Amos Tuck School of Business at Dartmouth College

Working Paper No. 03-26

Center for Research in Security Prices (CRSP) University of Chicago

Working Paper No. 550

The CAPM: Theory and Evidence

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August 2003

First draft: August 2003
Not for quotation
Comments solicited

The CAPM: Theory and Evidence

by

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The capital asset pricing model (CAPM) of William Sharpe (1964) and John Lintner (1965) marks the birth of asset pricing theory (resulting in a Nobel Prize for Sharpe in 1990). Before their breakthrough, there were no asset pricing models built from first principles about the nature of tastes and investment opportunities and with clear testable predictions about risk and return. Four decades later, the CAPM is still widely used in applications, such as estimating the cost of equity capital for firms and evaluating the performance of managed portfolios. And it is the centerpiece, indeed often the only asset pricing model taught in MBA level investment courses.

The attraction of the CAPM is its powerfully simple logic and intuitively pleasing predictions about how to measure risk and about the relation between expected return and risk. Unfortunately, perhaps because of its simplicity, the empirical record of the model is poor – poor enough to invalidate the way it is used in applications. The model's empirical problems may reflect true failings. (It is, after all, just a model.) But they may also be due to shortcomings of the empirical tests, most notably, poor proxies for the market portfolio of invested wealth, which plays a central role in the model's predictions. We argue, however, that if the market proxy problem invalidates tests of the model, it also invalidates most applications, which typically borrow the market proxies used in empirical tests.

For perspective on the CAPM's predictions about risk and expected return, we begin with a brief summary of its logic. We then review the history of empirical work on the model and what it says about shortcomings of the CAPM that pose challenges to be explained by more complicated models.

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I. The CAPM

The CAPM builds on Harry Markowitz' (1952, 1959) mean-variance portfolio model. In Markowitz' model, an investor selects a portfolio at time t-1 that produces a random return R_{pt} at t. The model assumes that investors are risk averse and, when choosing among portfolios, they care only about the mean and variance of their one-period investment return. The model's main result follows from these assumptions. Specifically, the portfolios relevant for choice by investors are mean-variance efficient, which means (i) they minimize portfolio return variance, $s^2(R_{pt})$, given expected return, $E(R_{pt})$, and (ii) they maximize expected return given variance.

The way assets combine to produce efficient portfolios provides the template for the relation between expected return and risk in the CAPM. Suppose there are N risky assets available to investors. It is easy to show that the portfolio e that minimizes return variance, subject to delivering expected return $E(R_e)$, allocates proportions of invested wealth, x_{ie} $(\sum_{i=1}^{N} x_{ie} = 1.0)$, to portfolio assets so as to produce a linear relation between the expected return on any asset i and its beta risk in portfolio e,

(1a) $$E(R_i) = E(R_{ze}) + [E(R_e) - E(R_{ze})]b_{ie}, \quad i=1,\ldots,N,$$

(1b) $$b_{ie} = \frac{Cov(R_i, R_e)}{s^2(R_e)} = \frac{\sum_{j=1}^{N} x_{je} Cov(R_i, R_j)}{\sum_{i=1}^{N} x_{ie} \sum_{j=1}^{N} x_{je} Cov(R_i, R_j)}.$$

In these equations, Cov denotes a covariance, $E(R_{ze})$ is the expected return on assets whose returns are uncorrelated with the return on e (they have $Cov(R_i, R_e) = 0$), and the subscript t that should appear on all returns is, for simplicity, dropped.

To interpret (1a) and (1b), note first that in the portfolio model, expected returns on assets and covariances between asset returns are parameters supplied by the investor. Equations (1a) and (1b) then say that given these inputs, finding the portfolio that minimizes return variance subject to having expected return $E(R_e)$ implies choosing asset weights (x_{ie}, i=1,…,N) that produce beta risks ($ß_{ie}$, i=1,…,N) that cause (1a) to be satisfied for each asset.

The beta risk of asset i has an intuitive interpretation. In Markowitz' model, a portfolio's risk is the variance of its return, so the risk of portfolio e is $s^2(R_e)$. The portfolio return variance is the sum of the weighted covariances of each asset's return with the portfolio return,

$$(2) \qquad s^2(R_e) = \sum x_{ie} Cov(R_i, R_e).$$

Thus, $ß_{ie} = Cov(R_i, R_e)/s^2(R_e)$ can be interpreted as the covariance risk of asset i in portfolio e, measured relative to the risk of the portfolio, which is just an average of the covariance risks of all assets.

Equation (1a) is the result of algebra, the condition on asset weights that produces the minimum variance portfolio with expected return equal to $E(R_e)$. The CAPM turns it into a restriction on market clearing prices and expected returns by identifying a portfolio that must be efficient if asset prices are to clear the market of all securities. Applied to such a portfolio, equation (1a) becomes a relation between expected return and risk that must hold in a market equilibrium.

Sharpe and Lintner add two key assumptions to the Markowitz model to identify a portfolio that must be efficient if the market is to clear. The first is complete agreement: Given market clearing prices at t-1, investors agree on the joint distribution of asset returns from t-1 to t. And it is the true distribution, that is, the distribution from which the returns we use to test the model are drawn.

The second assumption is that there is borrowing and lending at a riskfree rate, R_f, which is the same for all investors and does not depend on the amount borrowed or lent. Such unrestricted riskfree borrowing and lending implies a strong form of Tobin's (1958) separation theorem. Figure 1, which describes portfolio opportunities in the (E(R), s(R)) plane, tells the story. The curve abc traces combinations of E(R) and s(R) for portfolios that minimize return variance at different levels of expected return, but ignoring riskfree borrowing and lending. In this restricted set, only portfolios above b along abc are efficient (they also maximize expected return, given their return variances).

Adding riskfree borrowing and lending simplifies the efficient set. Consider a portfolio that invests the proportion x of portfolio funds in a riskfree security and 1-x in some portfolio g,

$$(3a) \qquad R_p = xR_f + (1-x)R_g, \qquad x = 1.0.$$

The expected return and the standard deviation of the return on p are,

(3b) $E(R_p) = xR_f + (1\text{-}x)E(R_g)$,

(3c) $s(R_p) = |1\text{-}x|\, s(R_g)$.

These equations imply that the portfolios obtained by varying x in (3a) plot along a straight line in Figure 1. The line starts at R_f (x = 1.0, all funds are invested in the riskfree asset), runs to the point g (x = 0.0, all funds are invested in g) and continues on for portfolios that involve borrowing at the riskfree rate (x < 0.0, with the proceeds from the borrowing used to increase the investment in g). It is then easy to see that to obtain the efficient portfolios available with riskfree borrowing and lending, one simply swings a line from R_f in Figure 1 up and to the left, to the tangency portfolio T, which is as far as one can go without passing into infeasible territory.

The key result is that with unrestricted riskfree borrowing and lending, all efficient portfolios are combinations of the single risky tangency portfolio T with either lending at the riskfree rate (points below T along the line from R_f) or riskfree borrowing (points above T along the line from R_f). This is Tobin's (1958) separation theorem.

The CAPM's punch line is now straightforward. With complete agreement about distributions of returns, all investors combine the same tangency portfolio T with riskfree borrowing or lending. Since all investors hold the same portfolio of risky assets, the market for risky assets does not clear at time t-1 unless each asset is priced so its weight in T is its total market value at t-1 divided by the total value of all risky assets. But this is just the asset's weight, x_{iM}, in the market portfolio of invested wealth, M. Thus the critical tangency portfolio must be the market portfolio. In addition, the riskfree rate must be set (along with the prices of risky assets) to clear the market for riskfree borrowing and lending.

Since the tangency portfolio is the market portfolio, the market portfolio M is efficient and (1a) and (1b) hold for M,

(4a) $$E(R_i) = E(R_{zM}) + [E(R_M) - E(R_{zM})]b_{iM}, \quad i=1,\ldots,N,$$

(4b) $$b_{iM} = \frac{\text{cov}(R_i, R_M)}{s^2(R_M)}.$$

Moreover, $E(R_{zM})$, the expected return on assets whose returns are uncorrelated with R_M, is the riskfree rate, R_f, and (4a) becomes the familiar Sharpe-Lintner CAPM risk-return relation,

$$E(R_i) = R_f + [E(R_M) - R_f)]\beta_{iM}, \quad i=1,\ldots,N. \tag{5}$$

In words, the expected return on any asset i is the riskfree interest rate, R_f, plus a risk premium which is the beta risk of asset i in M, β_{iM}, times the price per unit of beta risk, $E(R_M) - R_f$ (the market risk premium). And β_{iM} is the covariance risk of i in M, $cov(R_i, R_M)$, measured relative to the overall risk of the M, $\sigma^2(R_M)$, which is itself a weighted average of the covariance risks of all assets (see equations (1b) and (2)). Finally, note from (4b) that β_{iM} is also the slope in the regression of R_i on R_M. This leads to its commonly accepted interpretation as the sensitivity of the asset's return to variation in the market return.

Unrestricted riskfree borrowing and lending is an unrealistic assumption. The CAPM risk-return relation (4a) can hold in its absence, but the cost is high. Unrestricted short sales of risky assets must be allowed. In this case, we get Fischer Black's (1972) version of the CAPM. Specifically, without riskfree borrowing or lending, investors choose efficient portfolios from the risky set (points above b on the abc curve in Figure 1). Market clearing requires that when one weights the efficient portfolios chosen by investors by their (positive) shares of aggregate invested wealth, the resulting portfolio is the market portfolio M. But when unrestricted short-selling of risky assets is allowed, portfolios of positively weighted efficient portfolios are efficient. Thus, market equilibrium again requires that M is efficient, which means assets must be priced so that (4a) holds.

Unfortunately, the efficiency of the market portfolio does require either unrestricted riskfree borrowing and lending or unrestricted short selling of risky assets. If there is no riskfree asset and short-sales of risky assets are not allowed, Markowitz' investors still choose efficient portfolios, but portfolios made up of efficient portfolios are not typically efficient. This means the market portfolio almost surely is not efficient, so the CAPM risk-return relation (4a) does not hold. This does not rule out predictions about the relation between expected return and risk if theory can specify the portfolios that must be efficient if the market is to clear. But so far this has proven impossible.

In short, the central testable implication of the CAPM is that assets must be priced so that the market portfolio M is mean-variance efficient, which implies that the risk-return relation (4a) holds for all assets. This result requires the availability of either unrestricted riskfree borrowing and lending (the Sharpe-Lintner CAPM) or unrestricted short-selling of risky securities (the Black version of the model).

II. Early Tests

Tests of the CAPM are based on three implications of (4a) and (5). If the market portfolio is efficient,

(C1) The expected returns on all assets are linearly related to their market betas, and no other variable has marginal explanatory power;

(C2) The risk premium, $E(R_M) – E(R_{zM})$ is positive;

(C3) In the Sharpe-Lintner version of the model, $E(R_{zM})$ is equal to the riskfree rate, R_f.

Two approaches, cross-section and time-series regressions, are common in tests of (C1) to (C3). Both date to the early tests of the model.

Testing (C2) and (C3) – The early cross-section tests focus on (C2) and (C3), and use an approach suggested by (5): Regress average security returns on estimates of their market betas, and test whether the slope is positive and the intercept equals the average riskfree interest rate. Two problems in these tests quickly became apparent. First, there are common sources of variation in the regression residuals (for example, industry effects in average returns) that produce downward bias in OLS estimates of the standard errors of the cross-section regression slopes. Second, estimates of beta for individual securities are imprecise, creating a measurement error problem when they are used to explain average returns.

Following Blume (1970), Friend and Blume (1970) and Black, Jensen, and Scholes (1972) use a grouping approach to the beta measurement error problem, which becomes the norm in later tests. Expected returns and betas for portfolios are weighted averages of expected asset returns and betas,

(6) $$E(R_p) = \sum_{i=1}^{N} x_{ip} E(R_i)\,, \qquad b_{pM} = \frac{\text{cov}(R_p, R_M)}{s^2(R_M)} = \sum_{i=1}^{N} x_{ip} b_{iM}\,,$$

where x_{ip}, i=1,…,N, are the weights for assets in portfolio p. Since expected returns and market betas combine in the same way, if the CAPM explains security returns it also explains portfolio returns. And since beta estimates for diversified portfolios are more precise than estimates for securities, the beta measurement error problem in cross-section regressions of average returns on betas can be reduced by using portfolios. To mitigate the shrinkage in the range of betas (and the loss of statistical power) caused by grouping, Friend and Blume (1970) and Black, Jensen, and Scholes (1972) form portfolios based on ordered beta estimates for securities, an approach that becomes standard.

Fama and MacBeth (1973) provide a solution to the inference problem caused by correlation of the residuals in cross-section regressions that also becomes standard. Rather than a single regression of average returns on betas, they estimate monthly cross-section regressions,

(7) $$R_{pt} = g_{0t} + g_{1t} b_{pMt} + e_{pt}, \qquad p = 1,\ldots, P, \qquad t = 1,\ldots, t,$$

where P is the number of portfolios in the cross-section regression for month t, b_{pMt} is the beta estimate for portfolio p, and t is the number of monthly cross-section regressions.

Fama (1976, ch.9) shows that the slope $?_{1t}$ in (7) is the return for month t on a zero investment portfolio (sum of the weights equal to 0.0) of the left hand side (LHS) returns that has an estimated market beta, $ß_{pM}$, equal to 1.0. If the market portfolio is efficient, (4a) implies that the expected return on zero investment portfolios that have $ß_{pM}$ equal to 1.0 is the expected market premium, $E(R_M) - E(R_{zM})$. Inferences about the expected market premium can thus be based on the mean of the monthly estimates of $?_{1t}$ and its standard error. Likewise, $?_{0t}$ is the return on a standard portfolio (sum of the weights equal to 1.0) of the LHS returns whose estimated $ß_{pM}$ equals zero. The mean of the month-by-month intercepts, $?_{0t}$, can be used to test the prediction of the Sharpe-Lintner CAPM that the expected return on portfolios with $ß_{pM}$ equal to zero is the average riskfree rate. The advantage of this approach is that the month-by-month variation in the regression coefficients, which determines the standard errors of the means, captures all estimation error implied by the covariance matrix of the cross-section regression residuals. In effect, the difficult problem of estimating the covariance matrix is avoided by repeated sampling.

The second approach to testing the CAPM, time-series regressions, has its roots in Jensen (1968) and is first applied by Friend and Blume (1970) and Black, Jensen, and Scholes (1972). Jensen (1968) notes that if the Sharpe-Lintner risk-return relation (5) holds, the intercept in the time-series regression of the "excess" return on asset i on the excess market return,

$$R_{it} - R_{ft} = a_i + b_{iM}(R_{Mt} - R_{ft}) + e_{it}, \quad (8)$$

is zero for all assets i. Estimates of the intercept in (8) can thus be used to test the prediction of the Sharpe-Lintner CAPM that an asset's average excess return (the average value of $R_{it} - R_{ft}$) is completely explained by its realized CAPM risk premium (its estimated beta times the average value of $R_{Mt} - R_{ft}$).

The early cross-section regression tests (Douglas (1968), Black, Jensen and Scholes (1972), Miller and Scholes (1972), Blume and Friend (1973), Fama and MacBeth (1973)) reject prediction (C3) of the Sharpe-Lintner version of the CAPM. Specifically, the average value of $?_{0t}$ in estimates of (7) is greater than the average riskfree rate (typically proxied as the return on a one-month Treasury bill), and the average value of $?_{1t}$ is less than the observed average market return in excess of the bill rate. These results persist in more recent cross-section regression tests (for example, Fama and French (1992)). And they are confirmed in time-series regression tests (Friend and Blume (1970), Black, Jensen, and Scholes (1972), Stambaugh (1982)). Specifically, the intercept estimates in (8) are positive for low $ß_{iM}$ portfolios and negative for high $ß_{iM}$ portfolios.

When average return is plotted against beta, however, the relation seems to be linear. This suggests that the Black model (4a), which predicts only that the beta premium is positive, describes the data better than the Sharpe-Lintner model (5). Indeed Black's (1972) model is directly motivated by the early evidence that the relation between average return and beta is flatter than predicted by the Sharpe-Lintner model.

Testing (C1) – If the market portfolio is efficient, condition (C1) holds: Market betas suffice to explain differences in expected returns across securities and portfolios. This prediction plays a prominent role in tests of the CAPM, and in the early work, the weapon of choice is cross-section regressions. In the

Fama-MacBeth framework, one simply adds pre-determined explanatory variables $Z_{jp,t-1}$, j = 2,…, J, to the period-by-period cross-section regression (7),

$$(9) \qquad R_{pt} = g_{0t} + g_{1t} b_{pMt} + \sum_{j=2}^{J} g_{jt} Z_{jp,t-1} + e_{pt}, \qquad p = 1,\ldots, P, \qquad t = 1,\ldots, T.$$

Generalizing the interpretation of the one-variable cross-section regression (7), the OLS intercept in (9) is the return on a standard portfolio (sum of the weights equal to 1.0) of the LHS portfolio returns that has zero weighted average values of each of the other explanatory variables. And each regression slope is the return on a zero-investment portfolio (sum of the weights equal to 0.0) of the LHS returns that has a weighted average value of 1.0 for its explanatory variable and weighted average values of zero for other explanatory variables. (See Fama (1976, ch. 9.)) The average values of the period-by-period cross-section regression coefficients in (9) thus provide focused tests of the CAPM predictions (C1) to (C3).

If market betas suffice to explain expected returns (condition (C1)), the time-series means of the slopes $?_{jt}$ on the Z variables in (9) should not be reliably different from zero. For example, in Fama and MacBeth (1973) the Z variables are squared market betas (to test the prediction of (4a) that the relation between expected return and beta is linear) and residual variances from regressions of returns on the market return (to test the prediction of (4a) that market beta is the only measure of risk needed to explain expected returns). The tests suggest that these Z variables do not add to the explanation of expected returns provided by beta. Since the tests on $?_{1t}$ suggest that the average market premium is positive, the results of Fama and MacBeth (1973) are consistent with the hypothesis that their market proxy (an equal-weight portfolio of NYSE stocks) is efficient.

In the cross-section regression approach of (9), the alternative hypothesis is specific; a particular set of Z variables chosen by the researcher provides the alternative to the CAPM prediction (C1) that market betas suffice to explain expected returns. Because the alternative hypothesis is specific, t-tests on the average slopes for the Z variables provide tests of (C1) (though strictly speaking, a joint test on the average slopes for all the Z variables is more appropriate). The trick in this approach is to choose Z variables likely to expose any problems of the CAPM.

The way (C1) is examined in the time-series regression approach is generically different. The alternative hypothesis is vague. One estimates the time-series regression (8) for a set of left hand side (LHS) assets. One then jointly tests the vector of regression intercepts against zero. This step in effect asks whether there is anything about the regression intercepts that suggests there are unspecified omitted variables that add to the explanation of expected returns provided by market betas.

In principle, the vague alternative hypothesis of the time-series test allows it to detect any CAPM problems embedded in the returns on the LHS assets. But this generality has a cost. The joint test on the intercepts from (8) for a set of LHS assets is a multiple comparisons test and it can lack power. The test searches over combinations of the intercept estimates from (8) to find the portfolio of LHS assets that maximizes the probability of rejecting the hypothesis that the intercepts are all equal to zero. The p-value of the test must take into account that many combinations are implicitly examined to find the one that produces the strongest rejection, and this reduces the power of the test. Since more LHS assets imply more searching and a less powerful test, there is an incentive to restrict the number of LHS assets, which can result in lost information about shortcomings of the CAPM.

Gibbons (1982) and Stambaugh (1982) provide the initial tests of (C1) using time-series regressions. They use different joint tests on the intercepts from (8) that have the same asymptotic properties but different small sample properties, with no clear winner. This situation is resolved by Gibbons, Ross, and Shanken (1986). They provide an F-test for the intercepts (the GRS test) that has exact small-sample properties when asset returns are multivariate normal (also assumed in other tests). And they show that the test has an interesting interpretation. The test constructs a candidate for the tangency portfolio T in Figure 1 by optimally combining the market proxy and the LHS assets used to estimate (8). It then tests whether this tangency portfolio, along with the riskfree asset, provides an efficient set reliably superior to the one obtained by combining the riskfree asset with the market proxy alone. In other words, the GRS statistic tests whether the market proxy is the tangency portfolio in the set of portfolios that can be constructed from it and the specific LHS assets used in the test.

With the benefit of this GRS insight, one can see a similar interpretation of the cross-section regression tests of (C1). In this case, the test is whether the Z variables in (9) identify patterns in the returns on the LHS assets that are not explained by the assets' market betas. This again amounts to testing (but in a more restricted way) whether the market proxy is efficient in the set of portfolios that can be constructed from it and the specific LHS assets used in the tests.

It is clear from this discussion that time-series and cross-section regressions do not, strictly speaking, test the CAPM. What is literally tested is the efficiency of a specific proxy for the market portfolio. One might conclude from this that the CAPM has never been tested, and prospects for testing it are not good because data for the true market portfolio of invested wealth are likely beyond reach (Roll (1977)). But this criticism can be leveled at tests of any economic model when the tests use proxies for the variables called for by the model.

Like the early cross-section regression tests of the CAPM, the bottom line from the early time-series regression tests of Gibbons (1982) and Stambaugh (1982) is that various market proxies seem to be efficient – (C1) and (C2) seem to hold. This is good news for the central prediction of the model. But the Sharpe-Lintner prediction that $E(R_{zM})$ is equal to R_f is consistently rejected. The relation between average return and market beta is flatter than predicted by the Sharpe-Lintner CAPM, and the variant of the CAPM analyzed by Black (1972) seems more relevant.

The general success of the CAPM in early tests produced a consensus that the model, or at least the Black version, is a reasonable description of expected returns. The early empirical results, coupled with the model's simplicity and intuitive appeal, pushed the CAPM to the forefront of finance. Students were taught to use the model for many important applications, such as estimating a firm's cost of capital or the expected return on an investment manager's portfolio. And despite the more serious empirical failures discussed next, the CAPM continues to be a force among academics and practitioners alike.

III. Recent Tests

Starting in the late 1970s, there is a sequence of papers that challenge the CAPM prediction that the market portfolio is efficient. The evidence comes from tests of (C1); variables are identified that add to the explanation of expected returns provided by market beta. The first blow is Basu's (1977) evidence that when common stocks are sorted on earnings-price ratios, future returns on high E/P stocks are higher than predicted by the CAPM, and the returns on low E/P stocks are lower than predicted. Banz (1981) documents a size effect; when stocks are sorted on market capitalization (price times shares outstanding), average returns on small stocks are higher than predicted by the CAPM. Bhandari (1988) finds that high debt-equity ratios (book value of debt over the market value of equity, a measure of leverage) are associated with returns that are too high relative to their market betas. Finally, the relation between average return and the book-to-market ratio (B/M, the ratio of the book value of a common stock to its market value) also suggests that the market portfolio is not efficient. High B/M stocks have high average returns that are not captured by their betas, and the average returns on low B/M stocks are lower than implied by their betas (Statman (1980), Rosenberg, Reid, and Lanstein (1985)).

There is a common theme in the CAPM anomalies summarized above. Ratios involving stock prices have information about expected returns missed by market betas. This is not surprising. A stock's price depends both on the expected cash flows it will provide and on the expected returns that discount the expected flows back to the present. Thus, in principle the cross-section of prices has information about the cross-section of expected returns. The cross-section of stock prices is, however, arbitrarily affected by differences in scale (or units). But with a judicious choice of scaling variable X, the ratio X/P can reveal differences in the cross-section of expected stock returns. Such ratios are thus prime candidates to expose shortcomings of asset pricing models (Ball (1978)). The CAPM anomalies summarized above suggest that earnings-price, debt-equity, and book-to-market ratios play this role for the CAPM. Note, however, that the information in price ratios about expected returns is noisy because the cross-section of a price ratio also reflects information about the cross-section of expected cash flows.

Fama and French (1992) update and synthesize the evidence on CAPM anomalies outlined above. Using the cross-section regression approach of (9), they confirm that size, earning-price, debt-equity, and book-to-market ratios add to the explanation of expected returns provided by market beta. Fama and French (1996) reach the same conclusion using the time-series regression approach of (8) applied to portfolios of stocks sorted on the anomalies variables. They also find that different price ratios have much the same information about expected returns. This is not surprising given that price is the common driving force in the ratios, and the numerators are just scaling variables used to extract the information in price about expected returns.

Fama and French (1992) also confirm earlier evidence (Reinganum (1981), Stambaugh (1982), Lakonishok and Shapiro (1986)) that the relation between average return and beta for common stocks becomes even flatter after the sample periods used in the early empirical work on the CAPM. But this result is of little consequence given the strong evidence that various price ratios add to the explanation of expected returns provided by market beta. If market betas do not suffice to explain expected returns, the market portfolio is not efficient, the CAPM is dead in its tracks, and evidence on the size of the market premium cannot save or further doom it.

There is nothing new in Fama and French (1992). But bringing together the evidence on the CAPM anomalies discovered in earlier work serves as a catalyst, marking the point when it is generally acknowledged that the CAPM has potentially fatal problems. Research then turns to explanations.

One possibility is that the anomalies are the result of data dredging – publication hungry researchers scouring the data and unearthing CAPM anomalies that are sample specific results of chance. The standard response to this concern is out-of-sample tests. Chan, Hamao, and Lakonishok (1991) find a strong relation between book-to-market equity (B/M) and average return for Japanese stocks. Capaul, Rowley, and Sharpe (1993) observe a similar B/M effect in four European markets and in Japan. Fama and French (1998) find that the price ratios that produce problems for the CAPM in U.S. data show up in the same way in the stock returns of twelve non-U.S. major markets, and they are present in emerging market returns. Finally, Davis, Fama, and French (2002) extend the U.S. evidence on the relation between returns and B/M back to 1926, and find that it shows up in the earlier out-of-sample period.

This evidence produces general acceptance that the CAPM anomalies associated with price ratios are not sample specific. Two explanations emerge. On one side are the behavioralists. Their story is based on evidence that stocks with high ratios of book value (or earnings) to price are typically firms that have fallen on bad times, while low B/M is associated with growth firms (Lakonishok, Shleifer, and Vishny (1994), Fama and French (1995)). The behavioralists argue that sorting firms on B/M (or E/P) exposes investor overreaction to good and bad times. Investors over-extrapolate past performance, resulting in stock prices for growth firms that are too high and stock prices for troubled (value) firms that are too low. When the overreaction is eventually corrected, the result is high returns for value (high B/M) stocks and low returns for growth (low B/M) stocks. Proponents of this view include DeBondt and Thaler (1987), Lakonishok, Shleifer, and Vishny (1994), and Haugen (1995).

The second story for the CAPM anomalies associated with price ratios is that they point to the need for a more complicated asset pricing model. Fama and French (1993) argue that the higher average returns on small stocks and high B/M stocks are compensation for risk in a multifactor version of Merton's (1973) intertemporal capital asset pricing model (ICAPM). Consistent with this view, they document covariation in returns related to size and B/M beyond the covariation explained by the market return. Fama and French (1995) show that there are size and book-to-market factors in fundamentals (earnings and sales) like the common factors in returns. The acid test of a multifactor model is whether it explains differences in average returns. Fama and French (1993, 1996) propose a three-factor model that uses the market portfolio and diversified portfolios formed on size (market capitalization) and B/M to describe returns. They find that the model largely captures average returns on U.S. portfolios formed on size, B/M, and other price ratios known to cause problems for the CAPM. Fama and French (1998) show that an international version of their multifactor model seems to describe average returns on portfolios formed on scaled price variables in 13 major markets.

The behavioralists are not impressed by this evidence for a risk-based explanation of the CAPM anomalies. They concede that the Fama-French three-factor model captures common variation in returns missed by the market return and that it picks up much of the size and value effects in average returns left unexplained by the CAPM. But their view is that the return premium associated with the model's B/M factor – which does the heavy lifting in the improvements to the CAPM – is itself the result of investor overreaction that happens to be correlated across firms in a way that just looks like a risk story. In short, in the behavioral story, the CAPM flounders on irrational pricing and is not itself the problem. The market is trying to set CAPM prices, and violations of the CAPM are due to mis-pricing.

Our view is that the problem lies with the CAPM. For example, the assumption that investors care only about the mean and variance of distributions of one-period portfolio returns is extreme. Perhaps investors also care about how their portfolio return covaries with labor income and future investment opportunities, so a portfolio's return variance misses important dimensions of risk. If so, market beta is not a complete description of an asset's risk, and we should not be surprised to find that differences in expected return are not completely explained by differences in beta. In this view, the search should turn to asset pricing models that do a better job explaining average returns.

At this point, we face a timeworn impasse. Fama (1970) emphasizes that the information efficiency of capital markets (the hypothesis that prices properly reflect available information) must be tested jointly with a model of expected returns, like the CAPM. Intuitively, to test whether prices are rational, one must take a stand on what the market is trying to do in setting prices, that is, what is risk and what is the relation between expected return and risk. But the converse is also true. Asset pricing models like the CAPM assume that prices are rational. Indeed the complete agreement assumption of the CAPM (which is common to other standard asset pricing models) is basically a strong assumption about the rationality of prices. Thus, when tests reject the CAPM, one can't say whether the problem is irrational prices (the behavioral view) or violations of the other assumptions necessary to produce the CAPM (our position). Such is the state of the world.

IV. The Market Proxy Problem

It is possible that the CAPM holds, the true market portfolio is efficient, and empirical contradictions of the CAPM are due to bad proxies for the market portfolio. The model calls for the market portfolio of invested wealth, but the market proxies used in empirical work are almost always restricted to common stocks.

In response to this problem, one can lean on Stambaugh's (1982) evidence that tests of the CAPM are not sensitive to expanding the market proxy to include other assets, basically because the volatility of expanded market returns is dominated by stock returns. And it is unlikely that the CAPM problems exposed by price ratios like B/M are due to a bad market proxy. Portfolios formed by sorting stocks on price ratios produce little variation in betas calculated with respect to a market portfolio of stocks (Lakonishok, Shleifer, and Vishny (1994)). It seems unlikely that adding other assets to the market proxy will produce the spreads in betas needed to explain the value effect.

But there is no clean solution to the market proxy problem. And if standard market proxies cause tests of the CAPM to fail, they also cause problems in applications. Specifically, applications of the CAPM that use a standard market proxy to estimate expected returns will make systematic and predictable errors.

For example, finance textbooks often recommend using the Sharpe-Lintner CAPM risk-return relation (5) to estimate the cost of equity capital. The prescription is to estimate a stock's market beta and combine it with the riskfree rate and the average market premium to produce an estimate of the cost of equity. The large standard errors of estimates of the market premium and of betas for individual stocks probably suffice to make such estimates of the cost of equity meaningless, even if the CAPM holds and the estimates use the true market portfolio (Fama and French (1997), Pastor and Stambaugh (1999)). But if one of the common market proxies is used, the problems are compounded. Empirical work, old and new, tells us that the relation between beta and average return is flatter than predicted by the Sharpe-Lintner CAPM. As a result, CAPM cost of capital estimates for high-beta stocks are too high (relative to historical returns) and estimates for low-beta stocks are too low (Friend and Blume (1970)). Similarly, CAPM cost of equity estimates for high B/M (value) stocks are too low and estimates for low B/M (growth) stocks are too high.

The CAPM is also often used to measure the performance of actively managed portfolios. The approach, dating to Jensen (1968), is to estimate the time-series regression (8) for a portfolio (mutual funds are commonly studied), and use the intercept (Jensen's alpha) to measure abnormal performance. The problem is that, because of the CAPM's empirical failings, even passively managed (indexed) portfolios that assume rational pricing will produce abnormal returns if their investment strategies involve tilts toward CAPM anomalies (Elton, Gruber, Das, and Hlavka (1993)). For example, funds that concentrate on low beta stocks or value stocks will tend to produce positive abnormal returns relative to the predictions of the Sharpe-Lintner CAPM.

V. The Three-Factor Model

Merton's (1973) intertemporal capital asset pricing model (the ICAPM) is the natural extension of the CAPM. The ICAPM begins with a different assumption about investor objectives. In the CAPM, investors focus only on the wealth their portfolio produces at the end of the current period, time t. In the ICAPM, investors are concerned not only with their end-of-period payoff, but also with the opportunities they will have to consume or invest the payoff. Thus, when choosing a portfolio at time t-1, ICAPM investors worry about how their wealth at t might vary with future state variables, including (i) the prices of consumption goods and the nature of portfolio opportunities at t, and (ii) expectations about the consumption and investment opportunities to be available after t.

Like CAPM investors, ICAPM investors prefer high expected return and low return variance. But because their utility depends on state variables, ICAPM investors are also concerned with the covariances of portfolio returns with the state variables. As a result, optimal portfolios are multifactor efficient. Multifactor efficient portfolios are a subset of multifactor minimum-variance (MMV) portfolios. MMV portfolios have the smallest possible return variances, given their expected returns and the covariances of their returns with the state variables. Multifactor efficient portfolios are the subset of MMV portfolios that also have the largest possible expected returns, given their return variances and the covariances of their returns with the state variables.

Fama (1996) shows that the ICAPM generalizes the logic of the CAPM. Thus, with unrestricted short-selling of risky assets, market clearing prices imply that the market portfolio is multifactor efficient. If there is riskfree borrowing and lending, the ICAPM relation between expected return and beta risks is,

(10) $$E(R_i) - R_f = b_{iM}[E(R_M) - R_f] + \sum_{k=1}^{K} b_{ik}[E(R_k) - R_f],$$

where R_k, k = 1,…,K, are returns on "state variable mimicking portfolios," and the betas are slopes from the regression of $R_i - R_f$ on $R_M - R_f$ and $R_k - R_f$, k = 1,…,K. Equation (10) is the condition on asset weights in an MMV portfolio, applied to the market portfolio M. As in the CAPM, it becomes a market equilibrium risk-return relation because market-clearing implies asset prices that make the market portfolio MMV.

Fama and French (1993, 1996) propose that the contradictions of the CAPM exposed by sorts of common stocks on size and price ratios like B/M point to the need for a multifactor ICAPM. Specifically, we propose a three-factor model,

(11) $$E(R_i) - R_f = b_{iM}[E(R_M) - R_f] + b_{is}E(SMB) + b_{ih}E(HML),$$

where SMB (small minus big) is the difference between the returns on diversified portfolios of small and big stocks, HML (high minus low) is the difference between the returns on diversified portfolios of high and low B/M stocks, and the betas are slopes in the multiple regression of $R_i - R_f$ on $R_M - R_f$, SMB, and HML. One implication of (11) is that in the time series regression,

(12) $$R_{it} - R_{ft} = a_i + b_{iM}(R_{Mt} - R_{ft}) + b_{is}SMB_t + b_{ih}HML_t + e_{it},$$

the intercept a_i is zero for all assets i. Fama and French (1993, 1996) find that this prediction holds up well for portfolios formed on size, B/M, and other price ratios that cause problems for the CAPM.

From a theoretical perspective, the shortcoming of the three-factor model is its empirical motivation. The explanatory returns SMB and HML are not motivated by predictions about state variables of concern to investors. Instead they are brute force constructs meant to capture the patterns in average returns uncovered by sorts of stocks on size and B/M. But this is not fatal. The ICAPM does not require that the additional portfolios used to explain expected returns are state variable mimicking portfolios. The latter can be replaced by any MMV portfolios sufficiently different from the market portfolio to capture covariation in returns and variation in expected returns missed by the market portfolio. Thus, adding diversified portfolios that capture covariation in returns and variation in average returns left unexplained by the market is in the spirit of the ICAPM.

The three-factor model is widely used in empirical research that requires a model of expected returns. Estimates of a_i in (12) are used to calibrate how rapidly stock prices respond to new information, for example, Loughran and Ritter (1995), Mitchell and Stafford (2000). They are also used to measure the special information of portfolio managers, for example, in Carhart's (1997) study of mutual fund performance. And among practitioners, the model is offered as an alternative to the CAPM for estimating the cost of equity (for example, Ibbotson Associates).

It is interesting that the way one uses the three-factor model does not much depend on one's view about whether the average return premiums it captures are rational results of unknown state variable risks or the result of irrational investor behavior. Thus, in calibrating the response of stock prices to new information, one wants to account for known patterns in returns, rational or irrational. The same is true in evaluating the performance of managed portfolios. And the opportunity cost of equity capital depends on the premiums investors require to hold stocks, whether the premiums are rational or irrational.

Indeed, for studying price responses to information and for characterizing portfolio performance, it is not important that the average return premiums of the three-factor model are permanent features of expected returns rather than sample specific results of chance. In these applications one wants to account for known patterns in returns for the period examined, whatever their source. In contrast, the cost of equity capital is about expected returns so one is concerned only with robust expected return premiums.

The three-factor model is hardly a panacea, and there is clear evidence on its shortcomings. The most serious problem is the momentum effect of Jegadeesh and Titman (1993). Stocks that have done well relative to the market over the last three to twelve months tend to continue to do well for the next few months, and those that have done poorly continue to do poorly. This momentum effect is distinct from, and at least as large as, the value effect captured by book-to-market equity and other price ratios. And the momentum effect is left unexplained by the three-factor model, as well as by the CAPM. Following Carhart (1997), one response is to add a momentum factor (the difference between the returns on diversified portfolios of short-term winners and losers) to the three-factor model. This is again legitimate in applications where the goal is to abstract from known patterns in returns to uncover information-specific or manager-specific effects in average returns. But since it is a short-term phenomenon, the momentum effect is largely irrelevant for estimates of the cost of equity.

There is another strand of research that points to problems in both the three-factor model and the CAPM. Frankel and Lee (1998), Dechow, Hutton and Sloan (1999), Piotroski (2000), and others show that within portfolios formed on price ratios like B/M, stocks with higher expected cash flows have higher average returns that are not captured by the three-factor model (or the CAPM). These results are interpreted as evidence that market prices are irrational; they do not reflect available information about expected profitability.

As usual, however, one can't tell whether the problem is bad pricing or a bad asset pricing model. A stock's price is the present value of future cash flows discounted at the expected return on the stock. As a result, the cross-section of a scaled price ratio like B/M is in principle informative about both the cross-section of expected stock returns and the cross-section of expected cash flows (Campbell and Shiller (1989), Vuolteenaho (2002)). Moreover, the logic of valuation theory is that given B/M, expected return is positively related to expected cash flows; if two stocks have the same price, the one with higher expected cash flows must also have a higher expected return. In short, given B/M, a positive relation between expected profitability and expected return is a direct prediction of valuation theory. And it says nothing about whether the marginal relation between expected profitability and average return observed in recent work is the result of irrational pricing or the pricing of rational risks.

VI. Conclusions

The version of the CAPM due to Sharpe (1964) and Lintner (1965) has never been an empirical success. From the first, empirical work on the model consistently finds that the relation between average return and market beta is flatter (the risk premium per unit of market beta is lower) than predicted by the model. And this problem is serious enough to invalidate most applications of the model.

In the early empirical work, the Black (1972) version of the model, which can accommodate a flatter average return - beta tradeoff, has more success. But in the late 1970s, research begins to uncover variables like size, various price ratios, and momentum that add to the explanation of average returns provided by market beta. These findings cut to the heart of the CAPM's prediction that the market portfolio is efficient, so market betas suffice to describe expected returns. And again the problems are serious enough to invalidate most applications of the CAPM. Future work may show that the CAPM's problems disappear when better proxies for the market portfolio are found. But we judge this to be unlikely.

The CAPM, like Markowitz' (1952, 1959) portfolio model on which it is built, is nevertheless a theoretical tour de force. And its fundamental insights about risk and return carry over in generalized form to models like Merton's (1973) ICAPM. We continue to teach the CAPM, as an introduction to the fundamental concepts of portfolio theory and asset pricing, to be built on by more advanced models, and with warnings that despite its seductive simplicity, the CAPM's empirical problems probably invalidate its use in applications.

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Figure 1 -- Investment Opportunities