The Dividend Discount Model: A Primer

The dividend discount model provides a means of developing an explicit expected return for the stock market. By comparing this return with the expected return on bonds, as derived from a yield to maturity calculation, the investor can calculate a return spread between these two classes of securities that can be used to assess the relative attractiveness of each. Investors can also use these return data, along with risk data, to determine an optimal blend of assets—stocks, bonds, money market instruments or real estate—within an asset allocation framework.

Elaborations on the simple dividend discount model provide an important tool for comparing relative values across a sample of individual stocks. Returns derived from complex models may be combined with risk data to construct a "market line" benchmark. Securities that plot along the line may be considered fairly priced; those that plot below the line would be considered relatively unattractive; and securities that plot above the line presumably offer more return than would be expected, given their riskiness.

The dividend discount model may also be modified to provide an estimate of a stock's duration—its sensitivity to interest rate risk. Inasmuch as the measure of duration for stocks is similar to the measure of duration for bonds, stock and bond durations may be compared to determine the assets' relative sensitivity to interest rate changes. Similarly, the model provides a framework for comparing the sensitivities of stocks and bonds to unexpected changes in inflation rates.

The dividend discount model provides a means for developing explicit return estimates for both individual stocks as well as the aggregate market—essential inputs for appraising the relative attractiveness of individual stocks as well as for evaluating the attractiveness of the stock market within an overall asset allocation setting. In addition, the model offers a superior framework for understanding how risk factors such as interest rate variations and changing inflation rates affect stocks. This article describes the dividend discount model framework and illustrates the model's usefulness for determining stock market returns, assessing the relative attractiveness of individual stocks, evaluating the interest rate sensitivity of common stocks, and understanding the effect of inflation on common stocks.

Common Stock Valuation Concepts

The value of a bond at a given time can be defined as the present value of the stream of coupon payments plus the present value of the principal payment to be received at maturity, both discounted at the prevailing rate of interest for that maturity. Following analogous reasoning, the value of a common stock can be defined as the present value of the future dividend stream in perpetuity. This concept is consistent with the assumption that the corporation will indeed have a perpetual life, in accordance with its charter.

If the value of a stock is equivalent to the value for a perpetual annuity with a constant level of payments, this may be expressed math-
ematically as:

\[ V = \frac{D}{k} \]  

(1)

where

\[ V = \text{value}, \]
\[ D = \text{dividends per share and} \]
\[ k = \text{percentage discount rate} \]

If the dividends are assumed to grow at a certain constant rate, the formula becomes:

\[ V = \frac{D}{k - g}, \]  

(2)

where \( g \) represents annual constant percentage growth in dividends per share and \( D \) next year's dividends.

This model assumes that the growth rate for the corporation being analyzed is constant. It is thus most suitable for use in estimating the value of stable, mature companies (or, in the context of a more complex model, the residual value representing the mature phase of a currently more dynamic company). Companies with a more erratic or cyclical earnings pattern, or rapidly growing companies, require a more complex dividend capitalization model framework that can accommodate differing dividend growth patterns.

Although practical applications may require elaborate variations of the dividend capitalization model, the simplified form nevertheless provides a convenient means of analyzing the determinants of stock value. To begin with, the value of the stock should be greater, the greater the earning power and capacity of the corporation to pay out current dividends, \( D \). Correspondingly, the higher the growth rate of the dividends, \( g \), the greater the value of the corporation's stock. Finally, the greater the risk of the corporation (the higher the discount rate, \( k \)) the lower the value of the stock.

The discount rate is alternatively referred to as a required return. It is composed of two elements—a risk-free return and a risk premium. The risk-free return is, in turn, generally considered to consist of a real return component and an inflation premium. The real return is the basic investment compensation that investors demand for forgoing current consumption or, alternatively, the compensation for saving. Investors also require a premium to compensate for inflation; this premium will be high when the inflation rate is expected to be high and low when the inflation rate is expected to be low. Because the real return and the inflation premium comprise a basic return demanded by all investors, the risk-free return is a component of all securities.

The risk premium is made up of the following elements—interest rate risk, purchasing power risk, business risk and financial risk. The risk premium might be considered to be a function of the stock's systematic risk (beta), which is determined by these four fundamental risk factors. As securities differ in their exposure to these risk elements, the premium or return that investors require to compensate for risk will differ across securities.

**Appraising the Market**

The variable of most interest to investors is, generally, the stock's discount rate, \( k \). The price of the stock is readily found, and such variables as the current dividend and the growth rate can be estimated (albeit with varying degrees of ease). The simplified form of the dividend capitalization model can be rearranged to estimate the discount rate \( k \), as shown below:

\[ k = \frac{D}{P} + g. \]  

(3)

This equation says that a stock's discount rate is a function of two variables—the dividend yield, which is the year-ahead dividend, \( D \), divided by the stock price, \( P \), and the growth rate of the dividend, \( g \). Estimating the dividend and the growth rate of the dividend may be facilitated if we redefine these variables. Defining \( E \) as year-ahead earnings and \( 1 - b \) as a payout rate, we can think of dividends as a function of a payout rate and an earnings level such that:

\[ D = (1 - b)E. \]

By further defining \( b \) as a retention rate and \( r \) as a return on equity, or a measure of profitability, we can think of the growth rate of the dividend as a function of the retention rate and return on equity such that:

\[ g = br. \]

With these alternative definitions, the equation for determining the discount rate becomes:

\[ k = \frac{(1 - b)E}{P} + br. \]  

(4)

Note that the inputs for Equation (4) are estimates for the following variables—the level of earnings, \( E \); the retention rate, \( b \) (alternatively, the payout rate, \( 1 - b \)); and the basic level of
profitability, \( r \). The retention, or payout, rate is established by the management of the company. It can be assessed by examining past payouts of earnings or, more directly, from the stated policy of the corporation; for example, the management may have a policy of paying out 50 per cent of earnings over a long period of time. Estimates of the level of earnings, \( E \), and the productivity of retained earnings, \( r \), must be made by the fundamental analyst.

An Illustration

Suppose that we want to estimate the discount rate, or expected return, of the market as a whole. The model should be applicable to the total market, since the market is simply an aggregation of individual stocks; if it applies to the individual components, it should apply to the total. In fact, the simplified version of the dividend capitalization model may be more suitably applied to the market as whole than to individual stocks, because errors in measuring inputs tend to cancel out in the aggregate. That is, overestimates tend to be offset by underestimates.

Table I gives some relevant valuation data for a fairly representative index of the U.S. equity market—the Standard & Poor’s 500. This table shows, for the five-year period 1980–84, data on earnings, payout ratios, return on investment and retention rate times the return on investment. Note that the payout ratio averaged about 45 per cent while the return on investment averaged 15.6 per cent for the period.

The 1984 retention rate and return on investment imply a sustainable growth of 9 per cent. At year-end 1984, the dividend yield on the S&P 500 was 4.5 per cent. Combining this with a sustainable growth of 9 per cent indicates a discount rate, or expected return, for the S&P 500 of 13.5 per cent. Of course, this is an average; expected returns for individual companies will differ because of differences in risk.

Table I also compares the current “expected” return of 13.5 per cent with that earned by stocks over the 59-year period 1926–84. The current absolute return is higher than the historical return over the 1926–84 period. But the 9.5 per cent 1926–84 return was earned over a period when the rate of inflation averaged only 3.0 per cent, so the real return was 6.5 per cent. In the recent five-year period, inflation averaged 6.5 per cent. Using this as a naive proxy for the underlying rate of inflation implies a real return on stocks of 7.0 per cent (expected return of 13.5 per cent less underlying inflation of 6.5 per cent). The current expected real return on stocks is thus fairly close to that earned over the longer term. This apparent relative stability in real return might be helpful in developing a forecast of future return on stocks. In particular, one might build a return forecast by estimating the inflation rate and adding it to the real return.

The investor can also use rate of return information in comparisons with bonds and other non-stock classes of securities. For example, the expected return for stocks as derived from the dividend discount model may be compared with the expected return for bonds as derived from a yield to maturity calculation in order to determine a return spread between these two.

### Table 1  Expected Return on S&P 500

<table>
<thead>
<tr>
<th>Year</th>
<th>Earnings ((E))</th>
<th>Payout Ratio ((1 - b))</th>
<th>Return on Investment ((r))</th>
<th>Dividends ((D))</th>
<th>Retention Rate ((b))</th>
<th>Growth Rate ((br))</th>
</tr>
</thead>
<tbody>
<tr>
<td>1980</td>
<td>14.82</td>
<td>42%</td>
<td>17.8%</td>
<td>6.16</td>
<td>58%</td>
<td>10.3%</td>
</tr>
<tr>
<td>1981</td>
<td>15.36</td>
<td>43</td>
<td>17.0</td>
<td>6.63</td>
<td>57</td>
<td>9.7</td>
</tr>
<tr>
<td>1982</td>
<td>12.65</td>
<td>54</td>
<td>12.9</td>
<td>6.87</td>
<td>46</td>
<td>5.9</td>
</tr>
<tr>
<td>1983</td>
<td>14.04</td>
<td>55</td>
<td>14.0</td>
<td>7.09</td>
<td>45</td>
<td>6.3</td>
</tr>
<tr>
<td>1984</td>
<td>16.73</td>
<td>45</td>
<td>16.3</td>
<td>7.53</td>
<td>55</td>
<td>9.0</td>
</tr>
</tbody>
</table>

\[
\text{Expected return } E(R) = \frac{(1 - b)E}{P} + br
\]

\[
= 4.5\% + 9.0\% = 13.5\%
\]

Source: Standard & Poor’s Security Price Index Record, Standard & Poor’s Corporation, New York, N.Y.
classes of securities. The magnitude of this spread, relative to historical spreads and current market conditions, may be used to assess the relative attractiveness of stocks versus bonds. More formally, investors can use these return data along with risk data to determine an optimal blend of security classes—stocks, bonds, money market instruments or real estate—within an asset allocation framework.

**Appraising Individual Stocks**

The simplified form of the dividend discount model is also appropriate for companies that we might characterize as being of a stable, more mature variety—companies in such industries as electric and telephone utility, beverage, tobacco and food processing, retailing, banks and life insurance, and household products industries. For such companies, earning patterns as well as retention rates and returns on investment are fairly stable over time. This is because their investment opportunities, which are a prime consideration in setting the retention rate, are fairly constrained and their basic profitability is pretty constant over time.

The formula must be modified substantially, however, to deal with companies that have a highly cyclical operating pattern or exceptionally high rates of earnings growth. In the case of cyclical companies, the inputs to the model must be recast. For high-growth companies, an alternative, more complex form of the dividend discount model is needed.

More complex models typically provide for a yearly forecast for the next five years (usually based on expected results over a typical economic cycle); a transition period of five to 20 years' duration (used to link current expectations for growth, profitability and dividend payout in a corporate life cycle atmosphere to the mature state); and the residual, mature, or constant corporate phase. These complex models are comprehensible; they reflect the true theoretical value of a common stock; they provide an intellectual framework for comparing high-profit, high-growth companies with low-profit, low-growth firms; and they reflect the life cycle nature of firms and industries in a competitive environment. But these models may contain subtle but meaningful biases created by ground rules that appear very reasonable; they may be highly sensitive to very long-term forecasts; and they require a great deal of work.¹

Table II shows rates of return for 15 companies derived by Kidder Peabody from a three-stage variant of the dividend discount model described above. The table also gives the risk-sector rating for each stock; a "1" rating indicates lowest risk and a "5" rating indicates highest risk.

**Table II Expected Return and Risk for Selected Stocks**

<table>
<thead>
<tr>
<th>Company</th>
<th>Ticker</th>
<th>Expected Return</th>
<th>Risk</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Above-Average Expected Return</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Augat</td>
<td>AUG</td>
<td>18.0</td>
<td>5</td>
</tr>
<tr>
<td>Waste Management</td>
<td>WMX</td>
<td>16.8</td>
<td>4</td>
</tr>
<tr>
<td>Lochtite Corporation</td>
<td>LOC</td>
<td>16.7</td>
<td>3</td>
</tr>
<tr>
<td>Hospital Corp. America</td>
<td>HCA</td>
<td>16.0</td>
<td>2</td>
</tr>
<tr>
<td>Coca Cola</td>
<td>KO</td>
<td>15.7</td>
<td>1</td>
</tr>
<tr>
<td><strong>Average Expected Return</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Time Inc.</td>
<td>TL</td>
<td>16.2</td>
<td>5</td>
</tr>
<tr>
<td>Black &amp; Decker</td>
<td>BDK</td>
<td>15.8</td>
<td>4</td>
</tr>
<tr>
<td>Marriott</td>
<td>MHS</td>
<td>15.5</td>
<td>3</td>
</tr>
<tr>
<td>Weyerhaeuser</td>
<td>WY</td>
<td>15.2</td>
<td>2</td>
</tr>
<tr>
<td>Gillette</td>
<td>GS</td>
<td>14.7</td>
<td>1</td>
</tr>
<tr>
<td><strong>Below-Average Expected Return</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Data General</td>
<td>DGN</td>
<td>15.1</td>
<td>5</td>
</tr>
<tr>
<td>Caterpillar</td>
<td>CAT</td>
<td>14.5</td>
<td>4</td>
</tr>
<tr>
<td>Woolworth</td>
<td>Z</td>
<td>14.4</td>
<td>3</td>
</tr>
<tr>
<td>Penney</td>
<td>JCP</td>
<td>14.3</td>
<td>2</td>
</tr>
<tr>
<td>Procter &amp; Gamble</td>
<td>PG</td>
<td>13.5</td>
<td>1</td>
</tr>
</tbody>
</table>

Source: Kidder, Peabody.

¹. Footnotes appear at end of article.

**The Market Line**

Using this sort of risk and return data, Figure A plots the 15 stocks (designated by their ticker symbols) on a risk-return diagram. The line fitted to the 15 plots provides a benchmark known as the market line.² The upward slope of the line indicates that increasing risk should be accompanied by increasing return; alternatively, high risk stocks should offer higher prospective returns than low risk stocks.

The market line provides a way of evaluating whether stocks are providing returns that are more or less than proportional to their risk and thereby provides an explicit way of evaluating the relative attractiveness of individual stocks. For example, the five stocks that plot on the market line are offering returns in line with what would be expected, given their risk; they offer "fair values" in the context of the market line. The five stocks that plot above the line are offering higher returns than would be expected, given their risk, and the five stocks that plot below the line are offering lower returns than would be expected given their risks. Stocks plotting above the line would be considered relatively attractive and those plotting below the
line would be assessed as unattractive.

We would ideally prefer to construct a portfolio of stocks from those that plot above the market line. Portfolios constructed from such individually attractive stocks should offer prospective returns more than proportional to their risk. If the dividend discount mechanism has validity in identifying relatively attractive values, then one might anticipate that using such an approach would result in above-average risk-adjusted portfolio performance over time.

**Interest Rate Risk**

Investors commonly calculate the duration of fixed-income, finite-lived instruments—bonds—and use the magnitude of the calculated duration as a gauge of sensitivity to interest rate changes. In particular, long duration bonds would be expected to be highly sensitive to interest rate changes, whereas short duration bonds would be expected to have a low sensitivity to interest rate changes. Just as we can calculate duration for bonds, we can also calculate duration for stocks.

Dividend payments on stocks are presumed to continue over an indefinite period—that is, infinity. Developing a duration for stocks thus comes within the general category of developing duration for a perpetuity. For perpetuities such as preferred stocks, where dividend payments are fixed, the formula for calculating duration, \( d \), is:

\[
    d = \frac{1}{k}
\]  

(5)

As before, \( k \) represents the required return on the security, and the resulting expression is simply the inverse of the required return. Because we are dealing with perpetuities, the required return, \( k \), can be determined by merely observing the current yield of the security. For example, a preferred stock paying a $12 dividend and selling at $100 would have a current yield of 12 per cent. Assuming that this is representative of the required return on the security, we can use Equation (5) to calculate the duration of the preferred stock as follows:

\[
    d = \frac{1}{k} = \frac{1}{0.12} = 8.3 \text{ years.}
\]

Calculating the duration of a common stock is similar, except for the need to consider that common stock dividends are expected to grow
over time. Again using \( g \) to represent the growth rate of the dividend, we can amend the previous equation to account for the expected growth in dividends. The equation for calculating the duration of common stock is then:

\[
d = \frac{1}{k - g} \tag{6}
\]

Note that the denominator of the expression has the same form as that of the dividend capitalization model (see Equation (2)). Rearranging the dividend capitalization model, we see that:

\[
duration = \frac{1}{\text{dividend yield}}.
\]

This, in turn, indicates that stocks with low dividend yields have longer durations than stocks with high dividend yields and are relatively more sensitive to discount rate changes. High-growth stocks, which are generally characterized by relatively low dividend yields, would be more subject to this risk than low-growth stocks. We would expect high-growth stocks to carry a higher discount rate than lower-growth stocks in order to compensate for this risk.

Because the measure of duration for stocks is similar to the measure of duration for bonds, we can compare stock and bond durations to determine their relative sensitivities to interest rate changes. Using data for a 20-year government bond as of the end of 1984, we calculated a proxy bond duration using the standard bond duration formula. For stocks, we used S&P 500 year-end 1984 data in the duration formula presented above.

Table III shows the input data and calculated durations for stocks and bonds. At year-end 1984, stocks were yielding 4.5 per cent and showed a duration of 22 years, while a high-grade, 20-year government bond yielding 11.7 per cent showed a duration of eight years. Because of their perpetual life and positive growth character, stocks have a considerably longer duration than bonds, which have fixed maturity periods and, of course, no growth characteristics. Stocks should thus be considerably more responsive than bonds to changes in real interest rates and carry a correspondingly higher premium (via the discount rate).

**Purchasing Power Risk**

As noted, nominal returns contain both a real return component and an inflation premium that compensates for the inflation anticipated over an investment holding period. Inflation rates vary over time, however, and investors do not always correctly anticipate change in the rate of inflation. This results in a risk factor that might be termed "unanticipated inflation," which can cause securities' realized returns to diverge from the returns expected on the basis of the anticipated rate of inflation.

For securities such as bonds, whose cash flows (coupon payments) are fixed, an unanticipated increase in inflation results in a decline in price. The decline in price, combined with a fixed coupon, raises the expected return and compensates for the higher rate of inflation. Conversely, an unanticipated decrease in the rate of inflation lowers returns by increasing price.

Bonds and other fixed-income securities such as preferred stocks are thus highly vulnerable to accelerating inflation—that is, purchasing power risk. By the same token, they are highly desirable investments during periods of deflation or disinflation. In fact, bonds provided relatively high returns—7.0 per cent per annum—during the deflationary period from 1929-38, when the CPI declined an average 2.0 per cent per annum, and again provided an above-average return of 14.8 per cent per annum from 1981 to 1984, when inflation decelerated from 12.4 to 4.0 per cent.

For securities such as common stocks, whose cash flows (dividends) are flexible, the price of the security does not necessarily change in response to unanticipated inflation. Stock dividends may rise to offset an increase in the rate of inflation, precluding any need for price adjustment. The basic stock valuation Equation (2) provides a more specific illustration of the effect. To reflect an unanticipated increase in inflation, the equation is augmented by an in-
creased dividend growth rate, 1 + I, as follows:

\[ P = \frac{D(1 + I)}{k(1 + I) - g(1 + I)} \]  

(7)

Note that all three variables (dividends, growth and discount rate) have been augmented with the inflation factor 1 + I. When inflation increases, we would expect the discount rate to increase by (1 + I) to reflect the higher rate. If the corporate growth rate and dividend increased directly in line with the inflation increase, or to \( g(1 + I) \) and \( D(1 + I) \), the company would offset inflation entirely, and there should be no effect on price. In this case, we can factor out the 1 + I terms as shown below:

\[ P = \frac{(1 + I)D}{(1 + I)(k - g)} = \frac{D}{(k - g)}. \]

If the corporation cannot increase its rate of growth in line with inflation, or if there is only a partial adjustment, there should be a negative effect on stock prices. This happens because the discount rate increases more than the growth rate and dividend level, thus resulting in the application of a net higher discount rate. In the extreme situation where the corporation is completely unable to increase growth in the face of inflation, the dividend resembles a fixed coupon payment. In this case, stock price, like bond price, bears the full brunt of an increase in the discount rate.

Table IV illustrates the dividend growth adjustment under three different scenarios—a full dividend growth adjustment to inflation; a partial adjustment—in this case only a 50 per cent adjustment to an increase in inflation; and a zero, or bond-like, adjustment. Note that when inflation increases by 2 per cent, there is essentially no effect on stock prices if growth increases in tandem with inflation; however, stock prices depreciate by 35 per cent if there is no adjustment and by 21 per cent if there is only a partial adjustment for inflation. How stocks adjust to inflation—that is, which of the three scenarios seems to fit stocks best—is essentially an empirical question.

Table V provides some perspective on corporations' success in offsetting inflation over longer and shorter intervals. It shows price and dividend data from the S&P 500 and the CPI for selected dates from 1947 to 1980, giving the percentage changes in these variables at intervals over the period as an aid to evaluating the responsiveness of stocks to inflationary forces.

Table IV Inflation and Stock Prices

<table>
<thead>
<tr>
<th>Stock</th>
<th>Zero Inflation</th>
<th>50% Adjustment</th>
<th>100% Adjustment</th>
<th>Zero Adjustment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dividend</td>
<td>1.00</td>
<td>1.02</td>
<td>1.01</td>
<td>1.00</td>
</tr>
<tr>
<td>Growth Rate</td>
<td>0.05</td>
<td>0.0710</td>
<td>0.0605</td>
<td>0.05</td>
</tr>
<tr>
<td>Discount Rate</td>
<td>0.09</td>
<td>0.1118</td>
<td>0.1118</td>
<td>0.1118</td>
</tr>
<tr>
<td>Stock Price</td>
<td>$25.00</td>
<td>$25.00</td>
<td>$19.69</td>
<td>$16.18</td>
</tr>
<tr>
<td>Price Change, %</td>
<td>-</td>
<td>-0</td>
<td>-21</td>
<td>-35</td>
</tr>
</tbody>
</table>

Table V CPI versus S&P 500

<table>
<thead>
<tr>
<th>Year (1967 = 100)</th>
<th>Consumer Price Index</th>
<th>S&amp;P 500</th>
<th>Dividends</th>
</tr>
</thead>
<tbody>
<tr>
<td>1947–1950</td>
<td>7.7</td>
<td>33.0</td>
<td>75.0</td>
</tr>
<tr>
<td>1950–1955</td>
<td>11.2</td>
<td>123.0</td>
<td>11.6</td>
</tr>
<tr>
<td>1955–1960</td>
<td>10.6</td>
<td>27.7</td>
<td>18.9</td>
</tr>
<tr>
<td>1960–1965</td>
<td>6.5</td>
<td>59.1</td>
<td>39.5</td>
</tr>
<tr>
<td>1965–1970</td>
<td>23.2</td>
<td>-0.3</td>
<td>15.4</td>
</tr>
<tr>
<td>1974–1980</td>
<td>52.9</td>
<td>98.1</td>
<td>71.1</td>
</tr>
</tbody>
</table>

Note that during the 1947–65 period, the rate of increase in dividend income was on average considerably above the rate of increase in consumer prices. Stock prices also appreciated significantly, and the total return on stocks over the period provided a good hedge against inflation. But stocks did not provide a hedge against inflation during the 1965–74 period. Although dividend growth continued to be positive, it lagged the rate of increase in inflation, which had accelerated to the highest level of the postwar period. Stock prices also declined, providing a net return significantly below the inflation rate. More recently, dividend growth accelerated and stock prices appreciated; net return exceeded inflation, and stocks again provided a hedge against inflation.

These data indicate that, over the long term, corporations have been able to offset inflation and provide a significant real return to investors. Over shorter intervals, however, corporate performance has been less steady. On balance, it appears that stocks, while exposed to purchasing power risk, are less susceptible than long-term bonds or preferred stocks.

Footnotes

1. For a description of some of these models, see J.L. Farrell, Guide to Portfolio Management (New York:
2. The market line is the empirical counterpart to the Security Market Line (SML) of Sharpe et al. and was pioneered by William Fouse at Wells Fargo to provide a measure of the tradeoff between risk and return in the market at a given time.
3. This expression assumes continuous compounding, and for purposes of illustrating duration for perpetuities such as preferred and common stocks we'll consider that the assumption of continuous compounding is appropriate. When discrete compounding is assumed, the expression is:

\[ d = \frac{1 + k}{k} \]

This expression is only slightly more complex than the one in the text, but tends to obscure the analytical exposition.