They can only refer to buying and selling the government bonds. Buying bonds implies that they lend or save the money by market rate, selling bonds implies that they borrow or take the money by market rate. Then the prices of government bonds can be used to estimate the market rate with different maturities, which is generally called as the term structure of interest rate.

Generally, term structure is directly estimated from the prices of discount bonds, in which case the interest rate equals the yield to maturity of bond. This is the often case if there exist discount bonds in the market. But unfortunately, in China, all of the bonds that are publicly listed and traded in Shanghai and Shenzhen Stock Exchange are coupon bonds. For these bonds, traditional estimation method fails since the yield to maturity is not equivalent to interest rate any more in this case. McCulloch(1971), Lin and Yeh(2001), Carleton and Cooper(1976) all tried to settle this problem down and suggested some estimation methods, in which the spline approximation is one general method. Zheng and Lin (2002a) done the use of approximation method of McCulloch (1971) to compare the different methods of term estimations, select the suitable parameters in spline approximation method and estimate the term structure in China. Zheng and Lin (2002b,c) further discussed the problem of liquidity premium and credit risk premium. This paper will directly use the estimation result of Zheng and Lin (2002a) and empirically test the dynamic behavior of the market rate in China.

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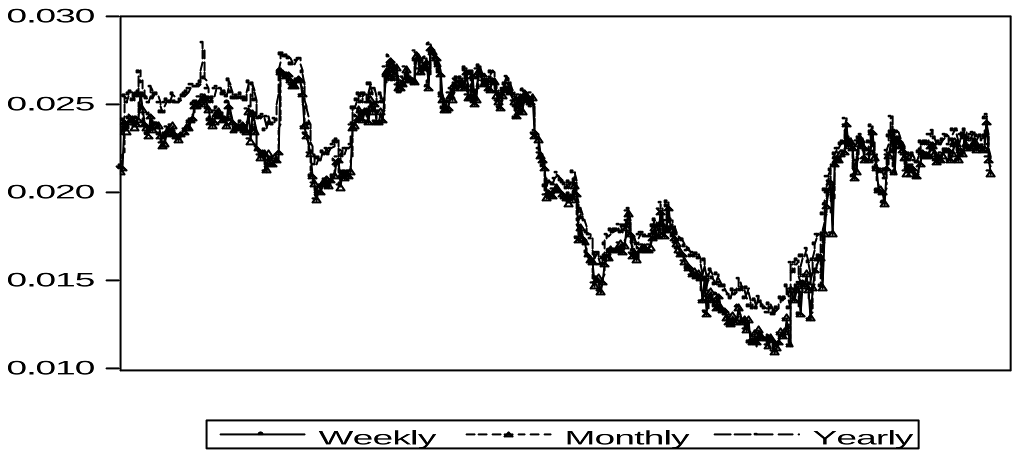
[5] We can see that the general diffusion model also meet the condition that $E(dr_t) = 0$.

In China, the development of bond market is far behind of stock market. The number and kinds of bonds are much less than those of stocks. The shortage of bonds limits the estimation of term structure a lot. Before May 2001, there are only five kinds of government bonds traded in stock exchange, the spline approximation method can not get the significant estimation results. We can only limit our estimation to the period from May 2001 to Dec 2002. During this period, the R-square of regressions are all above 95% and the significance level is all above 5%. Zheng and Lin(2002a) estimated the weekly data, but to analyze the dynamic behavior of market rate, we estimate daily data here and get the daily term structure from May 8, 2001 to Jan 3, 2003, 402 data totally. The behavior of market rate(weekly, monthly, yearly) is plotted as figure 1. The weekly rate almost coincides with monthly rate, and the yearly rate is slightly larger than monthly rate. We can intuitively find some evidence of mean reversion. The market rate changes around approximately 2%.

Fig 1. Dynamic Change of Market Rate:China



To analyze the dynamic behavior of the market rate in China, we use some different models, Vasicek Model, Vasicek-GARCH (1,1) model, CIR model, and CIR-GARCH(1,1) model[6]. The estimation results of different models on weekly, monthly and yearly rate are shown in table 3,4, 5 respectively[7].

We can see that, the parameter estimations of weekly data and monthly data are nearly the same. Vasicek model, Vasicek-GARCH (1,1), and CIR model are significant at different level, the estimation of CIR-GARCH(1,1) is not significant. But as a whole stochastic process, only Vasicek model and CIR model are significant at 10% (checked by the F value of every estimate). The log likelihood value of Vasicek model is much larger than the CIR model. For monthly data, only Vasicek and CIR model are significant and the log likelihood of Vasicek is also much larger than CIR model. It implies that Vasicek model performs is better than CIR model which is consistent with Xie and Wu (2002)[8]. The long term mean estimate of different models are stable. Long term mean for weekly rate is about 2.08%, monthly rate is 2.14%, and yearly rate is 2.25%.However, we can find that the R square of models is so small that it only explains little part of interest rate change.

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[6] Jump process and regime shift problem are neglected here since the time horizon is too short to consider them.
[7] For Vasicek model and CIR model, the estimate is done both by OLS and GMM, the results are similar. Vasicek-ARCH model and CIR-ARCH model is estimated by Maximum Likelihood Method.
[8] Their data is 1 month inter-bank interest rate. But the estimation of long term mean is more than 9%, which is a little unreasonable. They did not consider the estimation bias problem either.

Table 3. Dynamic Model of The market rate in China: Weekly Rate
Model 1. Vasicek Model: $\Delta r_t = k(u - r_t) + e_t, e_t \to N(0, s^2)$
Estimation Results: $\hat{k} = 0.019^{*}(1,94), \hat{u} = 2.08\%^{*}(1.90), s = 0.09\%$ $R^2 = 0.9\%, F = 3.78^{*}, \log = 2262.91$
Model 2. Vasicek-GARCH(1,1): $\Delta r_t = k(u - r_t) + e_t,$ $e_t = \sqrt{h_t} v_t, v_t \to i.i.d.(0,1),$ $h_t = k + a_1 h_{t-1} + b_1 e_{t-1}^2$
Estimation Results: $\hat{k} = 0.019^{*}(1.88), \hat{u} = 2.02\%^{*}(1.80),$ $\hat{k} = 2.08E-07^{***}(2.75), \hat{a}_1 = 0.60^{***}(5.20), \hat{b}_1 = 0.15^{***}(3.51)$ $R^2 = 0.9\%, F = 0.92, \log = 2268.8$
Model 3. CIR model: $\Delta r_t = k(u - r_t) + \sqrt{r_t} e_t, e_t \to N(0, s^2)$
Estimation Results: $\hat{k} = 0.018^{*}(1.94), \hat{u} = 2.06\%^{*}(1.99), s = 0.6\%;$ $R^2 = 1\%, F = 3.90^{**}, \log = 1473.95$
Model 4. CIR-GARCH(1,1) model: $\Delta r_t = k(u - r_t) + \sqrt{r_t} e_t,$ $e_t = \sqrt{h_t} v_t, v_t \to i.i.d.(0,1),$ $h_t = k + a_1 h_{t-1} + b_1 e_{t-1}^2$
Estimation Results: $\hat{k} = 0.009(1.14), \hat{u} = 1.78\%(1.04),$ $\hat{k} = 2.81E-05^{***}(2.75), \hat{a}_1 = -0.06(1.15), \hat{b}_1 = 0.35^{***}(4.81)$ $R^2 = 0.5\%, F = 0.58, \log = 1495.46$

Notes:*,,***indicate significance at 1%,5%,10% respectively.**

The number in parenthesis is t value of parameter.

Table 4. Dynamic Model of The market rate in China: Monthly Rate
Model 1. Vasicek Model: $\Delta r_t = k(u - r_t) + e_t, e_t \to N(0, s^2)$
Estimation Results: $\hat{k} = 0.018^{*}(1,92), \hat{u} = 2.14\%^{*}(1.98), s = 0.08\%$ $R^2 = 0.9\%, F = 3.68^{*}, \log = 2271.06$

Model 2. Vasicek-GARCH(1,1): $\Delta r_t = k(u - r_t) + e_t,$ $e_t = \sqrt{h_t} v_t, v_t \to i.i.d.(0,1),$ $h_t = k + a_1 h_{t-1} + b_1 e_{t-1}^2$
Estimation Results: $\hat{k} = 0.018^{**}(2.21), \hat{u} = 2.12\%^{**}(2.15),$ $\hat{k} = 1.64E-07^{***}(3.05), \hat{a}_1 = 0.60^{***}(7.14), \hat{b}_1 = 0.15^{***}(4.33)$ $R^2 = 0.9\%, F = 0.91, \log = 2280.32$
Model 3. CIR model: $\Delta r_t = k(u - r_t) + \sqrt{r_t} e_t, e_t \to N(0, s^2)$
Estimation Results: $\hat{k} = 0.017^{*}(1.92), \hat{u} = 2.14\%^{*}(1.97), s = 0.6\%;$ $R^2 = 1\%, F = 3.84^{**}, \log = 1481.20$
Model 4. CIR-GARCH(1,1) model: $\Delta r_t = k(u - r_t) + \sqrt{r_t} e_t,$ $e_t = \sqrt{h_t} v_t, v_t \to i.i.d.(0,1),$ $h_t = k + a_1 h_{t-1} + b_1 e_{t-1}^2$
Estimation Results: $\hat{k} = 0.008(1.02), \hat{u} = 1.55\%(0.83),$ $\hat{k} = 2.47E-05^{***}(12.35), \hat{a}_1 = -0.06^{**}(2.40), \hat{b}_1 = 0.43^{***}(4.92)$ $R^2 = 0.4\%, F = 0.36, \log = 1507.08$

Notes:*,,*** indicate significance at 1%,5%,10% respectively.**

The number in parenthesis is t value of parameter.

Table 5. Dynamic Model of The market rate in China: Yearly Rate
Model 1. Vasicek Model: $\Delta r_t = k(u - r_t) + e_t, e_t \to N(0, s^2)$
Estimation Results: $\hat{k} = 0.014^{*}(1{,}70), \hat{u} = 2.25\%^{*}(1.98), s = 0.07\%$ $R^2 = 0.7\%, F = 2.90^{*}, \log = 2343.85$
Model 2. Vasicek-GARCH(1,1): $\Delta r_t = k(u - r_t) + e_t,$ $e_t = \sqrt{h_t} v_t, v_t \to i.i.d.(0,1),$ $h_t = k + a_1 h_{t-1} + b_1 e_{t-1}^2$
Estimation Results: $\hat{k} = 0.014(0.85), \hat{u} = 2.26\%(0.85),$ $\hat{k} = 6.83E-08(0.26), \hat{a}_1 = 0.60^{**}(1.97), \hat{b}_1 = 0.15(1.44)$ $R^2 = 0.7\%, F = 0.72, \log = 2337.12$

Model 3. CIR model: $\Delta r_t = k(u - r_t) + \sqrt{r_t} e_t, e_t \to N(0, s^2)$
Estimation Results: $\hat{k} = 0.014^*(1.70), \hat{u} = 2.13\%^*(1.72), s = 0.48\%;$ $R^2 = 0.7\%, F = 2.97^*, \log = 1568.69$
Model 4. CIR-GARCH(1,1) model: $\Delta r_t = k(u - r_t) + \sqrt{r_t} e_t,\ e_t = \sqrt{h_t} v_t, v_t \to i.i.d.(0,1),\ h_t = k + a_1 h_{t-1} + b_1 e_{t-1}^2$
Estimation Results: $\hat{k} = 0.014(0.83), \hat{u} = 2.23\%(0.91),$ $\hat{k} = 6.54E-06(0.51), a_1 = 0.60^{**}(2.00), b_1 = 0.15(1.48)$ $R^2 = 0.7\%, F = 0.69, \log = 1586.62$

Notes: *,,*** indicate significance at 1%,5%,10% respectively.**

The number in parenthesis is t value of parameter.

5. Estimation Bias of Dynamic Models

Ball and Torous (1996) concluded by simulation that if the interest rate series follows close to unit root, all the estimation methods, including least squares, generalized method of moments, and maximum likelihood estimation, provide upward biased estimate of model's speed of adjustment coefficient, which is denoted by k in above models. In order to utilize the model to price the interest rate derivatives safely, we need to test whether the interest rate reject the hypothesis of unit root process.

Traditional unit root test estimation by ADF statistic and PP statistic is shown in table 6. Both statistics are not significant and the unit root hypothesis can not be rejected.

Table 6: Unit Root Test of The market rate in China: Traditional Method

ADF Test Statistic	**Weekly rate:-1.54**	**1% Critical Value***	**-3.4489**
	Monthly rate:-1.53	**5% Critical Value**	**-2.869**
	Yearly rate:-1.41	**10% Critical Value**	**-2.5708**
PP Test Statistic	**Weekly rate:-1.72**	**1% Critical Value***	**-3.4487**
	Monthly rate:-1.70	**5% Critical Value**	**-2.869**
	Yearly rate:-1.51	**10% Critical Value**	**-2.5707**

*MacKinnon critical values for rejection of hypothesis of a unit root.

However, Lai(1997) pointed out that traditional test of unit root had the problem of under rejection since it is restricted to integer order, I(1) or I(0), but did not take the I(d), 0<d<1 into consideration. Lai (1997) used the Geweke and Porter-Hudak(GPH(1983)) test method of spectral regression to analyze the fractional integration problem. This method can check the mean reversion not captured by traditional tests. So if we want to test the unit root problem of the market rate in China, traditional test is not enough.

Similar to Lai (1997), the GPH method involves estimation of the fractional integration order, d, using a spectral regression for the differenced series[9]:

[9] In Lai (1997), the real interest rate data is used. But here we use the nominal data since the time period is short

$$\ln(I(l_j)) = f_0 - f_1 \ln(4\sin^2(l_j/2)) + h_t,\ j = 1,2,3.....,n$$

where $I(l_j)$ is the periodogram at harmonic frequency $l_j = 2p\,j/T$, $n = T^d$ for $0 \prec d \prec 1$ is the number of ordinates used for regression. The least squares estimate of f_1 provides a consistent estimate of $1-d$, and hypothesis testing regarding the value of d can be conducted by traditional t test.

For a sample series with T data, the periodogram at harmonic frequency l_j, $I(l_j)$ is computed by:

$$\hat{I}(l_j) = \frac{1}{2p} \sum_{k=-T+1}^{T-1} \hat{g}_j e^{-il_jk}, i^2 = -1\text{,or equivalently,}$$

$$\hat{I}(l_j) = \frac{1}{2p}(\hat{g}_0 + 2\sum_{k=1}^{T-1} g_k \cos(l_j k)), ,$$

where g_j is order j auto-covariance of sample series,

$$\hat{g}_j = \begin{cases} T^{-1} \sum_{t=j+1}^{T} (y_t - \bar{y})(y_{t-j} - \bar{y}), j = 0,1,2,...,T-1 \\ \hat{g}_{-j}, j = -1,-2,...,-T+1 \end{cases}.$$

Similar to Lai(1997), we select $d = 0.60, 0.625, 0.65$ since the data number is not much different. The estimation is shown in table 7.

Table 7. Unit Root Test of The market rate in China: GPH Method[10]

Series	U	d	GPH Test based on empirical error variance	
			H0:d=1;H1:d<1	H0:d=0;H1:d>0
Weekly Rate	0.6	0.982	-0.135	7.383***
	0.625	0.955	-0.387	8.162***
	0.65	0.938	-0.609	9.160***
Monthly Rate	0.6	0.963	-0.28	7.241***
	0.625	0.935	-0.545	7.877***
	0.65	0.918	-0.791	8.818***
Yearly Rate	0.6	0.901	-0.801	7.278***
	0.625	0.892	-0.938	7.723***
	0.65	0.913	-0.86	9.040***

Note:* indicates significance level of 1%.**

After taking the spectral transformation and regression, d is still not significantly different from 1, I(1) process of the market rate still can not be rejected.

Different from most of the papers which can reject the unit root process of interest rate after some transformation, such as Lai(1997), Wang and Zhang(1997), Pesando (1979). We still can not

and the inflation will not change a lot.

[10] The GPH test critical value is shown in appendix.

reject the I(1) process of the market rate in China. This may be due to the short time horizon since we have only the transaction data of less than 2 years, but the mean reversion can only be checked by long time period. Another reason may be that in China, investors seldom pay attention to the change of market rate, so the historical information is not captured by current price change to make it behave like random walk.

But anyway, due to near I(1) process of market rates, the speed of adjustment to mean interest rate is biased upward and can not be used to simulate the interest rate paths when pricing the interest rate derivatives by Monte Carlo method. Furthermore, the R square is so small that great errors will take place during simulation.

Fortunately, the long term mean interest rate is not affected by the I(1) process. For the moment, to price the derivatives in China, such as the forthcoming stock index option, we may just hypothesize mean interest rate as constant interest rate so that the model misspecification can be avoided. This of course will bring some pricing error at the same time. However, if we want to price the interest derivatives, such as fixed-income securities, much more effort should be done on modeling specification and robustness test.

6. Impact of Government Rate Jump on Market Rate

In China, the government rate is the dominant rate and affects the market rate in a large part. Some would propose that after considering the change of government rate, the near unit root hypothesis would be rejected and then the mean reversion model would be stable. In order to check this, we readjust the market rate according to the government rate change, and then reanalyze it. During the period from May 8, 2001 to Jan 3, 2003, only one jump happened to government rate on Feb. 21,2002, which reduced the interest rate by 0.27%. Therefore, we readjuste the market rate by increasing the interest rate by the same degree since Feb 21,2002. The estimation results for traditional method and GPH method are shown in table 8 and 9 respectively.

Table 8:Unit Root Test of the adjusted market rate in China: Traditional Method			
ADF Test Statistic	Weekly rate:-1.70	1% Critical Value*	-3.4489
	Monthly rate:-1.69	5% Critical Value	-2.869
	Yearly rate:-1.57	10% Critical Value	-2.5708
PP Test Statistic	Weekly rate:-2.07	1% Critical Value*	-3.4487
	Monthly rate:-1.57	5% Critical Value	-2.869
	Yearly rate:-1.87	10% Critical Value	-2.5707

*MacKinnon critical values for rejection of hypothesis of a unit root.

Table 9. Unit Root Test of The adjusted market rate in China: GPH Method				
Series	U	d	GPH Test based on empirical error variance	
			H0:d=1;H1:d<1	H0:d=0;H1:d>0
Weekly Rate	0.6	0.962	-0.255	6.456***
	0.625	0.943	-0.441	7.310***
	0.65	0.921	-0.675	7.872***
Monthly Rate	0.6	0.944	-0.368	6.211***
	0.625	0.926	-0.561	7.015***
	0.65	0.899	-0.835	7.430***
Yearly Rate	0.6	0.96	-0.599	9.505***

	0.625	0.945	-0.938	10.396***
	0.65	0.892	-1.069	8.832***

Similarly, the near unit root problem still exists even considering the jump problem of government rate.

The history of market rate in China is so short that it is highly difficult to analyze the impact of government rate jump on dynamics of market rate exactly[11]. But intuitively from the dynamic change of market rate, we demonstrated the impact of government rate on market rate. Shortly before and after the decrease of government rate, market rate decreased too (fig.1). This implied that the market had forecasted the decrease of government rate to some degree before the decision became public. After the change, the market rate continued to declined, which means that the market rate did not adjust with the change at once. Both of these showed the inefficiency of bond market. Therefore, although the dynamics of market rate cannot reject the I(1) process, it is not efficient .

However, with time lapsed, the impact of government rate on market rate disappeared gradually, and the market rate returned to its long-term mean. This can partly testified the mean reversion of market rate.

7. Conclusion

This paper examined intuitively the dynamic behavior of two different kinds of interest rate in China. One is the government rate which is decided and published by central bank. It is simulated by pure jump process. Estimation of the jump intension was given out and by different robustness test, it keeps stable. The jump size has met the condition to make interest rate within reasonable bounds and showed some meaning of economic cycle behavior. The other is the market rate which is estimated by spline approximation based on the transaction data of government bonds. Several models, including Vasicek model, Vasicek-GARCH(1,1) model, CIR model, CIR-GARCH(1,1), were empirically tested and the best perform was found only by the simple Vasicek model.

However, in order to check the estimate bias problem due to the near unit root process, this paper tested the unit root, i.e., I(1) hypothesis of the market rate by traditional methods and GPH test. Neither test can reject the I(1) hypothesis, which is different from other papers. This problem may be due to the short time period of the development of bond market, or less attention that is paid to bond market. This will not result in big problem when we price the general derivatives such as stock index option and future which often regard the risk free rate to be constant. But in order to price fixed-income securities, much effort should be done on model specification and robustness test.

The government rate and market rate are highly relevant. The market rate can forecast the change of government rate to some degree. Furthermore, it is highly influenced by the sudden change of government rate. Considering the impact of government rate, I(1) hypothesis still can not be rejected. By intuitively analyzing the dynamics of market rate and government rate together, the forecast and reaction of market rate to government can be easily shown to demonstrate the inefficiency of bond market and mean reversion of interest rate.

Since the history of bond market in China is so short compared with that of the government

[11] We checked and tested the diffusion-jump model to possibly analyze the impact of government rate on market rate but failed to get the convergent results.

rate, the analysis on the government rate and the market rate is done independently. The joint analysis is intuitive. Jump analysis of the market rate failed and regime shift problem is neglected. The government rate is only estimated unconditionally. All these are further development and generalization of this paper. Furthermore, the integration between stock market and bond market is also neglected here. They are for future research.

Appendix

Table 10: Finite Sample Critical Value for GPH test (Lai (1997))

	GPH test based on empirical error variance			
U	5% test			10% test
Testing H0:d=1 against H1: d<1				
0.6	-1.805			-1.358
0.625	-1.783			-1.347
0.65	-1.272			-1.343
Testing H0:d=0 against d>0				
0.6	1.627			1.277
0.625	1.625			1.267
0.65	1.622			1.267

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