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THE EQUITY PREMIUM PUZZLE
AND THE RISKFREE RATE PUZZLE

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ABSTRACT

This paper studies the implications for general equilibrium asset pricing of a recently introduced class of Kreps-Porteus non-expected utility preferences, which is characterized by a constant intertemporal elasticity of substitution and a constant, but unrelated, coefficient of relative risk aversion.

It is shown that the solution to the "equity premium puzzle" documented by Mehra and Prescott [1985] cannot be found, for plausibly calibrated parameter values, by simply separating risk aversion from intertemporal substitution. Rather, relaxing the parametric restriction on tastes implicit in the time-addictive expected utility specification and adopting Kreps-Porteus preferences in the direction of "more realism" is likely to add a "riskfree rate puzzle" to Mehra's and Prescott's "equity premium puzzle."

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The purpose of this paper is to study the implications for general equilibrium asset pricing of the parametric class of Kreps-Porteus non-expected utility preferences introduced recently by Epstein-Zin [1987a] and myself (Weil [1987]).[1] These preferences generalize, in a non-expected utility framework, the commonly used time-additive, isoelastic expected utility specification to allow for an independent parametrization of attitudes toward risk and attitudes toward intertemporal substitution. They are characterized by a constant intertemporal elasticity of substitution and a constant, but unrelated, coefficient of relative risk aversion, and thus relax the well-known constraint intrinsic to time-additive, isoelastic expected utility that the intertemporal elasticity of substitution be the inverse of the constant coefficient of relative risk aversion.

Adopting this new class of preferences has clear benefits. Firstly, these preferences do not impose a behavioral restriction on tastes which is devoid of any theoretical rationale, and which has many unpleasant side-effects.[2] Secondly, the data reject the time-additive expected utility restriction, as established by Epstein and Zin [1987b]. Thirdly, from an analytical point of view, this class of Kreps-Porteus preferences is very simple to work with and a very natural generalization of isoelastic preferences to uncertainty.

Yet, despite these advantages, it is legitimate and necessary to wonder whether these new preferences contribute, even partially, to the resolution of any the many outstanding asset pricing or consumption theory puzzles. Do those puzzles disappear once preferences are "correctly" specified and the expected, time-additive utility restriction lifted? Can we conclude that the role we had attributed, in the empirical difficulties of frictionless asset pricing or permanent income theories, to incomplete markets or liquidity constraints was misplaced, and that the only problem was in reality one of mis-specification of preferences? It would be surprising that the

[1] See Kreps and Porteus [1978, 1979a, 1979b] for the axiomatic foundations of these preferences.

[2] Among them figure prominently: i) the impossibility of replicating the behavior of agents who are both moderately risk averse and yet very averse to intertemporal substitution (as most available empirical evidence suggests is the case), and ii) the difficulty, pointed out by Hall [1985], of determining whether regressions of log growth rates of consumption on mean log real interest rates provide an estimate of risk aversion or intertemporal substitution (see my [1987] paper on this point).

answer to these purposefully provocative questions be positive — and indeed it is not, as this paper will suggest.

The test to which this study submits this new parametric class of non-expected utility preferences is the one devised by Mehra and Prescott [1985]: can an artificial representative agent economy, calibrated with plausible parameter values and output process, replicate the average secular level of the riskfree rate (0.75 %) and of the risk premium on equity (6.20 %)? We know, from Mehra's and Prescott's original work, that a representative agent economy with CES, time-additive, expected utility preferences and complete markets cannot pass this test — because of the inability of the model to "fit" both the level of the riskfree rate and the discrepancy between the safe and average risky rates. Does, however, an economy in which agents are endowed with the Kreps-Porteus generalization of CES preferences perform substantially better with respect to the Mehra-Prescott touchstone? While intuition suggests that it might — these new preferences afford an additional degree of freedom — the answer to this question is negative.

As I will demonstrate below, the risk premium depends, for plausible calibrations of tastes and technology, almost exclusively on the coefficient of relative risk aversion. But this implies that relaxing the time-additive expected utility restriction on tastes does not substantially alter the fact, documented by Mehra and Prescott, that the model can replicate the risk premium only for astronomically high levels of risk aversion — thus leaving the equity premium puzzle intact. What emerges from the relaxation of this restriction and the appropriate calibration of tastes, is, as we shall see, an additional puzzle, centered around the riskfree rate: *why is it, if consumers are as averse to intertemporal substitution as some recent estimates suggest, that the riskfree is so low*?

The analysis proceeds as follows. Section 1 presents the model, and solves for equilibrium asset prices and rates of return. Section 2 establishes the main results of the paper: separating risk aversion from intertemporal substitution cannot, on its own, explain away the equity premium puzzle documented by Mehra and Prescott [1985], but instead highlights the existence of a "riskfree rate puzzle." The conclusion summarizes the paper, and outlines directions for further research.

1. The basic framework

The economy is similar, expect for the agents' preferences, to the one studied by Lucas [1978] and Mehra and Prescott [1985]. I first describe technology and consumer behavior, and then compute equilibrium asset returns.

1.1. Technology

There is one perishable consumption good, a fruit, which is produced by non-reproducible identical trees whose number is normalized, without loss of generality, to be equal to the size of the constant population. Let y_t the the number of fruits falling from a tree at time t, i.e., the "dividend" associated with holding a tree. It is assumed that the rate of growth of dividends, $\lambda_{t+1} \equiv y_{t+1}/y_t$, is random and Markovian over a finite state space, with transition probabilities given by[3]

$$\phi_{ij} = \text{Prob}\{\lambda_{t+1} = \lambda_j \mid \lambda_t = \lambda_i\}, \tag{1}$$

with $i, j = 1, 2, \ldots I < \infty$, $\lambda_j > 0$, and $\sum_{j=1}^{I} \phi_{ij} = 1 \ \forall i$. The uncertainty on dividends at $t+1$, and thus on y_{t+1}/y_t, is assumed to be resolved at the beginning of period $t+1$, before time $t+1$ consumption and savings decisions are made.

1.2. Consumers

The economy is inhabited by many identical infinitely-lived consumers. Let p_t, x_t and c_t denote, respectively, the fruit price of a tree at t, the number of (shares of) trees held at the beginning of period t, and consumption at t of a representative agent. The one-period budget constraint facing a representative consumer is then simply

$$c_t + p_t x_{t+1} = (p_t + y_t) x_t, \quad t \geq 0, \tag{2}$$

[3] The notation is purposefully similar to the one used by Mehra and Prescott [1985].

with $x_0 > 0$ given. Letting $R_{t+1} \equiv [p_{t+1} + y_{t+1}]/p_t$ denote the one-period (random) rate of return on a tree, and $w_t \equiv (p_t + y_t)x_t$ represent beginning-of-period wealth, the budget constraint (2) can be rewritten more compactly as

$$w_{t+1} = R_{t+1}(w_t - c_t). \tag{3}$$

I assume that agents are not indifferent to the timing of the resolution of uncertainty on consumption lotteries (as they are when preferences can be represented by a Von Neumann - Morgenstern (VNM) utility index), and that their preference ordering instead satisfies axioms 2.1, 2.2, 2.3 and 3.1 in Kreps and Porteus [1978].[4] From these authors' theorem 1 and corollary 4 [*ibid.*, pp. 192 and 199], and the results in Epstein-Zin [1987a], the representative consumer's preference ordering over uncertain consumption lotteries can be represented by sequence of functions V_t defined recursively by[5]

$$V_t \equiv U[c_t, E_t V_{t+1}] \tag{4}$$

where E_t denotes expectation conditional on information available at t. When the "aggregator" function $U(.,.)$ is linear in its second argument, (4) is the standard recursive equation characterizing the VNM time-additive expected utility index (it is linear in probabilities), and agents are indifferent to the timing of resolution of uncertainty over temporal consumption lotteries. As shown by Kreps and Porteus [1978, p.199], (4) also allows for preference for early (resp., late) resolution when $U(.,.)$ is convex (resp., concave) in its second argument.

Following initial results by Farmer [1987], Epstein-Zin [1987a] and myself [1987] have independently proposed the following functional form[6] for $U(.,.)$:

$$U[c, EV] \equiv \left[(1-\beta)c^{1-\rho} + \beta(EV)^{\frac{1-\rho}{1-\gamma}}\right]^{\frac{1-\gamma}{1-\rho}}, \tag{5}$$

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[4] The Kreps-Porteus axiomatization does not impose on ***temporal*** gambles the axiom of reduction of compound lotteries satisfied, original static VNM theory, by preferences over ***timeless*** gambles. It maintains, however, the so-called "independence" axiom, and axiomatically imposes the temporal consistency of optimal plans.

[5] Throughout this paper, and as in Farmer [1987], I assume "payoff history independence" — i.e., that today's tastes are independent of previously realized consumptions.

[6] Epstein and Zin [1987a] introduce a slightly different, but equivalent, parametrization.

with $\rho \in \mathbf{R}^+$, $\gamma \in \mathbf{R}^+$, and $\beta \in (0,1)$. $1/\rho$ can easily be shown to measure the constant intertemporal elasticity of substitution (IES), and γ to parametrize the constant coefficient of relative risk aversion (CRRA).[7] The VNM time-additive expected utility specification emerges, up to a monotone transformation, as the special case in which $\gamma = \rho$, i.e., as the special case in which the CRRA is restricted to be the inverse of the IES. The preferences defined by (4) and (5) are thus a very natural generalization of CES utility to uncertainty — one which does not impose the behaviorally groundless restriction of time-additive expected utility.

To characterize the optimal consumption plan of the representative consumer, denote by $V(w_t, \lambda_t)$ the maximum utility attainable by an agent who has wealth w_t when the state of nature at t, summarized by the realized growth rate of dividends, is λ_t. This value function is the solution to the following functional equation:

$$V(w_t, \lambda_t) = \arg\max_{c_t} U[c_t, E_t V(w_{t+1}, \lambda_{t+1})] \quad \text{subject to (3)}, \qquad (6)$$

where the aggregator function $U(.,.)$ is defined in equation (5). This functional equation of course reduces to the standard linear Bellman equation in the time-additive case $\gamma = \rho$. The first-order condition for the maximization problem in (6) is simply

$$U_{1t} = U_{2t} E_t[R_{t+1} V_{1t+1}], \qquad (7)$$

where U_{it} denotes the derivative of the aggregator function with respect to its i-th argument ($i = 1, 2$) evaluated at $(c_t, E_t V_{t+1})$, and V_{1t} the derivative of the value function with respect to wealth evaluated at (w_t, λ_t). Using the envelope theorem and (7), one finds:

$$V_{1t} = U_{2t} E_t[R_{t+1} V_{1t+1}] = U_{1t}. \qquad (8)$$

Notice, from (8), that while it remains true, with Kreps-Porteus preferences, that an optimum program is characterized by the equalization of the marginal utility of wealth, V_{1t}, to the marginal utility of consumption, U_{1t},[8] the latter depends (unless utility is time-

[7] This terminology is best justified by noting that, in the absence of uncertainty, γ plays no role [it can be eliminated by a monotone increasing transformation of (3)], in which case the standard time-additive CES utility obtains with sub-utility $c_t^{1-\rho}$ — so that $1/\rho$ is unambiguously the IES. Moreover, the proportional risk premium for a lottery on *permanent* consumption is proportional to γ, with the constant of proportionality reflecting the variance of the lottery (see Weil [1987]) — whence the usual interpretation of γ as being the CRRA.

[8] For a framework in which this equality is violated, see Grossman and Laroque [1987].

additive) on expected future value. In the present framework, changes in the marginal utility of wealth do not solely reflect, in optimal plan, changes in non-durable consumption, but also changes in expected future utility. Kreps-Porteus preferences thus introduce an effect very similar, at a formal level, to the one which would be associated with non-separabilities in consumer durables or government purchases. This is the reason why they are, *a priori*, a good candidate for explaining asset pricing or consumption theory puzzles.

Substituting (8) into (9) yields the following Euler equation:

$$E_t\left\{\frac{U_{2t}U_{1t+1}}{U_{1t}}\ R_{t+1}\right\} = 1, \tag{9}$$

which reduces to its familiar VNM form when the aggregator function is linear in its second argument, i.e., when U_{2t} is a constant. An analogous expression,

$$E_t\left\{\frac{U_{2t}U_{1t+1}}{U_{1t}}\ R_{kt+1}\right\} = 1, \tag{10}$$

can be shown to hold for any asset with rate of return R_{kt+1} which is held by our representative consumer, and can be used, without rewriting the budget constraint (3) or redefining wealth, to price, in equilibrium, any inside asset in zero net supply.

To complete the characterization of the optimal consumption program, it suffices to compute, from (3) and (5), the marginal rate of substitution $U_{2t}U_{1t+1}/U_{1t}$. After some tedious but straightforward computations (see the Appendix), one finds that, for any asset with rate of return R_{kt} which is voluntarily held, it must be the case that

$$E_t\left\{\left[\beta\left(\frac{c_{t+1}}{c_t}\right)^{-\rho}\right]^{\frac{1-\gamma}{1-\rho}}\ [R_{t+1}]^{\frac{1-\gamma}{1-\rho}-1}\ R_{kt+1}\right\} = 1 \tag{11}$$

an equation which holds, in particular, for the rate of return on trees ($R_{kt} = R_t$). As Epstein and Zin [1987a] emphasize, an equation such as (11) suggests that that the covariance between the return on asset k and the return on the market portfolio (for which R_{t+1} also stands for in this representative agent economy with only one type of tree and the other stores of value being inside assets) should be, in addition to covariance with consumption growth, a determinant of excess returns — unless, of course, utility is time-additive, in which case the "standard" consumption capital asset pricing model (C-CAPM) obtains. While this observation can in principle explain the unsatisfactory empirical performance of the C-CAPM relative to the portfolio based CAPM,[9] its general equilibrium implications, which I examine next, do not lend it much support: the particular departure from the C-CAPM embodied in (11) does not help solve the equity premium puzzle.

1.3. Equilibrium prices and returns

In equilibrium, each representative agent must hold one tree (remember the normalization of section 1.1), i.e., $x_t = 1$ for all t. By Walras' law, this requires, from (2), that the entirety of period t (perishable) output be consumed during that period, so that

$$c_t = y_t \quad \forall t. \tag{12}$$

Turning first to the determination of the equilibrium price of tree, and proceeding as in Mehra and Prescott [1985], I look for a "stationary" equilibrium such that $p_t = w_i y_t$ if the level output at t is y_t and i is the state of state of nature at t (the realized rate of growth of output between $t-1$ and t was λ_i). Note that this implies that the rate of return on a tree if state i is realized today and state j tomorrow is simply

$$R_{t+1} = \frac{p_{t+1} + y_{t+1}}{p_t} = \frac{w_j + 1}{w_i}\lambda_j. \tag{13}$$

Inserting this expression into the Euler equation (11), and using the market clearing condition (12) together with the specification (1) of the dividend process, one finds after a few straightforward manipulations that the w_i's ($i = 1, \ldots, I$), which fully characterize equilibrium, are the non-negative solution, if it exists,[10] to the

[9] See Mankiw and Shapiro [1986].

[10] Some restrictions on tastes and technology are of course necessary to ensure existence. They are henceforth assumed to be satisfied .

following system of I non-linear equations:

$$w_i = \beta \left\{ \sum_{j=1}^{I} \phi_{ij} \lambda_j^{1-\gamma} (w_j + 1)^{\frac{1-\gamma}{1-\rho}} \right\}^{\frac{1-\rho}{1-\gamma}} \tag{14}$$

for $i = 1, \ldots, I$.

The expected rate of return on a tree (i.e., on equity) if today's state is i is then simply, from (13),

$$ER^i = \sum_{j=1}^{I} \phi_{ij} \; \lambda_j \; \frac{w_j + 1}{w_i}. \tag{15}$$

I now turn to the computation of the riskfree rate, RF^i, prevailing if today's state is i. The Euler equation (11) implies that the price of a safe unit of consumption tomorrow if today's state is i, $1/RF^i$, is

$$\frac{1}{RF^i} = \beta^{\frac{1-\gamma}{1-\rho}} \left\{ \sum_{j=1}^{I} \phi_{ij} \; \lambda_j^{1-\gamma} \; \left(\frac{w_j + 1}{w_i} \right)^{\frac{1-\gamma}{1-\rho}} \right\}^{\frac{1-\rho}{1-\gamma}}. \tag{16}$$

The (proportional) equity premium, $\Pi^i \equiv ER^i/RF^i$, if today state is i is thus simply, using (14), (15) and (16),

$$\Pi^i = \frac{\left\{ \sum_{j=1}^{I} \phi_{ij} \lambda_j (w_j + 1) \right\} \left\{ \sum_{j=1}^{I} \phi_{ij} \lambda_j^{-\gamma} (w_j + 1)^{\frac{\rho-\gamma}{1-\rho}} \right\}}{\sum_{j=1}^{I} \phi_{ij} \lambda_j^{1-\gamma} (w_j + 1)^{\frac{1-\gamma}{1-\rho}}}. \tag{17}$$

Notice that in the absence of uncertainty we indeed have, as should be expected, $\Pi^i = 1$, i.e., that the (conventionally defined) net risk premium is zero.

2. Kreps-Porteus preferences, the equity premium puzzle, and the riskfree rate puzzle

I now turn to the implications of this framework for the analysis of one of the most striking asset pricing puzzles uncovered by the literature. In a thought provoking paper, Mehra and Prescott [1985] show that the Arrow-Debreu, representative agent framework with time-additive CES expected utility preferences cannot account, except for astronomical values of the CRRA (or, equivalently in that setting, extremely low values of the IES), during the period 1889–1978, for both the average *level* of the riskfree rate (0.75 %) and the *discrepancy* (6.20 %) between the average rate of return on equity (6.95 %) and on riskless securities.

Mehra and Prescott present this "puzzle" in the illuminating form of a dilemma. In the time-additive, expected utility framework which they consider (in which the CRRA is the inverse of the IES), a very high CRRA (of the order of 40 or 50) does make it possible to replicate the large secular premium on equity; yet, because it is synonymous with a very low IES, it also leads to the counterfactual prediction of an extremely high riskfree rate! Conversely, a low CRRA leads to a counterfactually low equity premium, although it does imply a relatively low riskfree rate.

It thus might seem that the major hurdle to be overcome in solving the equity premium puzzle is a purely technical one. Independently parametrizing the IES and the CRRA should provide the additional degree of freedom which is required to replicate both the level of the riskfree rate and the discrepancy between the safe and the average risky rate. The implicit reasoning is, of course, that the riskfree rate is mainly "controlled" by the magnitude of the IES, while the risk premium is a reflection of the CRRA.

I now demonstrate — both theoretically for the case of i.i.d. dividend growth processes, and numerically for the non-i.i.d. case — that this argument not only does not provide a solution to the equity premium puzzle, but also highlights the existence of a riskfree rate puzzle.

2.1. I.i.d. dividend growth

Suppose that the rate of growth of dividends is i.i.d., so that today's state of nature conveys no information as to future dividend, and hence consumption, growth.[11] We should, therefore, expect the fruit price of a tree, p_t, to be, relative to the "size" y_t of the economy, a state-independent constant. Formally, since the assumption of i.i.d.

[11] As Mehra and Prescott [1985] note, this is not a blatantly counterfactual assumption: the rate of growth of consumption only exhibits a small negative serial correlation over the period.

dividend growth is equivalent, from (1), to specifying

$$\phi_{ij} = \phi_j \quad \forall j, i \in \{1, \ldots, I\}, \tag{18}$$

it is obvious, from the equilibrium asset pricing formulæ given in (14), that

$$w_i = w \quad \forall i, \tag{19}$$

i.e., that the equilibrium price function is simply $p_t = wy_t$, where w is a constant determined by substituting (19) into (14); the price-dividend ratio is a constant when dividend growth is i.i.d.

An immediate implication of (19) is that, with i.i.d. dividend growth, the average risky and safe rates are state-independent [see equations (15) and (16)], and so is the equity premium Π^i. Its constant magnitude, denoted by Π, is simply, using (17) to (19),

$$\Pi = \frac{\left\{\sum_{j=1}^{I} \phi_j \lambda_j\right\} \left\{\sum_{j=1}^{I} \phi_j \lambda_j^{-\gamma}\right\}}{\sum_{j=1}^{I} \phi_j \lambda_j^{1-\gamma}}. \tag{20}$$

From equation (20)[12] can be drawn what is possibly the most striking result of this paper: *with i.i.d. dividend growth, the equity premium, when defined in relative terms, is independent of the IES*, and reflects only the properties of the dividend growth process and, of course, the magnitude of the CRRA.[13]

To understand this result, it suffices to remember that, with i.i.d. uncertainty, optimal consumption is a constant fraction of wealth.[14] The rate of growth of consumption is, therefore, proportional to R_{t+1}, the rate of return on the "market" portfolio [see equation (3)]. As a consequence, the marginal rate of substitution depends only on R_{t+1}, and the Euler equation (3) reduces to

$$E_t \left\{R_{t+1}^{-\gamma} \; R_{kt+1}\right\} = 1, \tag{21}$$

[12]Note that (20) confirms the interpretation given in the text of γ as the coefficient of relative risk aversion.

[13]Analogous results have been obtained by Barsky [1986] whithin a two-period framework based on Selden's [1978] "ordinal certainty equivalence" preferences.

[14]Preferences are homothetic — so that the ratio of consumption to wealth depends, at most, on the state of nature. Dividend growth is i.i.d., so that this ratio (equal to the marginal *and* average propensity to consume) is a constant — because today's state of nature conveys no information as to the future.

which is the condition characterizing the *static* optimal portfolio allocation chosen by an agent with a CRRA equal to γ! Equation (21) simply establishes[15] that an intertemporal program reduces, in practice, to a sequence of disconnected static problems when underlying uncertainty is i.i.d.[16] But then, it is not surprising that, as equation (20) shows, the risk premium on equity depends only (for a given output process) on γ, and that ρ be irrelevant — since intertemporal considerations play no role in the determination of the optimal program with i.i.d. uncertainty.

While this result confirms the intuitive argument presented *supra* (the CRRA controls the risk premium, and the IES the level of the riskfree rate), it also proves very decisively that relaxing the time-additive expected utility restriction cannot possibly help solve the Mehra-Prescott equity premium puzzle when dividend growth is i.i.d. For any γ, the equity premium is the same irrespective of whether ρ is equal to or different from γ , i.e., irrespective of whether the expected, time-additive utility restriction is satisfied or not ! Therefore, allowing attitudes towards risk to be parametrized independently from behavior towards intertemporal substitution cannot, with i.i.d. dividend growth, afford any improvement whatsoever over the results of Mehra and Prescott: independently of the value one might want to select for ρ (the inverse of the IES), one will still need implausibly (of the order of 40) high values of γ (the CRRA) to replicate the observed 6.20 % risk premium on equity. The equity premium puzzle ("why is the risk premium so large if consumers are only moderately risk averse?") thus remains intact.

Another implication of the foregoing results is that relaxing, for a given γ, the time-additive expected utility restriction in the direction of more plausible parameter values might, while leaving the equity premium unchanged, *deteriorate* the ability of the model to replicate the level of the riskfree rate. It is commonly estimated that the CRRA is in the range of 1 to 5, with both theoretical (Arrow [1965]) and empirical (Epstein-Zin [1987b]) grounds for thinking that it is in fact closer to 1. The implied values for the IES under the expected, time-additive utility restriction, 0.2 to 1, run counter

[15] See Huang and Litzenberger [1988] for a proof in the VNM case.

[16] See Giovannini and Weil [1988] for an elaboration of this and other related issues.

to the belief that consumers are in fact very averse to intertemporal substitution, and thus seem to overestimate the "true" intertemporal elasticity of substitution (i.e., underestimate the "true" ρ). But it is easy to show [from (16) and the assumption of i.i.d. growth] that increasing ρ while maintaining γ fixed may very well result, depending on the specification of the output process, in an increase in the predicted riskfree rate far over and above the already too high levels associated with the expected utility restriction! That this is indeed the case when the output process is calibrated to fit historical data is confirmed below.

The adoption of Kreps-Porteus preferences thus cannot in itself solve the equity premium puzzle. Instead, it highlights the existence of a new puzzle, focused on the riskfree rate: "why is it, if consumers are so averse to consumption fluctuations, that the riskfree rate is so low?"

I now turn to a numerical examination of the non-i.i.d. case, which confirms and amplifies the preceding results.

2.2. Non-i.i.d. dividend growth process

As the non-linear nature of equations (14) makes it clear, one has to resort to numerical methods to solve for the equilibrium price function, as summarized by the w_i's, when the dividend growth process is not i.i.d.

Mehra and Prescott observe that the evolution of the rate of growth of aggregate consumption (and thus, in this model, "dividends"), is well approximated, over the period 1889–1978, by a two-state stochastic process

$$\begin{aligned} y_{t+1}/y_t &= \lambda_1 = 1.054 \\ &= \lambda_2 = 0.984, \end{aligned} \tag{22}$$

with transition probabilities

$$\begin{aligned} \phi_{11} &= \phi_{22} = 0.43 \\ \phi_{12} &= \phi_{21} = 0.57. \end{aligned} \tag{23}$$

These magnitudes are used to solve for the w_i's in (14), as well as for the state-dependent rikless and expected risky rates.

The Markov process in (23)-(22) has a probability $\Phi = [1 - \phi_{11}]/[2 - \phi_{11} - \phi_{22}] = 1/2$ of being in the good state, λ_1, in the long-run. Given the three parameters β, γ and ρ which parametrize, respectively, consumers' attitudes toward impatience, risk, and intertemporal substitution, this ergodic probability, ϕ, can be used, as in Mehra and Prescott [1985], to compute the long-run average riskfree rate, $RF = \Phi RF^1 + (1 - \Phi)RF^2$, and the long-run average equity premium, $\Pi = \Phi\Pi^1 + (1 - \Phi)\Pi^2$, which are implied by the model. The results, reported in Table I for the case of $\beta = 95\%$,[17] $1/\rho$ between 2 and 0.1, and γ between 0.5 and 10, are as distressing for the representative agent, complete market model as Mehra's and Prescott's.

Under the expected time-additive utility restriction $\rho = \gamma$, decreasing the intertemporal elasticity of substitution amounts to increasing the coefficient of relative risk aversion, and results in the simultaneous rise of the risk premium and the riskfree rate — a property at the origin of the "dilemma" faced by Mehra and Prescott. The largest risk premium one can obtain under these circumstances in that parameter range is 2.33 %, a magnitude which not only falls far short of the observed 6.25 %, but also is associated with a riskfree rate way too large (17.87 % instead of 6.25 %)!

Relaxing the restriction $\rho = \gamma$ improves matters only slightly. For any intertemporal elasticity of substitution $1/\rho$, increasing the coefficient of relative risk aversion γ raises the risk premium and lowers the riskfree rate. For any coefficient of relative risk aversion γ, decreasing the intertemporal elasticity of substitution raises the riskfree rate, and the risk premium.[18]

This last result is very troublesome, in light of the fact that the riskfree rate is very sensitive, while the risk premium reacts very little, to declines in the IES.[19] It implies, as suggested above, that abandoning the time-additive expected utility restriction to adopt

[17] A subjective discount rate, β, closer to 1 improves the ability of the model to replicate the *level* of the riskfree rate, but only marginally so in light of the magnitude of the discrepancy between the predicted and observed magnitudes.

[18] While the explanation of the first effect is straightforward in each of these two comparative statics general equilibrium experiments, the rationale of the second result is more obscure. These results are moreover not independent of the specification of the output process.

[19] The rationale for those different reactions is, of course, that dividend growth is *almost* i.i.d. for the case $\phi_{11} = \phi_{22} = 0.43$ considered here.

more "plausible" parameter values in fact worsens the ability of the model to replicate historical rates. As Table I demonstrates, relaxing the $\rho = \gamma$ restriction in the direction of "more realism" (a smaller IES, i.e., $\rho > \gamma$) will, for any reasonable coefficient of relative risk aversion, dramatically increase the risk-free rate beyond its already large value, while only slightly increasing the risk premium. For instance, for $\gamma = 1$ (logarithmic risk preferences), the predicted risk premium is 0.12 % for $1/\rho = 1$ (i.e., when the VNM restriction is satisfied), and 0.45 % when $1/\rho = 0.1$ (the value estimated by Epstein and Zin [1987b]): both magnitudes fall far short of the observed 6.2 %. As for the riskfree rate, it rises from 7.03 % when $1/\rho = 1$ to 24.91 % when $1/\rho = 0.1$!

One must, therefore, conclude that, for empirically calibrated parameter values, the "equity premium puzzle" is in fact compounded by a "riskfree rate" puzzle, which the use of the Kreps-Porteus preferences (4) and (5) helped bring into light: why is it, if agents are as averse to intertemporal substitution as most empirical estimates suggest they are, that the riskfree rate is so much lower than the representative agent, complete market model predicts it should be?[20] Abandoning time-additive expected utility thus deepens the mystery and extends the puzzle outlined by Mehra and Prescott.

Conclusion

This paper has studied the implications for general equilibrium asset pricing of a recently introduced class of Kreps-Porteus non-expected utility preferences, which is characterized by a constant intertemporal elasticity of substitution and a constant, but unrelated, coefficient of relative risk aversion.

It has been shown that the solution to the "equity premium puzzle" documented by Mehra and Prescott [1985] cannot be found by simply separating risk aversion for intertemporal substitution. If

[20] Table I shows that the riskfree rate puzzle survives, but less strikingly, if one believes that the IES is not as small (0.1) as the estimates of Epstein and Zin suggest. The issue is, of course, which weight to attach to an aggregate estimate of the IES which implies a very negative interest elasticity of savings, and is thus in conflict with life-cycle estimates (which point to an almost zero aggregate interest elasticity of savings). Its resolution, which obviously involves questions related to Ricardian equivalence and bequest motives, is outside the scope of this paper — which is aggregative by design.

the dividend growth process is i.i.d, the risk premium, when appropriately defined, is independent of the intertemporal elasticity of substitution, and thus is be the same whether or not the time-additive, expected utility restriction is imposed. When the dividend growth process is non-i.i.d., relaxing the par restriction on tastes implicit in the time-additive expected utility specification and adopting Kreps-Porteus preferences in the direction of 'more realism' adds, if anything, a "riskfree rate puzzle" to Mehra's and Prescott's "equity premium puzzle."

From here, research may proceed into two directions. On the one hand, one might examine the further implications of the generalization of CES preferences achieved by Epstein-Zin [1987a] and myself [1987]. It is likely, as this paper already suggests, that the relaxing the time-additive, expected utility restriction will not solve all asset pricing and consumption theory puzzles. It might solve some of them, but one should not delude oneself into thinking that it will solve all of them; market imperfections undoubtedly submerge, in terms of empirical effects, the biases introduced this particular misspecification of preferences. On the other hand, one might concentrate on elucidating Mehra's and Prescott's "equity premium puzzle" and the "riskfree rate puzzle" highlighted in this paper. As suggested by recent work by Constantinides [1987] and Mason [1987] on habit formation, it might well be the case that other classes of preferences than the one explored in this paper might explain the puzzles explored in this study. But it is likely that market imperfections will, in the end, empirically hold center stage: both the equity premium and riskfree rate puzzles could be explained, for instance, by introducing undiversifiable idiosyncratic risk. The issue is, of course, to find a theoretically adequate way of rationalizing and explaining the presence of such imperfections.[21]

[21] See Ben Zvi and Sussman [1988] for a recent attempt.

References

Arrow, K. (1965): *Aspects of the Theory of Risk-Taking* (Yrjo Jahnsson Lectures), Yrjo Jahnsson Saatio, Helsinki.

Barsky, R. (1986), "Why Don't the Prices of Stocks and Bonds Move Together?" *National Bureau of Economic Research* Working Paper No. 2047.

Ben Zvi S. and O. Sussman (1988): "The Equity Premium and the Volatility of the Rate of Return on Stocks," *mimeo*, Center for the Study of the Israeli Economy, M.I.T.

Epstein L. and S. Zin (1987a): "Substitution, Risk Aversion, and the Temporal Behaviour of Consumption and Asset Returns I: A Theoretical Framework," *mimeo*, The University of Toronto and Queens' University, Ontario, Canada, July.

Epstein L. and S. Zin (1987b): "Substitution, Risk Aversion, and the Temporal Behaviour of Consumption and Asset Returns II: An Empirical Investigation," *mimeo*, The University of Toronto and Queens' University, Ontario, Canada, July.

Farmer, R. (1987): "Closed-Form Solutions to Dynamic Stochastic Choice Problems," *mimeo*, The University of Pennsylvania, May.

Giovannini A. and P. Weil (1988): "Market vs. Consumption Beta with Non-Expected Utility Preferences," *mimeo*, Columbia University and Harvard University, July.

Grossman S. and G. Laroque (1987): "Asset Pricing and Optimal Portfolio Choice in the Presence of Illiquid Durable Consumption Goods," *National Bureau of Economic Research* Working Paper No. 2369, August.

Hall, R. (1985): "Real Interest and Consumption," *National Bureau of Economic Research* Working Paper No. 1694, Cambridge, Mass.

Huang, C.-F. and R. Litzenberger (1988): *Foundations for Financial Economics*, North-Holland.

Kreps, D., and E. Porteus (1978): "Temporal Resolution of Uncertainty and Dynamic Choice Theory," *Econometrica*, 46, 185–200.

Kreps, D., and E. Porteus (1979a): "Dynamic Choice Theory and Dynamic Programming," *Econometrica*, 47, 91–100.

Kreps, D., and E. Porteus (1979b): "Temporal Von Neumann-Morgenstern and Induced Preferences," *Journal of Economic Theory*, 20, 81–109.

Lucas, R. (1978): "Asset Prices in an Exchange Economy," *Econometrica*, 46, 1426–1445.

Mankiw, N.G. and M. Shapiro (1986): "Risk and Return: Consumption Versus Market Beta," *Review of Economics and Statistics*, 452–459.

Mason J. (1987): "The Equity Premium and Time-Varying Risk Behavior," *mimeo*, Board of Governors of the Federal Reserve System, Washington, D.C.

Mehra, R., and E. Prescott (1985): "The Equity Premium: A Puzzle," *Journal of Monetary Economics*, 10, 335–359.

Weil, P. (1987): "Non-Expected Utility in Macroeconomics," *mimeo*, Harvard University, July.

Appendix: Derivation of the Euler equation

This appendix derives equation (11) in the text. The proof requires, from (10), computing the marginal rate of substitution

$$M_t \equiv U_{2t} U_{1t+1} / U_{1t}. \tag{A1}$$

It proceeds in two steps, exploiting the characteristics of the aggregator function (5) and of the optimal program.

A1. Value function and consumption function

Because of the homogeneity properties of the aggregator function, of the interpretation of γ as the CRRA, and of the fact that preferences are isoelastic, guess that the value function can be written in the form

$$V(w, \lambda) = \Lambda(\lambda) w^{1-\gamma}, \tag{A2}$$

where $\Lambda(.)$ is an unknown function, and that the consumption function is linear in wealth:

$$c_t = \mu(\lambda_t) w_t, \tag{A3}$$

where $\mu(\lambda_t)$ is the state-dependent marginal propensity to consume. It is easy to show, performing the maximization called for by (6), that the functions $\Lambda(.)$ and $\mu(.)$ are related by the following two conditions:

$$(1-\beta)[\mu(\lambda_t)]^{-\rho} = \beta\theta_t[1-\mu(\lambda_t)]^{-\rho} \tag{A4}$$

and

$$\Lambda(\lambda_t) = (1-\beta)^{\frac{1-\gamma}{1-\rho}} [\mu(\lambda_t)]^{-\rho\frac{1-\gamma}{1-\rho}}, \tag{A5}$$

where $\theta_t = E_t\{\Lambda(\lambda_{t+1})[R_{t+1}]^{1-\gamma}\}^{\frac{1-\rho}{1-\gamma}}$. Given the specification in (1) of a discrete state space, these expressions yield $2I$ equations in the $2I$ unknowns $[\Lambda_i, \mu_i] \equiv [\Lambda(\lambda_i), \mu(\lambda_i)]$, $i = 1, \ldots, I$, which can in principle be solved. Although the solution is closed in a only a few cases, the analysis which follows does not require the existence of such explicit forms.

Using the budget constraint (3) along with (A3), (A4) and (A5) imply that, along an optimal program,

$$\frac{\theta_t^{\frac{1-\gamma}{1-\rho}}}{\Lambda(\lambda_{t+1}) R_{t+1}^{1-\gamma}} = \beta^{\frac{1-\gamma}{1-\rho}} \left[\frac{c_{t+1}}{c_t}\right]^{\rho\frac{1-\gamma}{1-\rho}} R_{t+1}^{\frac{1-\gamma}{1-\rho}}, \tag{A6}$$

an expression which will be used infra.

A2. Computation of the marginal rate of substitution

From (5), it is straightforward to show that the marginal rate of substitution defined in (A1) is

$$M_t = \beta \left[\frac{c_{t+1}}{c_t}\right]^{-\rho} \left[\frac{E_t V_{t+1}}{V_{t+1}}\right]^{\frac{1-\rho}{1-\gamma}-1}, \tag{A7}$$

where V_{t+1} denotes the value function evaluated at (w_{t+1}, λ_{t+1}). Using the budget constraint (3) along with (A2) and (A3), one finds that

$$\frac{E_t V_{t+1}}{V_{t+1}} = \frac{\theta_t^{\frac{1-\gamma}{1-\rho}}}{\Lambda(\lambda_{t+1}) R_{t+1}^{1-\gamma}}, \tag{A8}$$

so that, substituting (A6) and (A8) into (A7), we find that

$$M_t = \left[\beta \left(\frac{c_{t+1}}{c_t}\right)^{-\rho}\right]^{\frac{1-\gamma}{1-\rho}} R_{Mt+1}^{\frac{1-\gamma}{1-\rho}-1} R_{it+1} \tag{A9}$$

an expression which, inserted in (10), yields the Euler equation (11).

IES	CRRA (γ)			
($1/\rho$)	.5	1	5	10
2	0.06 **6.16**	0.11 **6.12**	0.51 **5.79**	1.01 **5.40**
1	0.07 **7.10**	0.12 **7.03**	0.56 **6.60**	1.08 **6.06**
0.5	0.09 **8.96**	0.15 **8.87**	0.64 **8.14**	1.22 **7.27**
0.1	0.35 **25.32**	0.45 **24.96**	1.31 **21.68**	2.33 **17.87**

Note: the first number in each box is the average net risk premium, i.e., $100(\Pi - 1)$; the second number, in bold characters, is the average net riskfree rate, i.e., $100(RF - 1)$

Table 1: Net risk premium and riskfree rate, in percent, for selected values of γ and ρ ($\beta = 95$ %)