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An Empirical Test of a Valuation Model for American Options on Futures Contracts

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Abstract

Pricing models for American call and put options on futures contracts are derived herein. These models are used to investigate the efficiency of the market for options on Standard & Poor 500 and German Mark futures. The evidence presented here indicates that market prices for these options deviate substantially from their corresponding model prices. In addition, it is shown that a hedging strategy originated at prices that indicate a deviation of market from model is successful in translating the observed mispricing into excess profits after transactions costs. However, these net profits are eliminated if the origination of the strategy is delayed by one trade, or if bid-ask spreads are accounted for.

I. Introduction

One of the latest innovations in the financial markets that allows participants in the futures markets to hedge their exposure to adverse movements in spot and futures prices has been the option on futures contracts. Since the beginning of 1983, options on nine different futures contracts have been traded actively in the Chicago Board of Trade, the Commodity Exchange, the Chicago Mercantile Exchange, the New York Futures Exchange, and the Coffee, Sugar, and Cocoa Exchange. The underlying futures contracts include those on T-bonds, Gold, Silver, the NYSE Composite Index, the Standard and Poor's (S&P) 500, Sugar, the West German Mark, Soybeans, and Live Cattle.

The first author to deal with the valuation of options on futures contracts was Black [2]. He derives a pricing model for these options and shows that it is identical to the continuous dividend version of the Black-Scholes-Merton model with the risk-free rate replacing the continuous dividend yield. However, the Black model does not consider the early exercise feature that is attached to all traded options. The purpose of this paper is to derive and test a pricing model for American options on futures contracts, i.e., a model that accounts for a possible nonzero probability of early exercise. The valuation formula uses the techniques

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developed by Geske and Johnson [8] to price American options as a sequence of compound options.

The model discussed above is then used to test the efficiency of the market for options on S&P 500 and German Mark futures that are traded on the Chicago Mercantile Exchange. Empirical tests are performed to measure the extent to which market prices deviate from "equilibrium" (model) prices. In addition, this paper examines if a trading strategy can be used to translate any such deviations into positive excess (abnormal) profits.

The paper is organized as follows. Section II presents the analytic formulae and the polynomial approximation technique that is used to evaluate them. Section III describes the data used to examine the performance of the valuation models. Section IV discusses the variance estimators and the pricing biases of the models. Section V presents the empirical results of this study's tests of market efficiency and examines the ability of a trading strategy to produce excess profits when the option's market price deviates from its model price. These tests are performed under the assumption that the trading strategy can be originated immediately at the market price indicating a deviation from the model price. Section VI summarizes the results and presents the conclusions.

II. The Model

In this section, pricing models for American call and put options on futures contracts are presented. These options provide the holder with the right to buy/sell a prespecified amount of the underlying futures contract at the strike price before (or at) the maturity date. Under the assumptions of a constant risk-free rate, frictionless markets, and a lognormal distribution with a constant variance for the price of the underlying futures contract, Black [2] has shown that the partial differential equation that describes an option's equilibrium price path is given by

$$\frac{\delta V}{\delta t} = rV - \frac{1}{2}\sigma^2 F^2 \frac{\delta^2 V}{\delta F^2}, \tag{1}$$

where F is the underlying futures price,

r is the risk-free rate of interest,

σ^2 is the variance of the futures price, and

V is the value of the American option.

Equation (1) is subject to a variety of boundary conditions. These boundaries differentiate calls from puts, and American options from European options. Call option holders receive the maximum of the futures price minus the exercise price or zero at expiration,

$$C(F,0) = \max\ [0, F-X]\ , \tag{2}$$

and put option holders receive the maximum of the exercise price minus the futures price or zero at expiration,

(3) $$P(F,0) = \max\ [0, X-F]\ .$$

Since the options are American and can be exercised at any instant, equation (1) is subject to exercise conditions at each point in time t.[1] Thus if t^+ is the instant after, the value of the option at t would be equal to the maximum of the value of exercising it now, or its price at t^+. Therefore, the exercise boundary condition at any time t is given by

(4) $$C(F,t) = \max\ \left[F-X, C\left(F,t^+\right)\right] \text{ and}$$

(5) $$P(F,t) = \max\ \left[X-F, P\left(F,t^+\right)\right]\ .$$

The valuation equation for an American option is given by a function that solves equation (1) subject to the expiration and exercise boundary conditions—equations (2) and (4) for calls, and equations (3) and (5) for puts. Geske and Johnson [8] have shown that a solution (with infinitely many terms) exists to this partial differential equation when the free exercise boundary is considered to be an infinite set of discrete conditions.

The basic idea behind the Geske-Johnson approach is to value the option as if it can be exercised at discrete points in time. At each point in time, the holder of the call (put) receives the exercise value $F-X$ $(X-F)$ if the futures price is above (below) the critical futures price, and the option has not been exercised at any previous date. Thus, the American option can be considered as an infinite set of contingent claims. This approach implies that the American option can be valued as an infinite series of European compound options. Proceeding in this way, the value of an American call option on a futures contract is given by[2]

(6) $$C = FW_1 - XW_2,$$

where W_1 and W_2 are weighting factors comprised of infinite sums of the products of discount factors and conditional probability terms that reflect, at each instant, the present value of the exercise value conditioned on the probability that exercise did not occur at a previous instant.[3] Similarly, the value of a put option is given by

(7) $$P = XW_3 - FW_4,$$

where W_3 and W_4 are defined in a way similar to W_1 and W_2.

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[1] American options on futures contracts may be exercised early since the price of a European option converges to $(F-X)e^{-rt}$, a number less the exercise value $F-X$, as F becomes large.

[2] See [8] for the derivation of the model for equity puts, and [14] for an application of the Geske-Johnson model to foreign currency options.

[3] See the Appendix for detailed expressions for W_1 and W_2.

Equations (6) and (7) are complex in the form given above because they require evaluation of an infinite number of terms. Geske and Johnson [8] derive a technique based on Richardson's extrapolation to approximate the above infinite series by using a finite number of terms. This polynomial approximation involves using options that can only be exercised at a few discrete points in time to extrapolate to the price of an option that can be exercised at any date. The Geske-Johnson technique is more efficient than either finite differences or the binomial approximation because it involves the computation of only a few critical stock prices for penny accuracy. An equation using three critical stock prices for the evaluation of the analytic formula for American calls as defined in equation (6) is[4]

$$C = 1/2C_1 - 4C_2 + 9/2C_3, \tag{8}$$

where C_1, C_2, and C_3 are defined in the Appendix.

Similarly, a three point extrapolation for the American put equation is given by

$$P = 1/2P_1 - 4P_2 + 9/2P_3, \tag{9}$$

where P_1, P_2, and P_3 are similar in form to C_1, C_2, and C_3.

The inputs to the above models consist of five observable variables—the price of the underlying asset (the futures price F), the exercise price (X), the time to maturity (T), and the risk-free rate of interest (r), and one variable that is not observable, the volatility of the futures price (σ). The next section describes the sources used to collect data on the four observable variables, and the following section presents the techniques used here to obtain proxies for the unobservable volatility.

III. The Data

The tests of the models for options on futures contracts are based on data for the following two futures options that are traded on the Chicago Mercantile Exchange (CME)—options on S&P 500 Index Futures and options on West German Mark Futures. The data used consist of all transactions that took place on the floor of the CME for option maturities of June, September, and December 1983, and March, June, September, and December 1984 for S&P 500 Futures options and those of June, September, and December 1984, and March 1985 for German Mark Futures options. The data cover the period February 1, 1983, to September 30, 1984, for S&P futures options and February 1, 1984, to December 31, 1984, for German Mark futures options. These options transactions prices and the corresponding synchronous futures prices are obtained from a data base compiled by the CME. The data consist of a total of 76121 observations—44364 on S&P 500 futures options and 31757 on German Mark Futures options. However, the sample used here is smaller than this because of some deletions based on two criteria. First, to avoid problems associated with infrequent trading, all options are elimi-

[4] See [8] for more details.

nated that have less than 100 trades over their lifetime.[5] This criterion eliminates a total of 1687 observations from the sample. Second, 831 observations are deleted on call and put options that are deep out of the money, which are defined as options with prices less than five cents. These observations are eliminated because hedging strategies using these options are unrealistic since they require investment in 50 or more option contracts. This deletion procedure is presented in greater detail in Panel A of Table 1.

TABLE 1

Sample Classification

Panel A. Description of Deletions from Transactions Data

Underlying Security	Option Maturity	Observations in Original Data	Deletion Criteria: ≤ 100 trades[a]	Deletion Criteria: Price < 5¢	Deletions	% Deletions	Observations in Sample
S&P 500	8306	6506	131	48	179	2.75	6327
	8309	5916	157	18	175	2.96	5741
	8312	3912	242	6	248	6.34	3664
	8403	6619	133	92	225	3.40	6394
	8406	7972	184	50	234	3.15	7738
	8409	11000	70	27	97	0.88	10903
	8412	2439	127	0	127	5.21	2312
	TOTAL	44364	1044	241	1285	2.91	43079
German Marks	8406	8271	145	153	298	3.60	7973
	8409	6415	227	118	345	5.38	6070
	8412	12418	129	281	410	3.30	12008
	8503	4653	142	38	180	3.87	4473
	TOTAL	31757	643	590	1233	3.88	30524
Complete Sample		76121	1687	831	2518	3.31	73603

Panel B. Sample Breakdown by Option Type

Underlying Security	Option Maturity	Observations	Average Time between Future & Option Trade	Calls: Observations	Calls: Exercise Prices	Puts: Observations	Puts: Exercise Prices
S&P 500	8306	6327	16.07 secs	3372	5	2955	5
	8309	5741	25.46	3155	5	2586	4
	8312	3664	24.85	2008	4	1656	3
	8403	6394	21.94	3409	5	2985	5
	8406	7738	24.02	3559	5	4179	4
	8409	10903	17.39	6609	7	4294	6
	8412	2312	59.43	1412	5	900	5
	TOTAL	43079	23.03	23524	36	19555	32
German Marks	8406	7973	46.71	5601	6	2372	4
	8409	6070	72.98	4234	6	1836	4
	8412	12008	51.51	8866	8	3142	5
	8503	4473	144.68	3609	6	864	3
	TOTAL	30524	68.18	22310	26	8214	16
Complete Sample		73603	41.75	45834	62	27769	48

[a] Observations deleted based on this criterion are for options that have fewer than 100 trades over their lifetime.

[5] This deletion is based on the observation that hedging strategies involving such infrequently traded options would not be riskless since the holding period would be long.

The final sample consists of 73603 observations—45834 calls and 27769 puts. A breakdown of this sample by underlying security is provided in Panel B of Table 1. The above transactions data provide information on the futures price, the time to maturity on the option, and its exercise price. A fourth input required for the pricing of an option is the risk-free rate of interest. This risk-free rate is based on the yield of a Treasury bill with approximately the same maturity as the option being priced. The T-bill yield is obtained on a daily basis from *The Wall Street Journal*.

IV. Variance Estimation and Pricing Biases

A. Estimation of Exchange Rate Volatility

A key variable in the option pricing model defined in the earlier section is the volatility of the underlying futures contract. In this section, two alternative estimates are presented for the volatility of futures prices—a historical standard deviation (HSD) and an implied standard deviation (ISD).

The historical standard deviation (HSD) is estimated based on a sample of the 40 latest daily observations on futures prices and is calculated as follows,

$$\begin{aligned} HSD_t &= \left[\sum_{j=t-39}^{t-1} \left(R_j - \overline{R}\right)^2 \Big/ 38 \right]^{1/2} , \\ \text{where} \quad R_j &= \mathrm{Ln}\left[F(j)/F(j-1)\right] , \\ \overline{R} &= \sum_{j=t-39}^{t-1} R_j/39 , \\ \text{and} \quad F(j) &= \text{Futures Price on date } j . \end{aligned} \tag{10}$$

The other measure, ISD, is the standard deviation implied by actual option prices based upon the assumption that investors price options according to the models given by equations (6) and (7). Thus, option prices on day $(t-1)$ are used to obtain an implied standard deviation for date t. Although there are several techniques available to estimate the implied standard deviation, the minimum sum of squared errors technique is commonly used. Computational considerations preclude the use of this technique.[6] Instead the implied standard deviation derived from the option that is more sensitive to the specification of volatility is used.[7] In addition, since futures contracts of different maturities can exhibit different volatilities, a separate estimate of ISD is obtained for each future/options maturity. Finally, separate ISD estimates are obtained for call and put options to avoid inducing any interrelationships in the tests of the models for the two types of options.

[6] Given the complexity of the valuation equations and the size of the sample, the CPU time required for the computation of the implied standard deviation that minimizes the sum of squared deviations of market from model prices is exorbitant. For example, generating daily implied variances using only one option per futures contract required a total of approximately seven hours of CPU time.

[7] The options used to derive this estimate are the ones that are closest to being 'at the money,' i.e., those closest to $F/X = 1$ were picked.

Table 2 presents the means and standard deviations for HSDs and ISDs categorized by option type and maturity. As can be seen from this table, the means of both HSD and ISD vary considerably across option/futures maturities, but are fairly stable across option types. In addition, the results show that for this time period HSDs tend to be larger estimates of future price volatility as compared to ISDs.

TABLE 2

Summary Statistics for Estimates of Volatilities

Underlying Security	Option Maturity	Observations	HSD[a] Mean	HSD[a] SD[a]	ISD[a] Mean	ISD[a] SD[a]
Panel A. Calls						
S&P 500	8306	55	0.2012	0.0233	0.1507	0.0151
	8309	105	0.1944	0.0137	0.1598	0.0180
	8312	100	0.1582	0.0272	0.1371	0.0196
	8403	116	0.1381	0.0158	0.1358	0.0275
	8406	111	0.1537	0.0165	0.1275	0.0230
	8409	110	0.1712	0.0279	0.1478	0.0320
	8412	48	0.1963	0.0295	0.1525	0.0187
	TOTAL	645	0.1684	0.0309	0.1431	0.0259
German Marks	8406	90	0.1229	0.0067	0.1072	0.0124
	8409	131	0.1302	0.0155	0.1186	0.0126
	8412	122	0.1500	0.0244	0.1408	0.0218
	8503	79	0.1661	0.0227	0.1714	0.0150
	TOTAL	422	0.1327	0.0306	0.1325	0.0276
Panel B. Puts						
S&P 500	8306	55	0.2012	0.0233	0.1464	0.0275
	8309	101	0.1775	0.0129	0.1562	0.0214
	8312	88	0.1535	0.0260	0.1378	0.0282
	8403	95	0.1352	0.0130	0.1317	0.0361
	8406	108	0.1514	0.0149	0.1203	0.0263
	8409	111	0.1692	0.0269	0.1357	0.0261
	8412	64	0.1846	0.0385	0.1409	0.0160
	TOTAL	622	0.1645	0.0297	0.1375	0.0289
German Marks	8406	90	0.1229	0.0067	0.1063	0.0128
	8409	99	0.1261	0.0108	0.1176	0.0106
	8412	100	0.1538	0.0247	0.1469	0.0230
	8503	60	0.1684	0.0226	0.1687	0.0142
	TOTAL	349	0.1405	0.0251	0.1319	0.0280

[a] HSD = Historical Standard Deviation, ISD = Implied Standard Deviation, SD = Standard Deviation.

B. Comparison of Market and Model Option Prices

Table 3 presents the results of a comparison of put and call option market prices for each of the two estimates of volatility.[8] It can be observed from this table that for both measures of volatility, market prices deviate substantially from the corresponding model prices.

[8] The results reported here are for one standard contract. For S&P futures options, this implies multiplying the index price for 500, while for German Mark futures option the multiplicative factor is 1250.

For S&P 500 futures options, it is found that both measures of volatility tend to bias the model towards overvaluing call options.[9] The same bias is exhibited by HSD-based put option prices, while ISD-based puts tend to be undervalued. This indicates that HSD may be an upward-biased estimate of the true volatility of S&P futures prices.

The above pattern is completely reversed when the models are used to value options on German Mark futures. In particular, calls tend to be undervalued with HSD as well as ISD, while puts are undervalued with HSD but overvalued when ISD is used as a measure of volatility. Unlike S&P 500 futures options, the magnitude of the mispricing for these options is sensitive to the estimate of volatility used. Specifically, the use of ISD reduces the magnitude of the mispricing by approximately 66 percent. Here, it appears that HSD may be a downward-biased estimate of the true volatility of German Mark futures prices.

To test if the above deviations of market from model prices can be attributed to a new market, the overall sample is broken into two time periods, and the performances of the pricing model in the early part are compared with that in the later part. The mean magnitudes of the mispricing across the two subperiods indicate that the above patterns are stable over time, and therefore, cannot be attributed to inefficiencies in a new market.

To summarize, the extent and magnitude of mispricing shown in Table 3 may indicate inefficiency in these option markets and/or inaccuracy in the specification of the model. This proposition is subjected to further tests in Section V, where hedging strategies are used to determine whether an observed mispricing can be translated into arbitrage profits.

C. Tests for Biases in the Pricing Models

It has been well established that the Black-Scholes European option model exhibits systematic mispricing biases when used to value American call and put options (cf. [9], [10], [13], and [15]). This systematic mispricing is related to three different factors: the time to maturity of the option, the degree the option is in or out of the money, and the volatility of the underlying security. This systematic bias may be attributed to the fact that the probability of early exercise depends on these three factors (in addition to the interest rate and dividends). Thus, one would expect that these biases could be reduced or eliminated if an American option pricing model were used to obtain model prices.[10] Here the existence of time to maturity and exercise price biases are tested for in the pricing performance of the futures option pricing models developed in Section II.

Table 4 presents the results of these tests for both call and put options. The hypothesis that the model mispricing is linked to option maturity (T), and the degree to which the option is in the money or out of the money ($\ln F/X$) is tested using the following regression model,

$$\Delta_C = C_{mkt} - C_{mod} = a_0 + a_1 T + a_2 \mathrm{Ln}(F/X) \quad \text{and}$$
$$\Delta_p = P_{mkt} - P_{mod} = a_0 + a_1 T + a_2 \mathrm{Ln}(F/X) . \tag{11}$$

[9] An option is referred to as undervalued by the model if the market price is above the model price, i.e., $C_{mkt} > C_{mod}$ or $P_{mkt} > P_{mod}$.

[10] See [9] and [15] for empirical support of this proposition, and [13] for evidence against it.

TABLE 3

Comparison of Market and Model Prices[a]

Panel A. Calls

Underlying Security	Variance Estimate[b]	Time Period[c]	Observations[d]	Mean $\lvert\Delta\rvert$	Mean $\Delta > 0$	Mean $\Delta < 0$	% $\Delta > 0$	% $\Delta = 0$
S&P 500	HSD	I	23416	$143.90	$ 91.76	−$178.31	36.63	1.52
		II	8348	154.41	117.04	−177.65	34.58	1.28
		III	15068	138.07	78.93	−178.70	37.76	1.65
	ISD	I	23416	109.12	86.83	−126.34	37.25	1.98
		II	8348	92.39	108.77	−73.75	57.38	1.98
		III	15068	118.39	60.10	−142.81	26.10	1.98
German Marks	HSD	I	22232	114.04	129.26	−64.49	80.25	3.76
		II	8124	62.42	63.24	−74.78	66.09	6.33
		III	14108	143.77	157.68	−46.91	88.41	2.29
	ISD	I	22232	36.42	42.67	−38.19	58.42	11.48
		II	8124	33.52	38.27	−39.38	49.80	13.48
		III	14108	38.09	44.66	−37.23	63.38	10.33

Panel B. Puts

Underlying Security	Variance Estimate[b]	Time Period[c]	Observations[d]	Mean $\lvert\Delta\rvert$	Mean $\Delta > 0$	Mean $\Delta < 0$	% $\Delta > 0$	% $\Delta = 0$
S&P 500	HSD	I	19450	$115.68	$ 92.32	−$141.78	47.61	1.80
		II	6958	132.16	125.69	−143.03	51.52	1.37
		III	12492	106.50	71.23	−141.14	45.43	2.04
	ISD	I	19450	123.90	135.43	−106.12	68.15	2.07
		II	6958	142.66	161.35	−84.10	77.11	1.19
		III	12492	113.45	117.81	−113.89	63.16	2.56
German Marks	HSD	I	8166	86.99	105.08	−46.77	74.19	6.48
		II	3324	42.12	46.93	−46.63	61.94	10.08
		III	4842	117.80	135.01	−46.98	82.59	4.01
	ISD	I	8166	24.71	30.93	−28.97	44.37	17.70
		II	3324	24.66	31.25	−28.50	45.25	17.84
		III	4842	24.75	30.03	−29.28	43.76	17.60

[a] Δ represents the difference between market and model prices.
[b] HSD is historical standard deviation; ISD is implied standard deviation.
[c] For S&P 500 options, I refers to the period 2/1/83–9/30/84; II refers to 2/1/83–11/30/83; and III refers to 12/1/83–9/30/84. For German Mark Options, I refers to the period 2/1/84–12/31/84; II refers to 2/1/84–7/15/84; and III refers to 7/16/84–12/31/84.
[d] The number of observations here differs from that in Table 1 because all trades for the last two days in the sample are dropped to make these results compatible with those for the hedging strategy reported in subsequent tables.

Table 4 indicates that both time to maturity and exercise price biases exist for options on S&P 500 and German Mark futures for both measures of volatility. For S&P 500 futures options, it is observed that in the money put options are overvalued or less undervalued than those that are out of the money for both estimates of volatility. The exercise price bias for calls on S&P 500 futures depends on the measure of volatility used. HSD-based estimates of call prices exhibit the same bias as those for puts, while the bias reverses for ISD-based estimates. The time to maturity bias is consistent across option type, but varies with the volatility estimates. HSD biases the model toward overvaluing long maturity options and undervaluing short maturity options, while the opposite holds true with ISD-based estimates.

The exercise price biases for options on German Mark futures are similar to those for options on S&P 500 futures. In general, in the money options tend to be

TABLE 4

Time to Maturity and Exercise Price Biases in Pricing Models[a]

$$\Delta = a_0 + a_1 T + a_2 \text{Ln}(F/X)$$

Underlying Security	Variance Estimate[b]	Observations	a_0	$t(a_0)$	a_1	$t(a_1)$	a_2	$t(a_2)$	R^2
Panel A. Calls									
S&P 500	HSD	23524	0.062	13.13*	−1.547	−54.51*	−1.156	−13.32*	0.11
	ISD	23524	−0.139	−38.44*	0.386	17.78*	0.657	9.91*	0.02
German Marks	HSD	22310	0.016	12.28*	0.214	35.55*	−0.798	−46.50*	0.15
	ISD	22310	0.007	14.90*	−0.049	−22.13*	−0.602	−95.25*	0.29
Panel B. Puts									
S&P 500	HSD	19555	0.071	16.16*	−1.135	−41.32*	2.362	27.89*	0.09
	ISD	19555	0.055	11.55*	0.113	3.81*	2.703	29.64*	0.05
German Marks	HSD	8214	0.011	6.33*	0.290	27.64*	−0.119	−3.53*	0.09
	ISD	8214	0.003	3.83*	−0.011	−2.81*	0.128	10.35*	0.01

[a] Δ represents the difference between market and model prices, T is the option time to expiration, and F/X is the ratio of futures price to strike price.

[b] HSD is historical standard deviation; ISD is implied standard deviation.

* Indicates significance at the 5-percent level.

overvalued (or less undervalued) than those that are out of the money, except for ISD-based put prices, which exhibit an opposite bias. The time to maturity bias observed in this case is exactly the opposite of that observed for options on S&P 500 futures.

Finally, examination of the constant term (a_0) and its statistical significance indicates some other systematic biases may exist in the pricing models. The predominance of positive a_0s in Table 4 leads to the conclusion that the model tends to undervalue options even after exercise price and time to maturity biases are accounted for. The biases reported here for HSD-based option prices are identical to those reported for equity options in previous work with the one exception being the time to maturity bias exhibited by options on German Mark futures.

To conclude, in this section, it has been shown that the market prices of futures options deviate substantially from their corresponding American model prices for both historical and implied measures of futures price volatility. It was further found that the mispricing of these options is related to the time to expiration of the option and the degree to which the option is in or out of the money. This evidence may be viewed as support for the rejection of the hypotheses that the pricing models for American calls and puts as given in equations (6) and (7) are valid and/or that the market for these futures options is efficient.

The next section examines the question of market efficiency for these options in more detail by developing a trading strategy that could be used to exploit these pricing deviations.

V. Hedging Tests

To test for efficiency in these options markets, it is necessary to investigate whether any excess (abnormal) profit opportunities exist in the market. If these opportunities exist and persist over time, it could be argued that the market is inefficient for the period investigated. If such profitable opportunities do not materialize, then the market is efficient with respect to the trading rule used here.

Market efficiency is tested based on an *ex post* hedging strategy. The *ex post* trading rule assumes that the strategy can be originated at the market prices that indicate deviations from the corresponding model prices. The hedging strategy followed here is very similar to that used in earlier papers on equity options. Calls that are undervalued by the model (i.e., $C_{mkt} > C_{model}$) are written, and those that are overvalued by the model are brought along with a long or short position in the underlying futures contract, respectively. The number of calls for every futures contract in the above portfolio is assumed to be the reciprocal of the hedge ratio, i.e., $1/(\partial C/\partial F)$. The performance of this hedging strategy is examined for two possible liquidation points: a) one trade after execution, and b) two trades after execution.[11] The excess profits from these hedging strategies are calculated as the difference between the profit from the hedge portfolio and the amount that could have been earned by investing in a domestic bond.[12,13]

The results for the tests of the hedging strategy are presented in Tables 5 and 6. Table 5 reports the excess profits from a hedging strategy executed at transactions prices, and Table 6 presents the excess profits when the strategy is executed only if the option is mispriced by more than \$100.[14]

The profits reported in Table 5 indicate that for options on both futures contracts, the hedging strategy translates an observed deviation of market prices from their corresponding model prices into positive excess (abnormal) profits in over 50 percent of the cases. As a matter of fact, on the average, the strategy yields positive excess returns. The performance of the trading strategy is dependent on the measure of volatility used in the case of German Mark futures, where the ISD performs substantially better than HSD-based estimates in translating an observed mispricing into excess profits. However, both volatility estimates perform equally well when applied to S&P 500 futures and their corresponding options.

A comparison of the excess profits in Table 5 shows that they are on the average higher if the hedge is liquidated two trades after execution as compared to a liquidation at the next trade. This implies that delaying execution of the strategy by one trade would still result in positive excess profits before transactions costs.

The mean profits before transactions costs average \$17.88 for calls on S&P 500 futures and \$15.21 for the corresponding puts when ISD is used as a volatility estimate (Panel 1, Table 5), and the hedge is liquidated one trade after execution. These figures are marginally different for HSD-based estimates. The above profits are generated by buying/writing 3–5 options and futures contracts

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[11] The use of the two liquidation points provides information regarding the speed at which the market adjusts to eliminate the observed mispricing. In addition, the difference in the profits from the two liquidation points is equivalent to the profits from an *ex ante* strategy as defined in [6].

[12] This assumes that the portfolio when formed requires a cash outflow. If the portfolio results in a cash inflow, then the excess profits are based on the assumption that the proceeds are invested in a domestic bond.

[13] The hedging strategy for puts involves combining either a written undervalued put with a short position in the underlying security or a purchased overvalued put with a long position in the futures contract.

[14] See footnote 7.

TABLE 5

Excess Profits from Hedging Strategy

A. Calls

Underlying Security	Variance Estimate[a]	Mean |Δ|[b]	Number of Trades	# of Options per trade	Liquidation Time[c]: One Trade: Mean Profit	One Trade: % Positive	Two Trades: Mean Profit	Two Trades: % Positive
Panel 1: All Calls								
S&P 500	HSD	$146.12	23060	2.93	$15.16*	55	$17.30*	57
	ISD	111.33	22952	2.84	17.88*	56	20.14*	57
German Marks	HSD	118.50	21395	4.63	2.42*	53	2.33*	55
	ISD	41.15	19679	3.78	10.20*	58	12.28*	60
Panel 2: Undervalued Calls								
S&P 500	HSD	$ 91.76	8577	3.63	$22.85*	58	$26.93*	59
	ISD	86.83	8723	2.99	26.38*	59	30.38*	60
German Marks	HSD	129.26	17842	5.13	1.31	52	0.82	55
	ISD	42.67	12988	4.64	8.11*	56	9.82*	59
Panel 3: Overvalued Calls								
S&P 500	HSD	$178.31	14483	2.52	$10.60*	54	$11.60*	55
	ISD	126.34	14229	2.74	12.67*	55	13.86*	56
German Marks	HSD	64.49	3553	2.10	8.02*	57	9.93*	57
	ISD	38.19	6691	2.12	14.26*	61	17.07*	62

B. Puts

Underlying Security	Variance Estimate[a]	Mean |Δ|[b]	Number of Trades	# of Options per trade	Liquidation Time[c]: One Trade: Mean Profit	One Trade: % Positive	Two Trades: Mean Profit	Two Trades: % Positive
Panel 1: All Puts								
S&P 500	HSD	$117.80	19100	3.53	$16.17*	55	$21.33*	55
	ISD	126.52	19047	3.89	15.21*	55	19.22*	55
German Marks	HSD	93.02	7637	3.12	6.54*	54	8.39*	54
	ISD	30.03	6721	2.94	18.28*	62	22.33*	62
Panel 2: Undervalued Puts								
S&P 500	HSD	$ 92.32	9260	4.44	$19.57*	56	$26.89*	56
	ISD	135.43	13255	4.51	12.58*	54	16.45*	54
German Marks	HSD	105.08	6058	3.23	5.02*	53	6.75*	53
	ISD	30.93	3623	3.24	18.88*	62	24.64*	63
Panel 3: Overvalued Puts								
S&P 500	HSD	$141.78	9840	2.67	$12.96*	55	$16.09*	54
	ISD	106.12	5792	2.48	21.24*	58	25.66*	57
German Marks	HSD	46.77	1579	2.67	12.36*	60	14.68*	58
	ISD	28.97	3098	2.59	17.57*	63	19.63*	60

[a] HSD is historical standard deviation; ISD is implied standard deviation.
[b] Δ represents the difference between market and model prices.
[c] Results reported here are for situations in which the hedge is liquidated one and two trades after the execution of the hedging strategy.
* Represents significance at the 5-percent level.

per trade. Given a roundtrip transactions cost of $1 per contract, this implies that an average trade costs a maximum of $5.[15] Given the above mean profits, this translates into excess profits that are still positive and significant after inclusion of these costs. In addition, a glance at Table 5 shows that the above costs are

[15] It is assumed that this figure for transaction costs is representative of those faced by the floor members of the exchange.

sufficient to wipe out the excess profits if the strategy is executed one trade after the deviation is observed.

The corresponding profit figures before transactions costs for German Mark futures options are $10.20 for calls and $18.28 for puts when ISD is used as a volatility measure. This again represents opportunities for excess profits since transactions costs for the trading strategy are a maximum of $5. Here again, these profits disappear if execution of the trading strategy is delayed by one trade. Finally, unlike the results for options on S&P 500 futures, the profits using HSD-based estimates of model price are much lower than those obtained using ISD-based estimates. The profits, which average $2.42 for calls and $6.54 for puts, are probably not sufficient to cover transactions costs.

The above conclusions receive further support when the results of tests in which the hedging strategy is executed only if the deviation of market from model is more than $100 are viewed. Table 6 presents the results of these tests. The mean profits from the hedging strategy increase substantially with the implementation of the filter rule. In addition, as would be expected, the significance level of the excess returns increases when the hedging strategy is executed selectively.

The above tests indicate that the hedging strategy can yield abnormal profits, given full information on the prices at which the transactions are executed, and if the execution takes place at the prices that trigger the trading strategy. Since these profits are sufficient to cover transactions costs faced by floor members, these results indicate that the market for futures options may be inefficient from their viewpoint. Individual investors facing higher costs could also profit from these "inefficiencies" if they could selectively execute the hedging strategy at transactions prices. However, in practice, since an individual trading in this market will probably have to buy at offer prices and write/sell at the bid, statements regarding options market efficiency from their viewpoint would have to incorporate bid-ask spreads.[16] More specifically, if it is assumed that bid-ask spreads are $0.05 in terms of the quoted premium, all profits reported in Tables 5 and 6 would become negative.

VI. Summary and Conclusions

This paper presents an empirical test of a valuation model for American options on futures contracts. Using the technique developed by Geske and Johnson [8], these securities are priced as a sequence of compound options. Then, the model is used to measure the extent to which market prices differ from "equilibrium" values, and to test for the efficiency of the market for these options.

Substantial deviations of option market prices from their corresponding model prices were found. This mispricing is shown to be related to the time to maturity of the options and the degree to which the option is in or out of the money.

It is also shown that traders in this market for futures options can make significant abnormal profits if they can originate a hedging strategy at market transactions prices that indicate deviations from the corresponding model prices.

[16] Phillips and Smith [12] and Shastri and Tandon [14] have shown that many option market "inefficiencies" are illusionary since they fall within the bounds implied by transactions costs and bid-ask spreads.

TABLE 6

Excess Profits from Hedging Strategy
Strategy Executed Only if Option is Mispriced by $100 or More

A. Calls

					Liquidation Time[c]			
					One Trade		Two Trades	
Underlying Security	Variance Estimate[a]	Mean | Δ | [b]	Number of trades	# of Options per trade	Mean Profit	% Positive	Mean Profit	% Positive
Panel 1: All Calls								
S&P 500	HSD	$250.71	11153	2.53	$20.16*	57	$25.81*	58
	ISD	205.08	9419	2.23	24.32*	58	28.32*	59
German Marks	HSD	188.97	10279	4.62	7.74*	54	8.83*	56
	ISD	137.38	501	2.78	55.98*	70	73.20*	77
Panel 2: Undervalued Calls								
S&P 500	HSD	$176.16	2998	3.11	$38.77*	61	$50.68*	64
	ISD	182.86	2698	2.00	39.16*	62	46.61*	63
German Marks	HSD	191.03	9628	4.80	6.88*	54	7.76*	56
	ISD	135.38	260	3.60	60.45*	68	78.75*	77
Panel 3: Overvalued Calls								
S&P 500	HSD	$278.11	8155	2.32	$13.50*	55	$16.66*	56
	ISD	214.00	6721	2.32	18.36*	57	20.98*	58
German Marks	HSD	158.56	651	1.92	20.35*	64	24.78*	63
	ISD	139.52	241	1.78	51.15*	73	67.22*	78

B. Puts

Panel 1: All Puts								
S&P 500	HSD	$220.02	7807	3.17	$29.10*	58	$ 41.59*	59
	ISD	223.07	8401	4.01	26.29*	58	32.31*	58
German Marks	HSD	176.71	2813	3.03	7.40*	53	9.95*	53
	ISD	136.18	76	2.68	62.74*	71	73.05*	74
Panel 2. Undervalued Puts								
S&P 500	HSD	$195.26	3268	4.09	$43.39*	60	$ 61.77*	61
	ISD	219.26	6556	4.53	26.26*	57	32.81*	57
German Marks	HSD	177.94	2688	3.05	6.19*	52	7.76*	53
	ISD	140.23	55	2.87	61.15*	64	59.42*	69
Panel 3: Overvalued Puts								
S&P 500	HSD	$252.25	4539	2.52	$18.82*	56	$ 27.05*	57
	ISD	236.59	1845	2.19	26.40*	60	30.51*	59
German Marks	HSD	150.30	125	2.60	33.33*	67	56.95*	69
	ISD	125.60	21	2.19	66.89*	90	108.73*	86

[a] HSD is historical standard deviation; ISD is implied standard deviation.
[b] Δ represents the difference between market and model prices.
[c] Results reported here are for situations in which the hedge is liquidated one and two trades after the execution of the hedging strategy.
* Represents significance at the 5-percent level.

However, these profit opportunities are not sufficient to cover transaction costs if the strategy is originated one trade after the detection of the mispricing. Statements regarding the efficiency of the market would depend on how quickly traders can avail themselves of observed deviations of market from model prices.

Appendix

A. Definitions of W_1, and W_2

$$
\begin{aligned}
W_j = {} & e^{-rdt}N_1\left(d_{1j}\right) + e^{-r2dt}N_2\left(-d_{1j}, d_{2j}; -\rho_{12}\right) \\
& + e^{-r3dt}N_3\left(-d_{1j}, -d_{2j}, d_{3j}; \rho_{12}, -\rho_{13}, -\rho_{23}\right) \\
& + \ldots\ldots, \qquad j = 1,2;
\end{aligned}
$$

$$d_{i2} = \left[\mathrm{Ln}\left(F/\overline{F}_{idt} - 1/2\sigma^2 idt\right]\right/\sigma\, idt\ , \qquad i = 1,2,3,\ldots;$$

$$d_{i1} = d_{i2} + \sigma\, idt, \qquad i = 1,2,3,\ldots;$$

$$\rho_{ij} = \sqrt{i}/j, \qquad i,j = 1,2,3,\ldots; i \neq j;$$

$\overline{F}_{idt}$ = Critical Futures Price at idt, $\qquad i = 1,2,3, \ldots$; and

N_k = cumulative k-variate normal distribution.

B. Definitions of C_1, C_2, and C_3

The value of a call that can be exercised at time T only is

$$C_1 = Fe^{-rT}N_1\left(d_1 + \sigma\sqrt{T}\right) + Xe^{-rT}N_1\left(d_1\right),$$

$$\text{where} \quad d_1 = \left[\mathrm{Ln}(F/X) - 1/2\sigma^2 T\right]/\sigma\sqrt{T}\,.$$

The value of a call that can be exercised at time $T/2$ and at time T is

$$
\begin{aligned}
C_2 = {} & Fe^{-rT/2}N_1\left(d_2 + \sigma\sqrt{T/2}\right) \\
& + Fe^{-rT}N_2\left(-d_2 - \sigma\sqrt{T/2}\ , d_1 + \sigma\sqrt{T}\ ; -\sqrt{1/2}\right) \\
& - Xe^{-rT/2}N_1\left(d_2\right) - Xe^{-rT}N_2\left(-d_2, d_1; -\sqrt{1/2}\right),
\end{aligned}
$$

where $d_2 = [\mathrm{Ln}(F/\overline{F}_{T/2}) - 1/4\ \sigma^2 T]\ /\ \sigma\sqrt{T/2}$, and $\overline{F}_{T/2}$ solves $F - X = C_1(F, X, T/2, r, \sigma)$.

The value of a call that can be exercised at time $T/3$, $2T/3$, and T is

$$\begin{aligned} C_3 = {} & Fe^{-rT/3}N_1\left(d_3 + \sigma\sqrt{T/3}\right) \\ & + Fe^{-2rT/3}N_2\left(-d_3 - \sigma\sqrt{T/3}, d_4 + \sigma\sqrt{2T/3}; -\sqrt{1/2}\right) \\ & + Fe^{-rT}N_3\left(-d_3 - \sigma\sqrt{T/3}, -d_4 - \sigma\sqrt{2T/3}, d_1 \right. \\ & \left. + \sigma\sqrt{T}; \sqrt{1/2}, -\sqrt{1/3}, -\sqrt{2/3}\right) \\ & - Xe^{rT/3}N_1\left(d_3\right) - Xe^{-2rT/3}N_2\left(-d_3, d_4; -\sqrt{1/2}\right) \\ & - Xe^{-rT}N_3\left(-d_3, -d_4, d_1; \sqrt{1/2}, -\sqrt{1/3}, -\sqrt{2/3}\right) , \end{aligned}$$

where

$$d_3 = \left[\mathrm{Ln}\left(F/\overline{F}_{T/3}\right) - 1/6\ \sigma^2 T\right]\sigma\sqrt{T/3} ,$$

$$d_4 = \left[\mathrm{Ln}\left(F/\overline{F}_{2T/3}\right) - 1/3\ \sigma^2 T\right]\Big/\sigma\sqrt{2T/3} ,$$

$\overline{F}_{T/3}$ solves $F - X = C_2(F, X, 2T/3, r, \sigma)$, and

$\overline{F}_{2T/3}$ solves $F - X = C_1(F, X, T/3, r, \sigma)$.

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