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Abstract

This paper tests the ability of the American option pricing model proposed by Parkinson [18] or Mason [14] to explain the pricing of the foreign currency options traded on the Philadelphia Stock Exchange from February 28, 1983 to March 26, 1985. We find that the model underprices out-of-the-money options relative to at-the-money and in-the-money options. This relative underpricing is driven by an underpricing of out-of-the-money call options of short maturity. In addition, the degree of relative mispricing for most categories of options is shown to be a decreasing function of the time to maturity of the options. Longer maturity options appear to trade at similar levels of implied volatility whether they are in, at, or out of the money. Most of these biases appear consistent with the fact that the underlying spot currency rate follows a mixed jump diffusion process as described in [17].

I. Introduction

Recent empirical and theoretical articles on stock option pricing have stressed the importance of the premature exercise privilege inherent in any American option. In particular, Geske and Roll [10] have pointed out that the systematic differences between market prices and theoretical Black and Scholes values, which are related to the exercise price and the time to maturity, can be explained by the fact that the Black and Scholes model does not take into account the possibility of optimal early exercise of an American option. This result has been confirmed empirically by the studies of Whaley [27] and Sterk [25], which show that the standard American option pricing model[1] gives reasonably unbiased estimates of observed stock option prices. It also should be mentioned that other empirical studies, such as the one undertaken by Rubinstein [21], indicate that the American option pricing model is not sufficient to explain all the systematic

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[1] See, for example, [20], [22], [9], or [6].

differences that exist between market prices and theoretical Black and Scholes values.[2]

This paper will investigate whether an American option pricing model performs as well for the foreign currency options market of the Philadelphia Stock Exchange as it does for the stock options market. Our main purpose is to examine whether the American option pricing model exhibits significant biases when applied to this market.[3] The second section of the paper contains a description of the model that will be used in our tests. We will also derive in this section the theoretical biases that could be introduced in the tests if a European model were used to value these American options. In the third section, we will present the data and inputs to the model that we use in our empirical tests. The structure and results of the tests are presented in Section IV, which also compares our results and the empirical results obtained in the stock option pricing literature. Our conclusions are presented in the fifth section.

II. Valuation Model and Biases of the European Model

A. Valuation Model

The basic assumptions of the model used in our tests are identical to the assumptions made by Garman and Kohlhagen [8].[4] The markets are perfect, i.e., there are no transaction costs and no limits to short sales. In addition, the currency spot price, S, follows a geometric Wiener process with instantaneous variance $\sigma^2 S^2$ where σ, the volatility parameter, is a constant. Finally, the domestic interest rate, r, and the foreign interest rate, r_f, are both constants.

Given these assumptions, if $V(S,t)$ represents the value of a currency option at time t when the underlying currency spot price is S, we know from Garman and Kohlhagen that $V(S,t)$ will obey the following valuation equation,

$$\frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + \left(r - r_f\right) S \frac{\partial V}{\partial S} - rV + \frac{\partial V}{\partial t} = 0 \; . \tag{1}$$

If $V(S,t)$ represents the price of an American call option on a currency, $V(S,t)$ will satisfy equation (1) subject to the following two boundary conditions,

$$V(S,T) = \max \; \{0, S - E\} \; , \tag{2}$$

$$V(S,\lambda) = \max \; \left[V\left(S,\lambda^+\right), S - E\right] \quad \text{for all } \lambda = [t,T) \; , \tag{3}$$

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[2] For example, Rubinstein finds that for out-of-the-money options the shorter the time to expiration, the higher the option's implied volatility. This bias will not be corrected by the use of an American option pricing model.

[3] Shastri and Tandon [23] use a modified European model and closing prices to test the efficiency of the same market. They do not report the absolute and relative pricing biases exhibited by this modified European option pricing model, consequently our results are not directly comparable to theirs.

[4] This model follows directly from the stock option pricing model with continuous and proportional dividends of Merton [16]. Others, such as Feiger and Jacquillat [7] or Grabbe [11], have derived similar models for foreign currency options.

where T and E are, respectively, the maturity date of the option and its exercise price and where λ^+ represents the instant immediately after time λ. Boundary condition (2) accounts for the terminal value of the option, while equation (3) accounts for the possibility of premature exercise of the American call option on a foreign currency. The American call option valuation model (equations (1), (2), and (3)) is solved numerically through backward induction starting at the maturity date of the option. The numerical method that we use is an adaptation of the algorithm developed by Parkinson [18] and Mason [14].

Finally, if $V(S,t)$ represents the American put option value, $V(S,t)$ will be solved for numerically by solving valuation equation (1) subject to the following boundary conditions,

$$V(S,T) = \max\ [0, E-S] \tag{4}$$

$$V(S,\lambda) = \max\ \left[V\left(S,\lambda^+\right), E-S\right] \quad \text{for all } \lambda = [t,T)\ . \tag{5}$$

Equation (4) accounts for the terminal value of the put option, while equation (5) accounts for the possibility of premature exercise of the put option.

B. Biases of the European Model

Before testing the American option pricing model, it is valuable to point out the theoretical biases that may be introduced in the tests if the corresponding European option pricing model is used. We will do so by comparing, for different values of the parameters and variables, the numerical solution obtained for the American option value to the closed form solution proposed by Garman and Kohlhagen [8] for the European option value. Specifically, we consider a currency option written on a currency with a 10.00-percent annual volatility. In addition, we assume that the domestic and foreign rates of interest are both equal to 10.00 percent. We then vary each parameter by 5.00 percent to get a coarse estimate of the effect of a change in each parameter on the difference in value between American and European foreign currency options.[5] Results are presented in Table 1 for an option with 180 days remaining to maturity. From these results, we can observe that the undervaluation caused by the European model could be substantial for two categories of options:

—in-the-money and at-the-money put options when the underlying currency is a premium currency, i.e., when the foreign rate of interest is less than the domestic rate of interest;

[5] These values have been chosen to be roughly comparable to the actual values of these parameters in the market over the sample period. Ten percent is roughly comparable to the domestic interest rate and to the interest rates on short-term deposits denominated in British pounds and Canadian dollars while 5 percent is roughly comparable to the interest rates on short-term deposits denominated in German marks, Japanese yen, or Swiss francs. The average interest rates for three-month maturity deposits denominated in U.S. dollars, British pounds, Canadian dollars, German marks, Japanese yen, and Swiss francs were, respectively, 9.0 percent, 9.79 percent, 10.21 percent, 5.48 percent, 6.23 percent, and 3.55 percent over the sample period.

—in-the-money and at-the-money call options when the underlying currency is a discount currency, i.e., when the foreign rate of interest is greater than the domestic rate of interest.

It is also interesting to note that the difference between American and European option prices depends on the level of the volatility parameters. For in-the-money options, this difference is a decreasing function of volatility while it is an increasing function of volatility for out-of-the-money options. Finally, it must be pointed out that this difference increases with an increase in the time to maturity of the option.

TABLE 1

Difference in Value between American and European Foreign Currency Options*
(Values Scaled by the Exercise Price)

(r, r_f, σ)	Difference in Value for Put Options	Difference in Value for Call Options
Panel A: S/E = 0.90		
(0.10, 0.10, 0.10)	0.0033 (3.40%)	0.0000 (0.00%)
(0.05, 0.10, 0.10)	0.0000 (0.00%)	0.0001 (12.50%)
(0.15, 0.10, 0.10)	0.0241 (31.75%)	0.0000 (0.00%)
(0.10, 0.05, 0.10)	0.0222 (28.53%)	0.0000 (0.00%)
(0.10, 0.15, 0.10)	0.0000 (0.00%)	0.0001 (12.50%)
(0.10, 0.10, 0.05)	0.0048 (5.04%)	0.0000 (0.00%)
(0.10, 0.10, 0.15)	0.0024 (2.33%)	0.0001 (1.27%)
Panel B: S/E = 1.0		
(0.10, 0.10, 0.10)	0.0003 (1.12%)	0.0003 (1.12%)
(0.05, 0.10, 0.10)	0.0000 (0.00%)	0.0026 (15.48%)
(0.15, 0.10, 0.10)	0.0028 (17.07%)	0.0000 (0.00%)
(0.10, 0.05, 0.10)	0.0026 (15.48%)	0.0000 (0.00%)
(0.10, 0.15, 0.10)	0.0000 (0.00%)	0.0028 (17.07%)
(0.10, 0.10, 0.05)	0.0001 (0.75%)	0.0001 (0.75%)
(0.10, 0.10, 0.15)	0.0004 (1.00%)	0.0004 (1.00%)
Panel C: S/E = 1.10		
(0.10, 0.10, 0.10)	0.0000 (0.00%)	0.0030 (3.06%)
(0.05, 0.10, 0.10)	0.0000 (0.00%)	0.0227 (29.37%)
(0.15, 0.10, 0.10)	0.0001 (8.33%)	0.0000 (0.00%)
(0.10, 0.05, 0.10)	0.0001 (7.69%)	0.0000 (0.00%)
(0.10, 0.15, 0.10)	0.0000 (0.00%)	0.0246 (32.63%)
(0.10, 0.10, 0.05)	0.0000 (0.00%)	0.0048 (5.04%)
(0.10, 0.10, 0.15)	0.0001 (0.95%)	0.0023 (2.18%)

* Values in parentheses represent the difference in value as a percentage of the corresponding European option value.

The intuitive explanation for these results is as follows. A higher domestic interest rate increases the likelihood of premature put exercise and therefore increases the difference in value between the American and European options. For call options, an increase in the domestic rate of interest will increase the value of the option if alive and therefore decrease the difference in value between the American and European options. Since foreign interest payments are like dividend payments, the value of the call option if alive will decrease with an increase in the foreign interest rate. Therefore, American call options will be worth relatively more than their European counterparts for higher levels of foreign interest rates. For put options, higher foreign rates of interest will imply lower forward prices for the currency, thereby reducing the probability of early exercise and the difference in value between the American and European put options.

The effect of different volatility levels on the difference in value between American and European foreign currency options must be examined for two levels of the spot price to exercise price ratio. For high values of this ratio, call options are deep in the money and put options are deep out of the money. Therefore, lower levels of volatility will imply a higher probability of call early exercise and a lower probability of put early exercise. This implies that the difference in value between American and European options will be higher for the call option and lower for the put option relative to our benchmark case if the volatility decreases. The opposite will obtain if volatility increases. For low values of the spot price to exercise price ratio, call options are deep out of the money and put options are deep in the money. Lower volatilities imply a higher likelihood of put early exercise, whereas higher volatilities lower this likelihood. Hence, the difference in value between American and European put options will rise as the volatility falls. The opposite is true for calls when the spot price to exercise price ratio is low.

III. Inputs to the Models

A. The Data and Observable Inputs

The primary data base for this study is the transactions surveillance report compiled daily by the Philadelphia Stock Exchange.[6] This report contains the following information for each option trade: date of trade, currency, maturity date, exercise price, time of trade, number of contracts traded, option price, location of the current spot quote available from the Telerate data service, and the actual spot bid and ask quotes reported by Telerate from the interbank market. The data base contains a total of 105,525 option trades on the five major currencies from February 28, 1983, to March 26, 1985.[7]

For the domestic interest rate, we are using the yield of the Treasury bills that mature as close as possible to the option maturity dates, as in [21].[8] Daily measures of the yield of these Treasury bills have been computed from the average of the bid and ask discount rates published daily in the *Wall Street Journal*.

Measuring the foreign interest rate, however, is more difficult. Since we assume a world with no transactions costs, this rate should be such that interest rate parity holds between the corresponding forward contract and the currency spot rate, given our measure of the domestic interest rate. Consequently, given the measure of the domestic interest rate and the value of the currency spot rate at the time of one of the option transactions included in our sample, the foreign interest rate could be derived from the forward price of the currency if we had a transactions record of the forward prices of the currency for delivery on the different option maturity dates. Unfortunately, such a record does not exist. How-

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[6] See [4].

[7] The five currencies are: British pound, Canadian dollar, German mark, Japanese yen, Swiss franc. Options on the French franc were not included in our tests.

[8] The option contracts expire on the Friday preceding the third Wednesday of the months of March, June, September, and December.

ever, currency futures contracts maturing close to concurrently with the PHLX currency options are traded on the International Money Market. This market is liquid and the exchange keeps a record of all transactions, called the I.M.M. Statistics Department Quote Capture Report. Trade date, currency, maturity, time of trade, type of trade, and trade price are all available from this source. Therefore, although we are not able to match an option trade with a trade on the corresponding forward market, thanks to the I.M.M. Capture Report we are able to closely match this option trade with a futures price quotation on the I.M.M.[9] Since several studies[10] have shown that the difference between the futures price of a commodity and the corresponding forward price is small, we are making the assumption that interest rate parity holds with respect to the futures price. This assumption implies that the hypothesis being tested in this paper is the joint hypothesis that the American option pricing model correctly describes the observed structure of foreign currency option prices and that interest rate parity holds between the spot and futures markets for currencies. This assumption enables us to derive an estimate of the foreign interest rate that is consistent with our measure of the spot exchange rate, with our measure of the domestic interest rate, and with interest rate parity.

B. Volatility Estimate

The only input to the option valuation model that cannot be directly inferred from market prices is the volatility parameter. This parameter must be estimated. As in many other studies in the option pricing literature, we have used an implied volatility approach to do this estimation. Our estimate of volatility is computed daily for each currency according to the method proposed by Whaley [27]. Table 2 shows the average implied volatilities for all currencies over the sample period and their standard deviations. According to Whaley's method, the daily implied volatility estimate for a currency should be chosen to minimize the sum of squared deviations between market prices and model prices of the option trades on the same currency the previous day. Whaley uses the previous day of data in the implied volatility estimation to circumvent the biases mentioned by Phillips and Smith [19],[11] which would have adversely affected his market efficiency tests. Even though we do not run such tests, using the previous day implied volatility remains a valid procedure for the following reasons. First, it will give us some indication of whether volatility changes widely from one day to the next, since for wide fluctuations of the daily volatility the absolute relative and dollar pricing errors will be large. Secondly, it gives slightly more credibility to the model as a pricing tool since our volatility estimate does not use information that might be unavailable at the time of the option transactions in the sample under test. Also, to save on the computational cost, we do not use all of the trades from

[9] The futures price that we use is the average of the last available futures quote just prior to the option transaction and of the first futures quote just after the option transaction. The average time lag between the option transaction and the futures transaction prior to it is 5 minutes and 36 seconds, while the average time lag between the option transaction and the futures transaction following it is 5 minutes and 48 seconds.

[10] See, for example, [6].

[11] Transactions at the bid may be systematically considered as underpriced and transactions at the asked may be systematically considered as overpriced.

the previous day, but six randomly chosen trades. Finally, on those days in which the option trading in a currency has been so thin that less than four option trades are made, we use the previous day's implied volatility to be sure that we do not bias our estimate, in case these option trades were not representative of the total trade population.

TABLE 2

Average Implied Standard Deviations (ISD) and Standard Deviation of the ISD from February 28, 1983, to March 26, 1985

Currency	Average ISD	Standard Deviation of ISD
British Pound	0.116	0.034
Canadian Dollar	0.048	0.015
Deutsche Mark	0.127	0.029
Japanese Yen	0.104	0.019
Swiss Franc	0.114	0.026

Table 3 reports the composition of the sample used to compute all of the daily implied volatilities and the composition of the total population of option trades according to maturity classes and to in-the-money, at-the-money, and out-of-the-money classes.[12] These two samples show similar distributions between the different classes. Therefore, it is apparent from these results that our sampling procedure does not introduce any bias into the computation of the volatility in addition to those that are inherent to the method.[13]

IV. Structure and Results of the Tests

A. Structure of the Tests

Our tests are based on a sample of 20,000 trades randomly drawn from the 105,525 trades available in the data base. Table 3 shows the composition of this randomly drawn sample according to maturity classes and to in-, at-, and out-of-the-money classes as well as the composition of the total sample. It is clear from this table that the two samples have the same composition. Consequently, run-

[12] In this table, a call option is considered in the money and a put option is considered out of the money if the ratio of the spot price to exercise price is greater than 1.02; an option will be at the money if the ratio of the spot price to exercise price is included between 0.98 and 1.02; and, finally, a put option will be in the money and a call option wll be out of the money if this ratio is less than 0.98. The choice of 1.02 and 0.98 is arbitrary. However, since currencies are traded in one hundredth of a cent increments, defining at-the-money options by a spot price to exercise price ratio of one would result in practically no option trade being classified in this category. The results are not sensitive to this assumption. A range of 1.01 to 0.99 for the option trade to be at the money decreases the number of option trades classified as at the money but does not substantially modify the results.

[13] Any implied volatility estimation method introduces absolute pricing biases in the study. For example, Macbeth and Merville [13] force the at-the-money options to be priced correctly. Chiras and Manaster [5] tend to price out-of-the-money options more accurately since they use the elasticities of the option trade prices as weights to compute the implied volatility from the implied volatilities of all option trades. Finally, Whaley's method tends to price correctly the "average" option in the sample (since his estimation procedure depends on the sample composition) as well as the options in the sample with the greatest sensitivity to volatility. Consequently, our tests, as any other test using an implied volatility estimation procedure, are more suited to identifying relative mispricing of one class of options with respect to another class of options.

TABLE 3

Composition of the Different Samples According to Maturity Classes and to In-, At-, and Out-of-the-Money Classes

Maturity	In the Money	At the Money	Out of the Money
Panel A: Total Sample			
Less than 30 Days	2.6%	7.8%	2.6%
Between 31 & 90 Days	4.2%	15.9%	15.8%
Between 91 & 180 Days	4.9%	16.1%	21.4%
More than 180 Days	1.1%	3.0%	4.4%
Panel B: Sample Used to Compute Daily Volatilities			
Less than 30 Days	2.2%	7.8%	1.9%
Between 31 & 90 Days	4.6%	19.7%	15.3%
Between 91 & 180 Days	4.4%	17.1%	19.3%
More than 180 Days	0.9%	3.0%	3.7%
Panel C: Sample Used in the Tests			
Less than 30 Days	2.5%	7.6%	2.1%
Between 31 & 90 Days	4.3%	16.2%	16.1%
Between 91 & 180 Days	5.3%	16.0%	21.8%
More than 180 Days	1.2%	2.9%	3.9%

ning the tests on the randomly drawn subsample will not introduce any significant bias in our results and will substantially reduce the computation cost. The results are presented in three tables. Aggregate results are reported in Table 4. Tables 5 and 6 report results broken down by strike price and maturity classes, respectively. In each table, we present statistics that are based on the relative option pricing error and on the dollar pricing error. For a given option transaction, the dollar pricing error is equal to the difference between market and model prices, while the relative option pricing error is equal to the ratio of the dollar pricing error to the market price of the option.

Concerning the relative option pricing error, each table presents three statistics in every cell: the average of the relative option pricing errors assigned to the cell, the average of the absolute values of these relative option pricing errors, and the proportion of positive relative option pricing errors out of the total number of transactions assigned to this cell. The average of the relative option pricing errors is presented in the first row of each cell of the tables related to the relative pricing error. Positive relative option pricing errors imply option underpricing by the model, and negative errors imply overpricing. The average of the relative option pricing errors is followed in parentheses by its sample standard deviation. When this sample standard deviation implies that the average relative option pricing error lies at least two sample standard deviations above or below zero, the sample standard deviation is followed by an asterisk. The presence of the asterisk can be interpreted to represent poor model pricing. However, given a fair amount of asymmetry for the relative errors around zero, these results provide only an indicative significance test.

The average of the absolute values of the relative option pricing errors is presented in the second row of each cell of the tables related to the relative pricing error. It is followed by its sample standard deviation in parentheses. When the average of the absolute values of the relative option pricing errors lies two sample standard deviations above 5 percent, the sample standard deviation is followed by an asterisk. This statistic is presented to show whether large negative errors are cancelling out with large positive errors across options in a cell to yield small average relative option pricing errors. Like our previous test, these results provide only an indicative significance test.

Our final test statistics concerning the relative pricing error are presented in rows three and four of each cell of the tables dealing with the relative pricing error. These entries show the proportion of positive relative option pricing errors and the total number of errors. A sign test is then performed based on the number of positive errors to see if the number of positive and negative errors is consistent with equal probabilities for both positive and negative errors.[14] When the proportion of positive errors in the cell is significantly different from one half the total number of errors at the 0.95 level, this proportion is followed by an asterisk.[15]

As far as the dollar pricing error is concerned, the parts of the tables reporting the corresponding results give the average dollar pricing error and the standard deviation of the pricing error on the first line of every cell. As in the case of the relative pricing error, the standard deviation is followed by an asterisk when the average dollar pricing error lies at least two standard deviations above or below zero. The second line presents the average absolute dollar pricing error as well as the corresponding standard deviation in parentheses. Finally, the third line of every cell reports the price of the average option contract corresponding to this cell.[16]

B. Tests Results

Table 4 shows that, on average, the model overprices put options relative to call options. The combined effect across both put and call options results in mild overpricing by the model. Table 5 presents the same set of results as Table 4, except for the fact that the observations are now divided into three subgroups: in-the-money, at-the-money, and out-of-the-money options. The definition of these three subgroups is the same as in Table 3. Table 5 presents several interesting results. First, the model appears to overprice at-the-money and in-the-money options relative to out-of-the-money options. It should be noted that the overpricing of at-the-money options is more severe than the overpricing of in-the-money options, in relative terms. However, as in Shastri and Tandon [24], we find that the degree of overpricing by the model is greater for in-the-money options than for at-the-money options in dollar terms. This result is actually consistent with our estimation procedure for σ, since we are minimizing the sum of squared deviations between model and market prices, and since at-the-money options are more sensitive to a change in the value of σ than in-the-money options. Considering the results for put and call options separately, it appears that the results derived for out-of-the-money options are due to a slight overpricing of out-of-the-money put options by the model, combined with a close to insignificant underpricing of

[14] See [12] and [21] for a previous application of this test.

[15] This test is also valid for the dollar pricing error. In this case, the sign test would test whether or not the error is significantly different from $0.00 at the 0.95 level.

[16] A contract is an option to buy or sell 12,500 British pounds, 50,000 Canadian dollars, 62,500 German marks, 6,250,000 Japanese yen, or 62,500 Swiss francs.

out-of-the-money call options.[17] Finally, it should be stressed that the average of the absolute values of the relative pricing errors is a decreasing function of how deep in the money the option is. Since we are using transactions prices, this result is most likely due to the fact that the bid-ask spread represents a lower proportion of the option price as the option gets deeper in the money.

TABLE 4a

Aggregate Relative Pricing Errors**

	All Options	All Put Options	All Call Options
Error	−0.0184 (0.0021)*	−0.0587 (0.0036)*	−0.0009 (0.0026)
Absolute Error	0.1331 (0.0019)*	0.1239 (0.0033)*	0.1371 (0.0023)*
% of Positive Errors	45%*	35%*	50%
Cell Size	20000	6055	13945

** In each of the cells, the entries corresponding to the error and the absolute error give first the mean, then the standard deviation in parentheses. The standard deviation of the error is followed by an asterisk if the mean error lies at least two standard deviations above or below 0.0%. The standard deviation of the absolute error is followed by an asterisk if the mean absolute error lies at least two standard deviations above or below 5.0%. The percentage of positive errors is also followed by an asterisk if the percentage of positive errors is significantly different from 50% at the 95% confidence level.

TABLE 4b

Aggregate Relative Pricing Errors in Dollars**

	All Options	All Put Options	All Call Options
Error	−7.54 (0.44)*	−14.63 (0.72)*	−4.47 (0.54)*
Absolute Error	30.80 (0.38)	32.44 (0.62)	30.09 (0.48)
Average Contract Price	451.12	482.97	437.30

** For each cell, the first line corresponds to the average error and its standard deviation in parentheses, the second line corresponds to the average absolute dollar error and its standard deviation, while the third line corresponds to the value of the average contract in the cell (option to buy or sell 12,500 British Pounds, 50,000 Canadian Dollars, 62,500 German Marks, 6,250,000 Japanese Yen, or 62,500 Swiss Francs). The standard deviation of the error (not the absolute error) is followed by an asterisk if the mean error lies at least two standard deviations above or below $0.0.

Additional insights can be obtained on the performance of the model by classifying the results by maturity classes. Table 6 presents results for the entire sample of options classified by maturity classes and by in-, at-, and out-of-the-money classes. As far as in-the-money options are concerned, the results derived in Table 5 are confirmed by Table 6. It is also apparent from Table 6 that the overpricing by the model of the options in the sample tends to be a decreasing function of the time to maturity of the options, especially for out-of-the-money

[17] The average dollar pricing error does not seem to be different from zero, however the sign test indicates a significant underpricing.

and at-the-money options with a maturity greater than thirty days.[18] In addition, Table 6 shows that the relative underpricing of out-of-the-money options is driven in part by a very significant underpricing by the model of out-of-the-money options with a maturity of less than thirty days. This result is driven by a significant underpricing of out-of-the-money calls of shorter maturity, which dominates an overpricing of the corresponding puts in the aggregate results. Finally, Table 6 shows that the average of the absolute values of the relative pricing errors is also a decreasing function of the time to maturity of the option except for in-the-money options. This result is consistent with the reason given to explain that this average is a decreasing function of how deep in the money the option is.

TABLE 5a

Relative Pricing Errors According to In-, At-, and Out-of-the-Money Classes**

	In the Money	At the Money	Out of the Money
All Options			
Error	−0.0193 (0.0011)*	−0.0540 (0.0026)*	0.0166 (0.0040)*
Absolute Error	0.0349 (0.0010)	0.1153 (0.0024)*	0.1799 (0.0035)*
% of Positive Errors	33%*	38%*	56%*
Cell Size	2645	8561	8794
All Put Options			
Error	−0.0253 (0.0014)*	−0.0730 (0.0041)*	−0.0567 (0.0123)*
Absolute Error	0.0381 (0.0011)	0.1320 (0.0037)*	0.1926 (0.0112)*
% of Positive Errors	27%*	33%*	46%*
Cell Size	1371	3369	1315
All Call Options			
Error	−0.0129 (0.0018)*	−0.0417 (0.0034)*	0.0294 (0.0041)*
Absolute Error	0.0315 (0.0017)	0.1044 (0.0031)*	0.1777 (0.0036)*
% of Positive Errors	40%*	41%*	57%*
Cell Size	1274	5192	7479

** See footnote to Table 4a.

TABLE 5b

Pricing Errors in Dollars According to In-, At-, and Out-of-the-Money Classes**

	In the Money	At the Money	Out of the Money
All Options			
Error	−17.21 (1.20)*	−11.12 (0.65)*	−1.16 (0.67)
Absolute Error	34.82 (1.05)	33.31 (0.55)	27.15 (0.60)
Average Contract Price	1116.40	449.36	252.75
All Put Options			
Error	−21.94 (1.46)*	−14.92 (1.02)*	−6.27 (1.36)*
Absolute Error	35.50 (1.25)	34.37 (0.87)	24.30 (1.20)
Average Contract Price	1047.67	365.15	196.09
All Call Options			
Error	−12.11 (1.94)*	−8.65 (0.84)*	−0.26 (0.75)
Absolute Error	34.10 (1.72)	32.61 (0.72)	27.65 (0.68)
Average Contract Price	1190.36	503.99	262.72

** See footnote to Table 4b.

[18] Shastri and Tandon's [24] results concerning the time to maturity bias are difficult to interpret since they use different volatility estimates for different option maturities and do not indicate how the implied volatility estimates change across maturities.

TABLE 6a

Relative Pricing Errors According to In-, At-, and Out-of-the-Money Classes and to Maturity Classes**

Maturity	In the Money	At the Money	Out of the Money
Less than 30 Days			
Error	−0.0102 (0.0018)*	−0.0961 (0.0098)*	0.2863 (0.0238)*
Absolute Error	0.0283 (0.0014)	0.2063 (0.0087)*	0.4414 (0.0174)*
% of Positive Errors	37%*	34%*	76%*
Cell Size	506	1521	426
Between 31 & 90 Days			
Error	−0.0257 (0.0021)*	−0.0827 (0.0041)*	−0.0180 (0.0082)*
Absolute Error	0.0351 (0.0020)	0.1179 (0.0038)*	0.2265 (0.0072)*
% of Positive Errors	27%*	25%*	51%
Cell Size	858	3246	3228
Between 91 & 180 Days			
Error	−0.0178 (0.0019)*	−0.0147 (0.0027)*	0.0172 (0.0044)*
Absolute Error	0.0363 (0.0016)	0.0779 (0.0023)*	0.1347 (0.0039)*
% of Positive Errors	37%*	49%*	58%*
Cell Size	1051	3209	4351
More than 180 Days			
Error	−0.0226 (0.0045)*	−0.0011 (0.0054)	0.0091 (0.0067)
Absolute Error	0.0428 (0.0038)	0.0695 (0.0046)*	0.0977 (0.0057)*
% of Positive Errors	37%*	57%*	55%*
Cell Size	230	585	789

** See footnote to Table 4a.

TABLE 6b

Pricing Errors in Dollars According to In-, At-, and Out-of-the-Money Classes and to Maturity Classes**

Maturity	In the Money	At the Money	Out of the Money
Less than 30 Days			
Error	−7.63 (1.60)*	−9.07 (0.80)*	7.68 (0.74)*
Absolute Error	23.84 (1.24)	24.72 (0.55)	12.38 (0.57)
Average Contract Price	1014.47	208.23	41.26
Between 31 & 90 Days			
Error	−20.03 (1.77)*	−19.95 (0.85)*	−4.37 (0.73)*
Absolute Error	30.87 (1.58)	31.12 (0.74)	21.33 (0.63)
Average Contract Price	1013.68	363.64	151.75
Between 91 & 180 Days			
Error	−16.67 (2.02)*	−5.48 (1.30)*	−0.40 (1.12)
Absolute Error	38.36 (1.72)	37.23 (1.13)	30.22 (1.02)
Average Contract Price	1188.21	583.44	298.39
More than 180 Days			
Error	−30.21 (6.98)*	1.63 (3.34)	2.99 (2.90)
Absolute Error	57.57 (6.19)	46.20 (2.74)	42.03 (2.48)
Average Contract Price	1395.68	816.37	528.52

** See footnote to Table 4b.

C. Relationship to Existing Results in the Stock Option Pricing Literature

The exercise price bias that we have identified is clearly contrary to what Whaley and Sterk found for American stock options. The fact that the standard American option pricing model exhibits such a bias when applied to foreign currency options is most likely due to the fact that the rate of change in a foreign currency spot price is not normally distributed. It seems likely that the overpricing of at-the-money options and the underpricing of out-of-the-money call options can be explained by assuming that the rate of change in the foreign currency spot price is leptokurtic, as was done in [15]. An equivalent approach is to assume that the foreign currency spot price follows a mixed jump diffusion process as in [17].[19] Indeed, Merton shows that the standard Black and Scholes model would overprice at-the-money options and underprice out-of-the-money options if it were used when the underlying asset follows a jump diffusion model.[20]

The time-to-maturity bias generally exhibited by our results appears to be contrary to some of the results obtained in the stock option pricing literature, which shows that options with shorter maturities are underpriced as in [5]. However, results similar to ours have been found in the stock option pricing literature, for example by Rubinstein [21], and to some extent by Whaley [27].[21] Finally, it should be pointed out that the time-to-maturity bias exhibited in our results is consistent to a large extent with our explanation of the exercise price bias. Merton found that when the underlying spot price follows a mixed jump diffusion process, the overpricing of at-the-money options and the underpricing of out-of-the-money options by the standard option pricing model should decrease as the time to maturity increases.[22] We find similar results for at-the-money options and out-of-the-money call options of shorter maturity.[23]

V. Conclusion

We have shown in this paper that the standard American option pricing model does not explain the pricing of foreign currency options as well as it explains the pricing of stock options. In particular, our results show that this model underprices out-of-the-money options relative to at-the-money options and in-the-money options. This result may be explained partly by the fact that the distribution of the rate of change in the foreign currency spot price is leptokurtic or, equivalently, by the fact that part of the exchange rate risk comes from a jump component in the spot price process.

In addition, it is apparent from our results that the American model exhibits a definite time-to-maturity bias. As time to maturity increases, the overpricing of in-the-money and at-the-money options by the model decreases, while the underpricing of out-of-the-money options first decreases then increases slightly. The decreases in the underpricing of out-of-the-money call options of short maturity

[19] Beckers [2] points out that the distribution of the rate of change in the spot price is leptokurtic when the underlying spot price follows a mixed jump diffusion process.

[20] Even though we are using implied volatilities, our estimate of volatility will be biased upwards as in Merton if we use the standard option pricing model when the underlying currency spot price follows a mixed jump diffusion process.

[21] Though Whaley shows a slight time-to-maturity bias in the opposite direction, Whaley assumes our result by using a higher volatility estimate for long maturity options. The increasing volatility estimate was used by Whaley because longer maturity options may have greater implied volatilities since a greater amount of information uncertainty will be resolved during the lives of these options.

[22] Ball and Torous [1] have substantiated this point empirically for equity options.

[23] The results found for out-of-the-money options of longer maturity contradict the jump diffusion hypothesis. However, the mispricing of these options is quite small. In the same way, the overpricing of out-of-the-money put options of shorter maturity contradicts the jump diffusion hypothesis.

and in the overpricing of at-the-money options are both consistent with the possibility that part of the exchange rate risk comes from random jumps in the spot currency price. Given these results, it appears that Merton's mixed jump diffusion model or a model based on a leptokurtic probability distribution of the rate of change in the foreign currency spot price may be more appropriate for valuing foreign currency options. This could be a topic of future research.

Finally, it should be pointed out that, given the joint hypothesis tested in this paper, other potential explanations of the results may exist, in addition to the fact that the American model may be misspecified along the lines presented in this paper. It may be that the market prices are simply wrong or that interest rate parity does not hold with respect to the futures price. There is some empirical evidence, already mentioned, that interest rate parity holds, at least approximately with respect to the futures price in the case of the foreign exchange market. However, to address the issue of mispricing by the market would require the development of some sort of market efficiency test. While this was not the purpose of this paper, it could be the topic of future research.

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