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Further Evidence on the Risk-Return Relationship

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Further Evidence on the Risk-Return Relationship

Abstract

Recent tests of the capital asset pricing model by Fama and French (1992) showed that there is no significant relationship between the average return and systematic risk of common stocks. We propose two econometric methods to improve the efficiency of the estimation and provide more powerful test statistics: joint pooled cross-section and time-series estimation and generalized least squares. Using these techniques, we find a highly significant relationship between average portfolio returns and systematic risk.

In a recent article, Fama and French (1992) (FF) estimated the relationship between stock returns and beta, the measure of systematic risk, in order to test the capital asset pricing model (CAPM) of Sharpe (1964), Lintner (1965), Mossin (1966) and Black (1972). They found no significant relationship between return and beta even when beta was the only regressor in the return equation, hence casting doubt on the validity of the CAPM The press coverage of this study concluded that "beta ... is dead".[1] In this paper we suggest that the press reports of beta's death were greatly exaggerated.

Our results show that the absence of a significant relationship between average stock returns and beta is due to the estimation methodology of Fama and MacBeth (1973) (FM), which produces tests that are not sufficiently powerful. We develop the methodology of Amihud and Mendelson (1986, 1989) (AM) to test the CAPM, and find that the power of the resulting tests is greater than that of FM's tests. This methodology is applied to examine the return-beta relationship using data for the period 1953-1990. When we employ the FM methodology, we find an insignificant relationship, consistent with FF. However, when we apply the improved AM methodology, we find a positive and highly significant return-beta relationship. We conclude that beta is still alive and well.

In what follows, we present the test methodologies in section I. Section II describes our empirical study and presents the results. Concluding remarks are offered in section III.

[1] *New York Times*, February 18, 1992. See also "Beta beaten" (*The Economist*, March 7, 1992) and "Bye-bye to Beta" (*Forbes*, March 30, 1992). Recently, Chan and Lakonishok (1992), employing a similar methodology as FF, found that for data spanning over 1932-1992, the return-β relationship was weakly significant.

I. Econometric Issues

A. Pooled joint time-series and cross-section estimation

Consider a general empirical model for the analysis of the cross-sectional variation in expected asset returns, specified as

$$r_y = e\gamma_{0y} + X_y\gamma_{1y} + \epsilon_y, \tag{1}$$

where r_y is a P-vector of returns on P assets in period y, X_y is the $P \times K$ matrix of regressors and e is a column vector of ones. The coefficient γ_{0y} is the regression constant, γ_{1y} is the vector of K coefficients of the K regressors, and ϵ_y are the P-vectors of errors. The period index is y, $y = 1, 2, \ldots, Y$.

In the FM procedure for testing the CAPM, stocks are aggregated into P portfolios in each period y, $y = 1, 2, \ldots, Y$. For each portfolio p, $p = 1, 2, \ldots, P$, the mean portfolio return in period y is given by r_{py} and the regressor is $x_{py} = \beta_{py}$, the systematic risk (β) coefficient for portfolio p. The regression intercept γ_{0y} is interpreted as the risk-free rate of return and γ_{1y} is the market risk premium. The systematic risk coefficients β_{py} are estimated from the market model for each portfolio using the return time series on the stocks in portfolio p and on the market over some time preceding period y. The use of portfolios increases the precision of the estimated β and resolves the "errors in the variables" problem encountered when using estimates for individual stocks.[2]

In each period y, an ordinary least squares (OLS) cross-sectional regression of model (1) produces estimates $(\hat{\gamma}_{0y}, \hat{\gamma}_{1y})$. The estimates $\hat{\gamma}_{1y}$ are viewed as the sampled values of a variate representing the market risk premium, and the focus of the test is on whether its

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[2] FF generally follow the FM procedure, except that β is estimated for portfolios and the cross-sectional regressions are estimated on individual stocks with the portfolio-β assigned to all stocks in a portfolio. This is done because FF add to the regressors the company's book-to-market equity variable, which is not studied in FM nor in this paper.

mean, γ_1, is positive and significantly different from zero. To this end, the Y estimates of γ_1 are averaged, producing $\bar{\gamma}_1 = \frac{1}{Y}\sum_y \hat{\gamma}_{1y}$. The estimated standard error of $\bar{\gamma}_1$ is given by $\hat{\sigma}(\bar{\gamma}_1) = (\frac{1}{Y(Y-1)}\sum_y(\hat{\gamma}_{1y} - \bar{\gamma}_1)^2)^{\frac{1}{2}}$, and tests of significance are carried out using the statistic $t = \bar{\gamma}_1/\hat{\sigma}(\bar{\gamma}_1)$. In further tests by FM, the matrix X_y contained more regressors than β alone, and γ_{1y} was a K-element vector for K regressors. The estimation and test procedures are similar for all regressors. For the FM procedure to be meaningful, the slope coefficients must have a common mean vector γ_1, which is the parameter of ultimate interest, estimated by $\bar{\gamma}_1$. The FM test is for this parameter.

In view of the common parameter γ_1, it is natural to consider explicitly the joint time-series and cross-sectional model as given by

$$r = (I_Y \otimes e)\gamma_0 + X\gamma_1 + \epsilon. \tag{2}$$

Here, the YP-vector r stacks the Y cross-sectional return vectors, i.e., the y^{th} subvector is r_y, and similarly for ϵ. The Y-vector γ_0 stacks the intercepts γ_{0y}, the $YP \times K$ matrix X stacks the matrices X_y, I_Y is the $Y \times Y$ identity matrix and $\otimes$ denotes the Kronecker product. This is the pooled model, employed by AM. In this model, the test is on whether $\tilde{\gamma}_1$, the estimator of γ_1, is greater than zero, using a standard t-test.

A pooled, joint estimation of time-series and cross-section is known to improve efficiency (see, e.g., Judge et al. (1980, Chapter 13)). It follows from the Gauss-Markov theorem that if the assets are serially and cross-sectionally homoskedastic and uncorrelated, then the OLS estimator in the joint pooled model is optimal. In particular, *it is more efficient than the FM estimator and leads to a more powerful test on* γ_1.

To illustrate, consider the (mean-adjusted) model

$$r_y = X_y\gamma_{1y} + \epsilon_y. \tag{3}$$

Here, $var(\hat{\gamma}_{1y}) = \sigma_\epsilon^2(X_y'X_y)^{-1}$, where σ_ϵ^2 is the residual variance. It follows that

$$var(\bar{\gamma}_1) = \frac{\sigma_\epsilon^2}{Y^2}\sum_{y=1}^{Y}(X_y'X_y)^{-1}. \tag{4}$$

Similarly, the estimator of γ_1 from the joint pooled estimation, $\tilde{\gamma}_1$, may be computed by OLS in

$$(5) \qquad r = X\gamma_1 + \epsilon,$$

obtained by stacking (3), and

$$(6) \qquad var(\tilde{\gamma}_1) = \sigma_\epsilon^2 \left(\sum_{y=1}^{Y} X_y' X_y \right)^{-1}.$$

Evidently, the joint pooled estimation dominates the FM estimation, that is, $var(\tilde{\gamma}_1) \leq var(\bar{\gamma}_1)$ in the ordering of positive semi-definite matrices. To see this, use (4) and (6) in this inequality and multiply through by Y, yielding

$$(7) \qquad (\frac{1}{Y} \sum_{y=1}^{Y} X_y' X_y)^{-1} \leq \frac{1}{Y} \sum_{y=1}^{Y} (X_y' X_y)^{-1},$$

which is a consequence of Jensen's inequality. Relation (7) says that the mean of the inverse exceeds the inverse of the mean and follows since the inverse operation is convex (see Farrell (1985) for details on the matrix case). The inequality is strict unless $X_y' X_y$ is the same for all y, $y = 1, 2, \ldots, Y$.

B. Generalized Least Squares (GLS)

The use of OLS in the joint pooled procedure would be optimal if the residuals ϵ_y were cross-sectionally uncorrelated, and if they were homoskedastic across assets (or portfolios) and over time. However, the variance-covariance matrix in each period y does not satisfy the Gauss-Markov assumptions, and in addition, the variances change over time. While the OLS estimated coefficients are still unbiased and consistent under these violations of the Gauss-Markov assumptions, the estimates are inefficient. In addition, the standard errors estimated under the Gauss-Markov assumptions are biased and inconsistent, and so are the resulting test statistics. Under these circumstances, GLS is the proper estimation method.[3]

Assume that $var(\epsilon_y) = E(\epsilon_y \epsilon_y') = V_y$ is a positive definite matrix, and consider a matrix W_y such that $V_y^{-1} = W_y' W_y$. Premultiplying the data in period y by W_y, we replace r_y by $W_y r_y$ and the regressors — by $W_y e$ and $W_y X_y$. The resulting model has the same structure as the pooled model (2) (except that now e is not a constant vector), but the error terms of the transformed model have a spherical variance-covariance matrix. Hence, OLS on the transformed data is consistent and efficient in the joint pooled cross-sectional and time-series regression. Thus, our procedure solves two problems: first, it yields consistent test statistics, and second — it increases the power of the tests, as discussed in section A.

A similar GLS methodology can be employed in the FM procedure without pooling. Given $var(\epsilon_y) = V_y$ and $V_y^{-1} = W_y' W_y$, we can transform the data for period y by premultiplying it by W_y. The resulting model will have the same structure as model (1), but the residuals will now adhere to the Gauss-Markov assumptions, thereby increasing the efficiency of the estimation and the power of the significance test for $\bar{\gamma}_1$.

To summarize this section, the joint pooled cross-section and time-series estimation improves on the FM procedure under classical conditions. These conditions can be violated: the estimation residuals for each period are cross-sectionally correlated and heteroskedastic across portfolios, and the variances may differ across periods. Then, a GLS estimation increases efficiency and leads to a more powerful test of the CAPM. This applies to both the joint pooled estimation and to the FM estimation. By the analysis in section A, the GLS estimation of the joint pooled time-series and cross-section model provides a more

[3] See, e.g. Kmenta (1971). In the context of the CAPM, see Brown and Weinstein (1983), AM, Shanken (1992). In addition, as argued by Brown *et al.* (1992), GLS estimation, which effectively standardizes the observations by the variance-covariance matrix which reflects both the residual dispersion and their cross-sectional dependence, is an effective way to mitigate the survivorship bias.

powerful test than that provided by the FM-GLS procedure. Yet, the FM-GLS procedure is more powerful than the classical FM procedure.

II. The Empirical Study

A. Data and methodology

In this section we present tests of the CAPM using a number of methodologies. First, we estimate the return-β relationship employing two procedures: that of FM, and the joint pooled time-series and cross-section method, as in AM. Next, we apply the GLS procedure to both the joint pooled estimation and to the FM method and present the results. We then extend the set of regressors to include, in addition to the portfolio β, the portfolio residual standard deviation and the (logarithm of) firm size.

The empirical work follows the methodology described in both FM and AM. We used the monthly return database of the Center for Research in Securities Prices (CRSP) of the University of Chicago for stocks traded on the New York Stock Exchange over the years 1946 through 1990. For each of the years 1953 through 1990 — a total of 38 years — stocks were selected if they had return data for that year and for the preceding seven years, unless there were more than two consecutive months at a time with missing data during the seven year period. The deletion of stocks that were delisted during the year of study (and therefore had missing data for the rest of that year) naturally leads to the survivorship bias discussed by Brown *et al.* (1992); we address this problem later. The return data used include the return series r_{it} for each stock i in month t, and the equally-weighted market return, R_{mt}. We call these return series "raw" returns. We also followed the CAPM studies of Black, Jensen and Scholes (1972), Litzenberger and Ramaswamy (1979), Miller and Scholes (1982), among others, and used *excess* returns over the risk-free rate: the return series r_{it} and R_{mt} were replaced by their excess returns $r_{it} - R_{Ft}$ and $R_{mt} - R_{Ft}$, where R_{Ft} is the three-month Treasury-bill rate for month t (source: Citibase). In what follows, the same procedure was followed for both raw and excess returns, and we present the results for both (our description here applies to the raw returns, but its extension to the case of excess returns is obvious).

Each eight-year period was divided into three subperiods: I of three years, II of four years and III (the test period) of one year. In subperiod I, the market model was estimated for each stock by regressing the 36 monthly returns r_{it} on the market return R_{mt}. This provided estimates of the stocks' β coefficients. Next, all stocks were ranked and divided into 6 portfolios by size, i.e., the number of shares outstanding times the price per share at the end of subperiod II, and the stocks within each size-portfolio were ranked by their estimated β (from subperiod I) and divided into 6 beta-portfolios. We thus have 36 (6×6) portfolios of stocks ranked by size and beta. Our selection procedure admitted between 805 and 1838 stocks in each year, and thus the number of stocks in each portfolio ranged between 22 and 51.

For each year y in the test period (subperiod III), we have a preceding four-year subperiod II. For these four years, we calculated r_{pt}, the average return for each portfolio p in each month t, and estimated the market model

$$r_{pt} = \alpha_p + \beta_p R_{mt} + \epsilon_{pt},$$

obtaining an estimate of the portfolio risk measure β_{py}. In addition, we retained the subperiod-II standard deviations of the regression residuals ϵ_{pt}, which we denote by SD_{py}. The last portfolio characteristic we calculated was the portfolio size variable SZ_{py}, the logarithm of the average size of the firms in portfolio p at the end of subperiod II. Thus, each portfolio p in the test period (i.e., subperiod III) y is characterized by three attributes based on subperiod-II data: β_{py}, SD_{py} and SZ_{py}.

Finally, we calculated the averages of the annual returns of the stocks in each portfolio p over the test period y (subperiod III). Annual buy-and-hold returns[4] were shown by

[4] Annual returns in tests of the CAPM were also used by Handa, Kothari and Wasley (1992) and Kothari, Shanken and Sloan (1992).

Blume and Stambaugh (1983) and Roll (1983) to overcome the problem of upward biases in return averages resulting from trading noise and bid-ask spreads because then the spread effect appears only once and its effect is negligible compared to the annual return. The result is an estimate of the holding period return on a realistic portfolio of an investor who decides at the beginning of a year, after having observed the stocks' risk and size parameters, to invest an equal amount in each stock and hold this portfolio for a year.

After completing this procedure, we have for each year y, $y = 1, 2, \ldots, Y$, $Y = 38$ the following data: r_{py}, β_{py}, SD_{py} and SZ_{py}, $p = 1, 2, \ldots, P$, $P = 36$. Note that the last three variables, i.e., the portfolio characteristics that can be used to predict the return in year y, are known *before* the beginning of year y.

B. Survivorship Adjustment

According the above procedure, stocks which were delisted in the middle of the test period (subperiod III) were not admitted to the sample of that year. If the model is to simulate the investor's decision at the beginning of subperiod III, then he or she could not know at that point whether a stock would be delisted during that year. This results in a potential *survivorship bias* (see Barry and Brown (1984), Brown *et al.* (1992)). If, for example, high-risk stocks are more likely to be delisted due to bankruptcy, excluding them from the sample makes the average return of the surviving stocks higher than the average return of all stocks in that risk group, when accounting for the loss due to bankruptcy. Thus, the survivorship bias may create the appearance of a positive risk-return relationship where none exists. On the other hand, small companies' stocks are also more likely to be delisted due to mergers and acquisitions which can result in very high returns prior to delisting. Thus, the final effect of the exclusion of delisted stocks is unknown.

To simulate the investor's decision more accurately without giving him the benefit of *hindsight*, we reconstructed the data to obtain a survivorship-adjusted sample. It includes all stocks that were traded at the beginning of period III and satisfied the data requirement of subperiods I and II. The resulting sample size ranged between 818 and 1935 stocks per year. The estimation procedures over subperiods I and II are identical to those described for the original sample in subsection A above.

The stock returns in subperiod III for the survivorship-adjusted sample were computed as follows. During subperiod III, we distinguish between three components of each delisted stock's annual return: The returns prior to the delisting month, which need no adjustments; the return in the delisting month, which we evaluated using the delisting segment data of the CRSP; and the post-delisting returns, for which the CRSP data are generally insufficient.

Specifically, if a stock was delisted during subperiod III, the annual return for that stock was computed by compounding the monthly returns before delisting, then using the "Delisting Return" from the delisting segment of CRSP for the delisting month, and finally using the monthly market returns (R_{mt}) for the rest of the year. If the CRSP files had no delisting return but the delisting price was available, the return over the delisting month was computed from this price and the preceding (last) trading price, and again we compounded at R_{mt} after the delisting month. If the delisting price was also missing, we started compounding at R_{mt} in the case of unavailable prices, mergers or exchanges from the point at which prices were unavailable and on, but assigned an annual return of –100% in the case of liquidation, or delisting by an exchange or by the SEC.

This methodology is conservative, and it could bias the results *against* finding a significant return-β relationship, because we assign to the delisted stock returns that are commensurate with $\beta = 1$, rather than a return associated with the stock's own β.[5] In addition, stocks delisted from an exchange due to bankruptcy do not always result in a zero value to their holders as we assume here, because they sometimes continue to trade OTC (usually classified as "pink sheet" stocks). If these stocks are more likely to belong to riskier groups, we in fact underestimate the returns on the high-risk portfolios.

[5] An alternative rule could be to simulate a situation by which the investor reinvested in a portfolio of the same β and size as those of the delisted stock.

Another related aspect of the survivorship bias is addressed by our GLS procedures. As discussed in Brown *et al.* (1992), the implicit option value in the evaluation of average returns leads to a positive mean return by virtue of survivorship, and this bias is an increasing function of the asset's volatility. We address this problem by employing GLS.

All our estimations were performed using both the original return data set, unadjusted, and the survivorship-adjusted returns.

C. Implementation of the GLS procedure

The implementation of the GLS procedure requires an estimator of the variance-covariance matrix V_y in each period y. We assume the structure

$$V_y = \sigma_y^2 V,$$

where $\sigma_y^2 > 0$ is a scalar that allows for heteroskedasticity across years, and V is a positive definite matrix allowing for cross-sectional correlation and heteroskedasticity across portfolios. Given the estimated OLS residuals $\hat{\epsilon}_y$, $y = 1, \ldots, Y$ from the pooled model (2), the scalar σ_y^2 is estimated by

$$\hat{\sigma}_y^2 = \frac{1}{P}\hat{\epsilon}'_y\hat{\epsilon}_y.$$

The next step would be to construct scaled residuals $\hat{v}_y$ by dividing $\hat{\epsilon}_y$ by $\hat{\sigma}_y$ and estimate V by $\hat{V} = \sum_{y=1}^{Y} \hat{v}_y\hat{v}'_y/Y$, and thus the variance-covariance matrix in period y would be estimated by $\hat{\sigma}_y^2\hat{V}$. However, since for each year $\hat{\epsilon}_y$ is orthogonal to the regressors and in particular to e (the vector of ones), $\hat{V}e = 0$ and the variance-covariance matrix thus estimated would be short-ranked. Our GLS procedure resolves this problem by using the scaled residuals $\hat{v}_y^J = \hat{\epsilon}_y^J/\hat{\sigma}_y$, where $\hat{\epsilon}_y^J$ is obtained from $\hat{\epsilon}_y$ by eliminating the Jth portfolio (coordinate). We then construct

$$\hat{V}^J = \frac{1}{Y}\sum_{y=1}^{Y} \hat{v}_y^J \hat{v'}_y^J,$$

which is of full rank. Let similarly r_y^J, e^J and X_y^J be obtained from r_y, e and X_y by deleting the J^{th} row. A $(P-1)\times(P-1)$ matrix W is selected so that $(\hat{V}^J)^{-1} = W'W$, where

W is obtained by the Choleski decomposition method. The data are then transformed to $r_y^* = Wr_y^J/\hat{\sigma}_y$, $X_y^* = WX_y^J/\hat{\sigma}_y$ and $e_y^* = We^J/\hat{\sigma}_y$. The pooled GLS estimator is computed by applying OLS to the transformed data,

$$r^* = e^*\gamma_0 + X^*\gamma_1 + \epsilon^*,$$

where X^* is of dimension $Y(P-1) \times K$, r^* and e^* are of dimension $Y(P-1) \times 1$, and r^*, e^* and X^* stack r_y^*, e_y^* and X_y^*, respectively.

To implement the GLS version of the FM procedure, we transform the data similarly, using the period-y residuals $\hat{\epsilon}_y$ from (1). The year-by-year cross-sectional regressions

$$r_y^* = e_y^*\gamma_{0y} + X_y^*\gamma_{1y} + \epsilon^*$$

are then subjected to the usual FM procedure, the overall FM-GLS slope estimate being the average of the cross-sectional estimates and the t-tests employing the sample variance of the estimated coefficients $\hat{\gamma}_{1y}$.

D. Empirical Results

We present the results of four estimation methods using four data sets. The estimation methods are the FM method and the joint pooled time-series and cross-section estimation, and for each we apply the ordinary least squares (OLS) and the generalized least squares (GLS) estimation methods. We applied our estimations both to the original sample (requiring stocks to have data through the end of the test period) and to the survivorship-adjusted sample, using both the raw returns and the excess returns (over T-bill rates). The annual return data for the cross-sectional tests are for the period 1953–1990.

We first consider the case where beta is the only explanatory variable, i.e., the matrix X_y of explanatory variables contains only the values β_{py}. Table I shows the OLS results for the entire 38-year period, 1953–1990, and for the two equal 18-year subperiods, 1953–1971 and 1972–1990.

INSERT TABLE I

Based on the FM methodology, the results show that γ_1 (estimated by $\bar{\gamma}_1$) is positive as expected, but the t statistic implies that it is insignificantly different from zero. The results are similar for all four data sets used.[6] This suggests that there is no relationship between average stock return and β, consistent with the results of FF.

The results are substantially different under the joint pooled cross-section and time-series estimation, which is more efficient. Under this method, the estimated γ_1 is statistically significant. This result is similar for all four data sets used. While the joint pooled time-series and cross-section estimation produces a lower point estimate of γ_1, its variance is considerably lower and it can be more reliably distinguished from zero. This demonstrates the benefit of using joint pooled estimation compared to the method of FM.

While the OLS coefficients are unbiased, they are inefficiently estimated and their variance estimates are biased. Consequently, the test statistics are biased and the power of the tests is low. This calls for a GLS estimation of the model.

INSERT TABLE II

Table II presents the GLS estimation results for both methods – the FM and the joint pooled time-series and cross-section – and for the four data sets. The application of GLS requires the removal of one portfolio (see section C above); the results presented correspond to the elimination of portfolio $J = (3,3)$, a "middle" portfolio. This is the third (out of six) β portfolio in the third (out of six) size portfolio, where portfolio (1,1) is that of the largest firms and within it the largest β. Table II also reports the GLS results when we eliminate instead either of the extreme portfolios: $J = (1,6)$ (large firms, small β) or $J = (6,1)$ (small firms, large β). We reestimated the GLS model eliminating in turn each of the 36 portfolios, and the results remained qualitatively unchanged.

The results show that β is an important factor in pricing capital assets, consistent

[6] We also considered the case where portfolios were formed by sorting on β first and then on size, which should lead to stronger significance of the β-coefficient. However, the FM methodology still indicated an insignificant return-β relationship.

with the CAPM. Under the joint pooled-GLS method, the coefficient of β is positive and highly significant. The hypothesis $\tilde{\gamma}_1 = 0$ is strongly rejected in favor of the alternative hypothesis $\tilde{\gamma}_1 > 0$ at significance levels greater than 0.001 (usually greater than 0.0001), fifty times greater than the standard benchmark of 5%. The results are similar for all four data sets, and remain unchanged when eliminating portfolios $J = (1,6)$ or $J = (6,1)$.[7] This demonstrates the robustness of the β-effect. Notably, the point estimates of γ_1 under the joint pooled time-series and cross-section GLS are generally similar to those obtained under the corresponding OLS estimation procedure, but their statistical significance is substantially improved under the GLS.

Even the less-efficient FM estimation method produces more significant results when applying GLS: the hypothesis that $\bar{\gamma}_1 = 0$ which could not be rejected when estimated by OLS is rejected at the standard level of significance for three of the four data sets in favor of the alternative hypothesis $\bar{\gamma}_1 > 0$. In both OLS and GLS estimates, the risk premium $\bar{\gamma}_1$ is of the same order of magnitude while the significance of the GLS estimates is greater. This means that when we account for the heteroskedasticity across years as well as the cross-portfolio heteroskedasticity and correlations, we obtain lower-variance estimates of γ_1 that are highly significant. Clearly, the power of the test obtained by the joint pooled cross-section and time-series GLS estimation is even greater.

Our conclusion is that there is a robust, strongly significant positive relationship between systematic risk and stock returns.

E. Effects of Size and Standard Deviation

Tests of the CAPM (as in FM) often examine the effect of unsystematic risk – the standard deviation of the market-model residuals. Under the CAPM, unsystematic risk should not

[7] The results were essentially the same when other portfolios were eliminated. For example, in the case of raw returns calculated for our original data, the 36 estimates of $\tilde{\gamma}_1$ had a mean of 0.0319 and their t-statistics were between 3.47 and 5.36. The detailed results are available upon request.

be priced by well-diversified investors. Another company-specific factor affecting stock returns is the size of the company's equity value, which was found to have a significant negative effect on stock returns.[8] In this Section we apply our methodology to examine these relationships, because it could be argued that the β-effect reflects the effects of these other variables. Given that β is negatively correlated with size, the documented positive relationship between β and stock returns could proxy for the size effect. Similarly, given the positive correlation between systematic and unsystematic risk, the effect of β may well reflect that of the unsystematic risk. Thus, we test whether the inclusion of these explanatory variables in the regression equation obliterates the positive and significant relationship between β and average portfolio returns, documented in Section D above. We present results for the survivorship-adjusted data and concentrate on the most powerful and efficient method, the joint pooled time-series and cross-section GLS estimation.

INSERT TABLE III

First, we consider the addition of the standard deviation of the market-model residuals, SD, which were estimated from the market-model regressions in subperiod II for each portfolio. There are three predictions on the relationship between expected return and SD (see discussion in AM (1989)): By the CAPM, SD should have no effect on expected returns for well-diversified investors; however, if diversification is constrained, the return-SD relationship will be positive because risk-averse investors expect a compensation for the undiversified risk. On the other hand, Constantinides and Scholes (1980) show that assets with higher standard deviations provide investors with a more valuable tax trading option, implying a negative return-SD relationship.

For maximal efficiency, the variance-covariance matrix $\hat{V}_y$ should be based on residuals from the correctly specified model. When introducing SD, we test the null hypothesis that this variable is in fact superfluous, hence we base $\hat{V}_y$ on residuals from the null, i.e., from

[8] See Banz (1981), Reinganum (1981) and recently FF. AM argued that the size effect proxies for a liquidity effect.

the model including only β. If we reject the null, we re-estimate $\hat{V}_y$ based on residuals from the model that includes both β and SD.

The results in row 1 of Table III show that the coefficients of both β and SD are significant. Thus, $\hat{V}_y$ was reestimated from the residuals of a model that includes both variables; the GLS estimation is presented in row 2 of Table III. The coefficient of SD is negative and highly significant, consistent with the proposition of Constantinides and Scholes (1980).

We then added the variable SZ_{py}, the logarithm of the average size of firms included in portfolio p at the end of the year preceding y. FF found that the return-size relationship is negative and significant, consistent with other studies. Moreover, they found that when SZ is included in the equation, $\bar{\gamma}_1$ is *negative* and insignificantly different from zero. AM found, however, that the effect of β survives the inclusion of SZ in the equation.

Our estimation results show that the β effect remains positive and significant even when the estimation model includes two additional regressors, SD and SZ. We first use the covariance matrix $\hat{V}_y$ under the null, based on residuals from the model with β and SD. The results in row 3 of Table III show that the coefficients of both these variables remain highly significant even though SZ is included in the equation. Given the significance of SZ, we reestimated the GLS model using a covariance matrix $\hat{V}_y$ based on residuals from the complete model. The resulting coefficients of SZ and SD, shown in row 4 of Table III, are highly significant. Most importantly, the coefficient of β remains positive and highly significant.

We conclude that β performs uniformly well under our empirical methodology. Even the inclusion of variables such as size and the residual standard deviation in the model does not render β insignificant.

III. Conclusion

In this paper we presented two econometric techniques to test the capital asset pricing model (CAPM) that improve on the commonly-used Fama-MacBeth (1973) methodology: (1) a joint pooled cross-section and time-series estimation procedure, and (2) the use of generalized least squares estimation. This method of estimation (used in Amihud and Mendelson (1986)) produces more efficient estimates and more powerful tests than those obtained by the FM methodology. A recent study by Fama and French (1992) which applied, in the main, the FM method found an insignificant return-β relationship. This implies that there is no support for the major dictum of the CAPM. However, using our methodology we reach different conclusions.

Replicating the FM methodology, we found that the return-β relationship is insignificant, consistent with FF. However, using the same data and employing the joint pooled time-series and cross-section estimation, we obtained a significantly positive coefficient of average return on β. An additional improvement in statistical significance was obtained when we applied GLS to account for heteroskedasticity over time and across portfolios as well as cross-portfolio correlations. The joint pooled GLS estimation method produced positive and highly significant estimates of the coefficient of average return on β. The results were highly robust to the data used. In particular, we used raw and excess returns (over the T-bill rates), and adjusted the data for a possible survivorship bias. The FM methodology itself was improved when we applied to it the GLS estimation procedure: we found, again, that the return-β relationship was positive and generally significant.

The return-β relationship remained highly significant after including in the model two additional regressors, the standard deviation of the market model residuals and the (logarithm of) firm size. Both variables had negative and significant coefficients. However, the coefficient of the average return on β remained positive and highly significant.

We conclude that β remains an important factor in asset pricing.

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Table I: OLS Results

Estimated slope coefficients of β from regressions of average portfolio returns r_{py} on portfolio beta β_{py}, allowing for a separate intercept in each year (t-values are in parentheses). Results are presented for both raw returns and for excess returns over the T-Bill rate. The portfolios are indexed by $p = (i, j)$, where i is the size group ($i = 1, 2, \ldots 6$; $i = 1$ for the largest-size group) and j is the β group ($j = 1, 2, \ldots 6$; $j = 1$ for the largest β). y is the year index, 1953 through 1990. In constructing portfolios for each year y, stocks were ranked by size (total capitalization at the end of year $y - 1$) then by β (estimated from the market-model from year $y - 7$ through the end of year $y - 5$). β_{py} of each portfolio p in year y is estimated from the market-model regression over 48 months from year $y - 4$ through $y - 1$. The annual returns r_{py} are compounded monthly returns for each stock in year y, averaged across the stocks in the portfolio. Stocks delisted during the test year y were excluded from the "Original Data" but included in the "Survivorship-Adjusted Data," in which case returns were estimated using the delisting segment of the CRSP for the return on the delisting month and the market return through the rest of the year.

The coefficients are OLS estimates using the FM methodology and the joint pooled time-series and cross-section model. The FM coefficients are the averages of the yearly cross-sectional regression coefficients, and the corresponding t-values are based on these coefficients' standard deviation. The results for the pooled method use the joint pooled time-series and cross-sectional model (2).

Method	Period	Raw Returns		Excess Returns	
		Original Data	Survivorship Adjusted Data	Original Data	Survivorship Adjusted Data
FM	1953–90	.053	.051	.053	.049
		(1.198)	(1.164)	(1.185)	(1.126)
Pooled	1953–90	.037	.033	.037	.031
		(3.215)	(2.868)	(3.176)	(2.733)
FM	1953–71	.082	.081	.081	.080
		(1.273)	(1.277)	(1.264)	(1.250)
Pooled	1953–71	.051	.049	.051	.047
		(3.004)	(2.895)	(2.975)	(2.804)
FM	1972–90	.025	.019	.025	.018
		(.409)	(.333)	(.401)	(.311)
Pooled	1972–90	.022	.016	.022	.014
		(1.436)	(1.032)	(1.416)	(.935)

Table II: GLS Results

Estimated coefficients from GLS regressions of portfolio returns r_{py} on portfolio beta, β_{py}, allowing for a separate intercept in each year, applying both the FM and the joint pooled time-series and cross-section estimation methodologies (t-values in parentheses; see Table I for a description of the data). The variance-covariance matrices $\hat{V}_y$ for the GLS procedure were estimated from the OLS residuals, deleting one portfolio so that $\hat{V}_y$ are non-singular. Unless otherwise stated, the deleted portfolio is (3,3). Results are also presented when two other portfolios are deleted: portfolio (1,6) of large size, low beta stocks, and (6,1) of small size, large beta stocks. For FM-GLS, $\hat{V}_y$ are based on the FM-OLS residuals, and for the joint pooled GLS, they are based on the joint pooled OLS residuals. The results are presented for both raw returns and excess returns over the T-bill rate, for the original and the survivorship-adjusted data.

Method	Period	Raw Returns		Excess Returns	
		Original Data	Survivorship Adjusted Data	Original Data	Survivorship Adjusted Data
FM	1953–90	.039	.035	.042	.024
		(2.138)	(2.242)	(2.295)	(1.574)
Pooled	1953–90	.035	.027	.034	.027
		(5.146)	(3.939)	(4.345)	(4.054)
FM	1953–71	.075	.040	.077	.046
		(3.087)	(1.870)	(2.803)	(3.225)
Pooled	1953–71	.044	.029	.048	.032
		(5.481)	(3.365)	(4.967)	(3.976)
FM	1972–90	.004	.029	.007	.001
		(.171)	(1.315)	(.323)	(.044)
Pooled	1972–90	.012	.023	.007	.016
		(.965)	(2.022)	(.548)	(1.358)
FM (1,6)	1953–90	.034	.040	.040	.036
		(1.595)	(2.496)	(1.995)	(2.169)
Pooled (1,6)	1953–90	.031	.026	.029	.031
		(3.997)	(3.836)	(3.667)	(4.680)
FM (6,1)	1953–90	.037	.047	.033	.035
		(1.839)	(3.042)	(1.422)	(2.174)
Pooled (6,1)	1953–90	.033	.029	.027	.032
		(4.848)	(4.325)	(3.370)	(4.840)

Table III
Pooled GLS Results for Size and Residual Standard Deviation

Estimated coefficients from joint pooled cross-section and time-series GLS regressions of portfolio returns r_{py} on the following variables: portfolio beta, β_{py}, and portfolio residual standard deviation, SD_{py}, both estimated from the market model over the 48 months preceding year y, and the logarithm of portfolio size, SZ_{py}, as of the end of the year preceding year y. The estimation is over the period 1953-1990 and allows for a separate intercept in each year y (t-values in parentheses; see Table I for a description of the data). The estimated variance-covariance matrices $\hat{V}_y$ are based on OLS residuals from a joint pooled model containing the variables indicated in the $\hat{V}_y$ column and a separate intercept in each year, deleting portfolio (3,3) for the feasible GLS procedure. The results are for survivorsip-adjusted data and for both raw returns and excess returns over the T-Bill rate.

	Raw Returns			Excess Returns		
$\hat{V}_y$	β	SD	SZ	β	SD	SZ
β	.027	–.394	.	.026	–.291	.
	(3.920)	(–2.481)		(3.965)	(–2.186)	
(β, SD)	.036	–1.290	.	.036	–1.219	.
	(5.332)	(–8.871)		(5.593)	(–11.085)	
(β, SD)	.038	–1.369	–.0023	.039	–1.285	–.0014
	(5.618)	(–9.116)	(–2.065)	(5.842)	(–11.006)	(–1.676)
(β, SD, SZ)	.018	–2.327	–.0117	.023	–2.206	–.0112
	(2.928)	(–18.552)	(–12.647)	(3.486)	(–21.955)	(–12.083)