Dan Galai
Hebrew University and University of California, Los Angeles

The Components of the Return from Hedging Options against Stocks*

The returns from hedging strategies with options are decomposed to a few components. Three elements are presented: first, the riskless return on the investment; second, the return from the discrete adjustment of the hedge; and third, the return from the change in the deviation of the actual price from the model price. The empirical analysis reveals that the contribution of the first two elements is quite small on average. The procedure to be suggested can be used in evaluating the performance of the investment in options.

Introduction

In a previous paper (Galai 1977) I estimated the profits that could have been made by following a hedging strategy which consisted of a position in an option traded on the Chicago Board Options Exchange (CBOE) and an offsetting position in a fraction of a share of the underlying stock. The paper concluded that: "1. The trading strategies based on the Black-Scholes model (Black and Scholes 1973) perform well in tests of the ex post hedge return. . . . 2. The market did not seem perfectly efficient to market makers. The ex ante returns, while usually statistically significant, show a strong tendency to be positive" (p. 195).

In this article the profits from the hedging strategies with options are further analyzed. The returns are decomposed to a few components in order to achieve a better understanding and additional insight into their character. Two main factors will be shown to combine to give the hedge return: the profits (or losses) from discrete ad-

* This paper is based on my Ph.D. dissertation, "Pricing of Options and the Efficiency of the Chicago Board Options Exchange," University of Chicago, 1975. This work was partially supported by a grant from CIVITAS Research Program in Regulation of Financial Markets, University of California, Los Angeles. I am indebted to Al Madansky and an anonymous referee for their helpful comments.

(*Journal of Business*, 1983, vol. 56, no. 1)

0021-9398/83/5601-0004$01.50

justments of the position and, second, the profits (or losses) from exploiting the changes in the differences between the actual price and the model price. The latter factor is the dominant one in the tests. The decomposition will help in making inferences about the quality of the Black-Scholes model and in evaluating the effects of the discrete daily adjustments of the hedged positions. Moreover, the procedure to be suggested can be used in evaluating the performance of the investment in options.[1]

Decomposition of the Hedge Return

The hedge position is based on daily closing prices. For each trading day $t - 1$ the value of the options is calculated according to the Black-Scholes model. The model price, C^M_{t-1}, is then compared to the actual closing price for the option, C^A_{t-1}. If the call is considered to be undervalued ($C^A_{t-1} < C^M_{t-1}$) it is assumed to be bought and, at the same time, a proportion $N(d_{1,t-1})$ of the underlying stock is sold.[2] Hence, the actual investment is given by

$$I^A_{t-1} \equiv C^A_{t-1} - N(d_{1,t-1})\, V_{t-1} \tag{1}$$

and the actual hedge dollar return by

$$R^A_{Ht} = (C^A_t - C^A_{t-1}) - N(d_{1,t-1})\,(V_t - V_{t-1}). \tag{2}$$

Define the following dollar returns:[3]

$$R_{Vt} \equiv V_t - V_{t-1};$$
$$R^A_{Ct} \equiv C^A_t - C^A_{t-1};$$
$$R^M_{Ct} \equiv C^M_t - C^M_{t-1};$$

and $G_t \equiv C^A_t - C^M_t$ will denote the difference between the actual and the model price of the option. By substituting these definitions in equation (2) the actual hedge return can be written as follows:

$$R^A_{Ht} = R^A_{Ct} - N(d_{1,t-1})\, R_{Vt} \tag{2'}$$
$$= R^M_{Ct} - N(d_{1,t-1})\, R_{Vt} + G_t - G_{t-1}$$
$$= R^M_{Ht} + R_{Gt}, \tag{2''}$$

where $R^M_{Ht} \equiv R^M_{Ct} - N(d_{1,t-1})\, R_{Vt}$ is the model hedge return, and $R_{Gt} \equiv G_t - G_{t-1}$ is the change in the deviation of the actual price of the option

1. It will be assumed that the reader is familiar with the methodology and testing procedures described in Galai (1977). The description of the data is also available in my previous paper. For a list of notation, see Appendix.
2. The term $N(d_{1,t})$ is a component of the Black-Scholes model and $(\partial C/\partial V) = N(d_1)$, where V is the value of the underlying stock (see Black and Scholes 1973; Galai 1977).
3. Superscripts A and M denote actual and model price, respectively.

from the model's value. Expression (2″) is the basis for the decomposition of the hedge return, which will be further analyzed below.[4]

The term R^M_{Ht} should be equal to the dollar return earned at the riskless interest rate on the model investment, $I^M_{t-1} = C^M_{t-1} - N(d_{1,t-1}) V_{t-1}$, if the time period between $t - 1$ and t is very small. This property is the basis for developing the Black-Scholes model (see Black and Scholes 1973, pp. 641–42). In notation form, the model hedge return can be expressed as follows:

$$R^M_{Ht} = I^M_{t-1}\, r\Delta t + \eta_t \tag{3}$$

where r is the riskless rate of interest, η_t is the residual return due to the risk associated with a discrete adjustment of the hedge position, and $\eta_t \rightarrow 0$ when $\Delta t \rightarrow 0$.

For adjustments made more infrequently, the volatility of η_t and therefore of R^M_{Ht} will be greater, because the chances of big price changes in the underlying stock over the (discrete) time period are greater. The effects of a discrete adjustment are due to discrete changes in the price of the underlying stock, R_{Vt}, and to the change of the time to maturity. The latter will cause losses if the option is held long, ceteris paribus, and will reduce the profit of the combined position of option and stock.[5] The effect of the change in the stock price can be seen to produce profits if the option is held long and the stock short.[6] The net effect is impossible to determine in general.

4. It should be noted that if the option is overvalued ($C^M_{t-1} < C^A_{t-1}$) and therefore being held short, the decomposition can be done in a similar way. Only the signs in the equation for R_{Ht} will be changed, and $R^A_{Ht} = -(R^M_{Ht} + R_{Gt})$.

5. The effect of the time to maturity is also a function of the variance of the stock rate of return. Hence, an error in estimating the variance may affect η through its relationship with the time to maturity.

6. This result can be read from the plot of C on V (see fig. 1). Ignoring the time factor, an increase in V will produce greater absolute change in C than in the stock holding (in proportion $N[d_1]$), while a decline in V will cause a smaller change in C than in the stock holding.

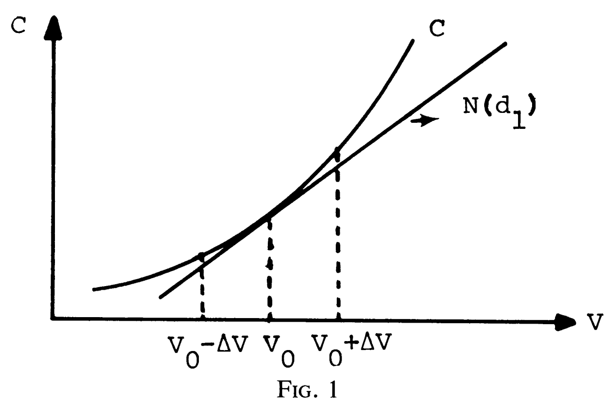

Fig. 1

The second component in the decomposition of the actual hedge return is R_{Gt}, the change in the deviation of the actual option price from the model's price. For undervalued (or overvalued) options, an increase (decrease) in the deviation from $t - 1$ to t ($G_t < G_{t-1}$) will increase the profits on the hedge position. If G_t remains constant for a certain period, the hedge return will not be affected by the deviation between the model and the actual price. This suggests that any systematic bias in the market or model prices of the option will be reduced substantially by canceling the effect of $G_t - G_{t-1}$. Only a change, over time, in the deviation will affect the return on the hedge.

This also suggests a way to analyze the results of Black and Scholes (1972). In their test of the model, they have imputed the returns on the option position for every day except the initial day. In the above notation, their procedure means that $G_t = 0$ for $t > t_0$ and therefore $R_{Gt} = G_t - G_{t-1} = 0$ for $t > t_0 + 1$. In calculating the average of the hedge return, they also include the term $G_{t_0} = C^M_{t_0} - C^A_{t_0}$. The only other source that will contribute to the average return is R^M_{Ht}, the model hedge return (including the opportunity cost). The results below from the decomposition analysis for the CBOE options reveal that the magnitude of the average model hedge return, $\bar{R}^M_H$, is relatively small. It might be the case that the average deviations from equilibrium pricing Black and Scholes report for the over-the-counter options are due mainly to G_{t_0}.

Let us look carefully at the characteristics of $\bar{R}_G$. On any given day R_{Gt} can take on any sign. But since at maturity $C^A_T = C^M_T$, as long as the hedge return is calculated up to the maturity date of the option, $\bar{R}^A_H$ will be greater than $\bar{R}^M_H$ and the average hedge return, $\bar{R}_G$, will be positive. (This explains why $\bar{R}_G > 0$ in table 1 for the July 1973 and October 1973 options. Indeed $\bar{R}_G$ was positive for all options with these expiration dates for each stock in our analysis, which ranged over the period April 26, 1973–November 30, 1973.)

But if the average hedge return is calculated up to a point t' in time prior to maturity, it may happen that $\bar{R}_G$ will be negative. This can only occur if the inequality between $C^M_{t_0}$ and $C^A_{t_0}$, on which the simulated hedge is based, never reverses in the period (t_0, t'), for example, if the option is continuously undervalued relative to the model price. Such a phenomenon occurs with the FNC option prices. Note that $\bar{R}_G < 0$ for FNC in table 2. But FNC data were available only for the January 1974 and April 1974 expirations, beginning with t_0 = November 1, 1973, and ending with t' = November 30, 1973, and the option was continuously undervalued relative to the model in that period.

It is easy to show otherwise that if the sign of the difference $C^A_{t'} - C^M_{t'}$ differs for some $t_0 < t' < T$ from that of $C^A_{t_0} - C^M_{t_0}$, $\bar{R}_G$ will be positive for that period. (Thus $\bar{R}_G > 0$ for the January 1974 and April 1974 expirations in table 1 and for all but FNC in table 2.)

TABLE 1 Decomposition of Average Hedge Return per Maturity with No Lag in Execution

Parameter	Maturity			
	July 1973	October 1973	January 1974	April 1974
$\bar{R}_H^A$	.100	.094	.094	.109
$\sigma(R_H^A)$	.494	.488	.589	.736
$\sigma(\bar{R}_H^A)$	.013	.007	.008	.011
$\bar{R}_H^M$	−.004	−.000	.000	.000
$\sigma(R_H^M)$	.097	.070	.071	.070
$\sigma(\bar{R}_H^M)$	.003	.001	.001	.001
$\bar{R}_G$	.104	.094	.094	.109
$\sigma(R_G)$	.494	.489	.588	.728
$\sigma(\bar{R}_G)$	.013	.007	.008	.011
Number of observations (N = 15, 397)	1,488	4,290	5,942	3,677
Number of options (N = 202)	28	46	66	62
Number of options with significant $\bar{R}_H^A$ (N = 71)	14	28	19	10

NOTE.—r = 10%, $\hat{\sigma}$ of Scholes (see Black and Scholes 1973).

The same technique of decomposing the hedge return can be applied to the ex ante return.[7] If on day t it is found that $C_{t-1}^M > C_{t-1}^A$, then the hedge return on $t + 1$ can be written as follows:

$$\begin{aligned} R_{Ht+1}^A &= (C_{t+1}^M - C_t^M) - N(d_{1,t-1})(V_{t+1} - V_t) + (G_{t+1} - G_t) \\ &= R_{Ht+1}^M + R_{Gt+1}; \end{aligned} \tag{4}$$

and the right-hand side of (4) will change signs if $C_{t-1}^M < C_{t-1}^A$. On the investment of $\pm\ [C_t^A - N(d_{1,t})V_t]$ on day t, we expect to yield the riskless rate of interest, approximately.

In the ex ante test, the discrete element is being strengthened as the model is being used on day $t - 1$ to establish a hedge, but the actual trading is taking place only on t (and the hedge is liquidated at $t + 1$). The implication of the discrete adjustment is ambiguous with regard to the mean, but we should expect the return on the model to fluctuate more widely for the ex ante test than for the ex post test.

The approach suggested above can be used to measure the performance of the option trader. The dollar daily return, $R_{Ct}^A \equiv C_t^A - C_{t-1}^A$, can be written as

$$R_{Ct}^A = R_{Ct}^M + R_{Gt}; \tag{5}$$

7. By the ex ante test it is meant that the hedging strategy is based on prices at $t - 1$ but the initial trading is based on prices at t which are unknown at $t - 1$. In the ex post tests the information on prices at which options and stocks can be bought or sold is given in advance (see Galai 1977).

TABLE 2 **Decomposition of Average Hedge Return per Stock without and with Lag in Execution**

	Without Lag			With Lag		
Stock	$\bar{R}_H^A$	$\bar{R}_H^M$	$\bar{R}_G$	$\bar{R}_H^A$	$\bar{R}_H^M$	$\bar{R}_G$
ARC	.195	−.000	.195	.105	.016	.089
AVP	.098	−.002	.101	.050	−.001	.051
BC	.071	−.001	.072	.020	−.007	.027
BS	.079	.004	.076	.042	.004	.038
EK	.153	−.007	.160	.080	−.002	.081
F	.019	.002	.016	.020	−.002	.022
FNC	−.002	.003	−.005	−.008	−.003	−.006
GW	.066	−.003	.069	.040	.003	.037
GWF	.039	.003	.039	.002	−.008	.010
HR	.038	.003	.035	.010	.000	.010
IBM	.780	.060	.720	.171	.030	.142
INA	.072	−.007	.078	.037	−.004	.041
ITT	.037	.005	.032	.013	−.008	.020
KG	.064	.004	.059	.014	.003	.010
KMG	.165	.003	.162	.088	−.009	.097
LTR	.079	−.006	.085	.009	−.002	.012
MCD	.096	−.016	.112	.072	−.017	.090
MMM	.095	.015	.080	.092	.225	.067
MRK	.088	.015	.073	.053	.008	.045
MTC	−.011	−.054	.043	.008	−.024	.031
NWA	.024	−.003	.027	.004	−.003	.007
PRD	.212	−.019	.231	.094	−.024	.119
PZL	.043	.002	.041	.011	.003	.009
RCA	.024	.005	.019	.011	.001	.009
S	.097	.026	.071	.024	.019	.004
SY	.101	−.001	.101	.062	−.000	.062
T	−.002	−.026	.024	.004	−.015	.018
TXN	.237	.015	.222	.131	.017	.115
UPJ	.165	−.005	.170	.095	−.012	.107
WY	.151	.016	.135	.093	.008	.085
XON	.069	.001	.069	.043	−.001	.044
XRX	.170	−.001	.179	.121	−.007	.128
Average	.098	.000	.098	.050	−.001	.051
SD	.600	.073	.598	.604	.105	.597

NOTE.—r = 10%, $\hat{\sigma}$ of Scholes.

by using the definition of R_{Ht}^M it can also be written as

$$\begin{aligned} R_{Ct}^A &= R_{Ht}^M + R_{Gt} + N(d_{1,t-1})\,R_{Vt} \\ &= I_{t-1}^M r\Delta t + \eta_t + R_{Gt} + N(d_{1,t-1})\,R_{Vt}. \end{aligned} \tag{5'}$$

Equation (5) shows how much of the return is expected, given the changes in the price of the underlying stock and in the time to maturity, and what part is due to finding a mispriced option. Equation (5′) gives a finer decomposition of the option return. The first component is the riskless return on the model investment. The second gives the compensation for not eliminating the option's risk continuously. The third component, R_{Gt}, measures the ability to find an under- or overvalued option. The last term on the right-hand side of (5′) is the part which is due to the change in value of the underlying stock and as such can indicate the ability of the option buyer to select the right stock. The term R_{Vt} can be further analyzed along the lines suggested by Fama (1972).

The Empirical Results

Table 1 corresponds to panel C of table 1 in Galai (1977). Here the per maturity $\bar{R}_H^A$ of the ex post test is decomposed. In the estimation I have used 10% for the riskless interest rate and the estimates of variance supplied by Scholes. The table contains the estimates of the averages $\bar{R}_H^A$, $\bar{R}_H^M$, and $\bar{R}_G$,[8] their standard deviations, and the standard deviation of the distribution of the returns on all hedges for a given maturity. The results strongly show that the distribution of $\bar{R}_H^M$ is not as dispersed as that for $\bar{R}_H^A$ and its mean is much lower than for the latter—it is very close to zero.

The average $\bar{R}_H^M$ will deviate from its expected value, the riskless interest rate on the investment, if the discrete adjustment affects its systematic risk or if the hedge portfolio has nonzero systematic risk due to errors in estimating the variance of the stock's rate of return. The results indicate that the effect of the discreteness of the adjustment is negligible. If it can also be established that the systematic risk of $\bar{R}_H^M$ is zero, then the hypothesis that the parameters of the model are well specified will gain support.

The effect of aggregation over different options can be partially evaluated by inspecting the information in table 2. In table 2, the decomposition is on a firm basis and, therefore, still contains aggregation over the several options written on the stocks of the specific firm. While $\bar{R}_{Hi}^M$ per company i is not as close to zero as the grand mean, it is still small relative to $\bar{R}_{Hi}^A$. Looking at the time series of R_{Hij}^M for each option j (on stock i) will reveal that most of the model profits were insignificantly different from zero. Further decomposition of $\bar{R}_H^M$, along the lines of equation (3), reveals that the contribution to its mean is due mainly to the imputed interest rate on the average model investment, $\bar{I}\, r\, \Delta t$.[9] The average residual return, $\bar{\eta}$, which is associated with the risk

8. Bars will denote averages, e.g., $\bar{R}_{Hij}^A$ is the time-series average of the hedge return for option j on stock i; $\bar{R}_{Hi}^A$ is the cross-sectional average of the average hedge return for all options on stock i; $\bar{R}_H^A$ is the cross-sectional average over all options on all stocks.

9. For the options in the sample, in no case is the actual average investment per hedge above \$6,000 (or below −\$6,000). At 10% annual interest rate, the daily interest expenses on an investment of \$6,000 is \$1.60. Most options had an average investment of about \$1,000–\$2,000, which implies a cost of \$0.20 or \$0.40 per hedge. These numbers

TABLE 3 Decomposition of Average Hedge Return per Maturity with Lag in Execution

Parameter	Maturity			
	July 1973	October 1973	January 1974	April 1974
$\bar{R}_H^A$	.053	.040	.052	.049
$\sigma(R_H^A)$	.492	.484	.589	.751
$\sigma(\bar{R}_H^A)$	.013	.007	.008	.012
$\bar{R}_H^M$	−.010	−.002	.001	−.001
$\sigma(R_H^M)$	.136	.105	.101	.097
$\sigma(\bar{R}_H^M)$	.004	.002	.001	.002
$\bar{R}_G$	.062	.043	.052	.050
$\sigma(R_G)$	.488	.484	.586	.740
$\sigma(\bar{R}_G)$	.013	.007	.008	.012
Number of observations	1,460	4,244	5,876	3,612
Number of options (N = 20)	28	46	66	61
Number of options with significant $\bar{R}_H^A$ (N = 12)	1	3	7	1

NOTE.—r = 10%, $\hat{\sigma}$ of Scholes.

of the discrete adjustment in the hedge position, is very close to zero; its volatility, however, determines that of $\bar{R}_H^M$.

It becomes clear from the analysis that the main contributor to $\bar{R}_H^A$ is the $\bar{R}_G$. The opportunity cost and the returns on the discrete adjustment are marginal. This result is important for two reasons. First, it shows that discrete adjustments at one (trading) day intervals apparently do not affect the returns in a significant way and can thus be regarded as operational for hedging activity. Second, given the validity of the model and the accuracy of the estimated parameters, the ability of the trading rule, which is based on the model, to find ex post "winners" is emphasized, as this ability is described fully by $\bar{R}_G$. The rule was able, on average, to capture the changes in G_t by buying undervalued or selling overvalued options and hedging the option position by trading in stocks in accordance with $N(d_1)$.

Equation (4), which is the basis for the decomposition of the ex ante hedge return, was also empirically estimated. A comparison of the results in table 3 with those in table 1 confirms our expectations. While the volatility of R_H^A is approximately the same for both tables, the volatility of R_H^M is much higher for the ex ante test. But still, the average of the model hedge return is very small and close to zero. The most

should be compared to the $10 average daily hedge return per contract. Note, however, that daily fluctuations were greater and for a few options some big investments exceeding $8,000 were observed. In 581 observations out of more than 16,000 the investment exceeded ±$8,000.

noticeable change is the decline in $\bar{R}_{Gi}$ for all firms, except for one (F). These results suggest that the model is less good in finding ex ante profits but it is still able to select "winning" positions on average.[10]

The standard deviation of $\bar{R}_H^A$ suggests that the positions lost substantially on some days and gained substantially on others. The volatility of R_H^A is approximately the same in absolute terms for the ex post and ex ante tests, but the mean for the latter is about half of the former. Therefore, the coefficient of variation is about twice as much for the latter as for the former. The significance of the averages declines, and this is noticeable especially when applied to the time series of R_{Hjt}^A for each option j. The number of significant averages falls from 71 for the ex post test to 12 for the ex ante test.

Summary and Conclusions

To sum up, the decomposition analysis reveals the interplay of different factors that contribute to the return on hedging activity. Three elements were presented: first, the riskless return on the investment; second, the return from discrete adjustment of the hedge; and third, the return from the *change* in the deviation of the actual premium from the model premium. The empirical analysis revealed that the contribution of the first two elements was quite small, on average, for both the ex post and ex ante tests. In general, the effect of increasing the holding period on the mean of R_H^M cannot be assessed, but we expect its volatility to increase with the holding period. This hypothesis was confirmed by the evidence. The effect of the riskless return was found to be small on average because the averaging is done while we allow changes in the position, so that a net long position is held for some part of the period and net short positions for the rest of the period. By aggregating over different options, a similar effect of canceling long and short positions will be found. It is possible to point out the factor by which the model's performance should be evaluated. This third element, $\bar{R}_G$, was found to be the dominant one in determining $\bar{R}_H^A$ for both types of tests but was much smaller for the ex ante test versus the ex post test.

The decomposition can be extended to analyze spread positions, whereby a position in one option is offset by holding an opposite position in another option on the same underlying stock but with different maturity and/or striking price (see Galai 1975, pp. 135–40). Moreover, the decomposition can be very helpful in analyzing the returns realized by the options trader whether he holds a hedge, spread, or naked position in the option. The performance analysis for options can be coupled with the one suggested for common stocks by Fama (1972) in

10. The test corresponds to panel C of table 4 in Galai (1977).

order to separate further the elements that contributed to the performance of the trader.

Appendix

Notation Used in This Work

K = striking price;
σ^2 = variance of the stock's rate of return distribution;
r = riskless interest rate;
C_t = price of the option on day t;
C_t^A = realized price of the option on day t;
C_t^M = model price of the option on day t;
T = maturity date;
τ = time to maturity ($= T - t$);
$N(\cdot)$ = cumulative standard normal frequency distribution;
V_t = price of the stock on day t;
R_{Ht} = profit on the hedged position at t;
R_{Ht}^A = realized profit on the hedged position at t;
R_{Ht}^M = model profit;
R_{Ht}^F = opportunity cost based on riskless interest rate r;
R_{Vt} = return on the security ($= V_t - V_{t-1}$);
R_{Ct}^A = realized return on the option ($= C_t^A - C_{t-1}^A$);
R_{Ct}^M = model return on the option ($= C_t^M - C_{t-1}^M$);
I_t = investment on the hedged position on day t;
G_t = deviation of the actual option price from the model price ($= C_t^M - C_t^A$);
R_{Gt} = change in the deviation ($= G_t - G_{t-1}$).

Additional superscripts or subscripts will be added when necessary to indicate time period t or firm i, etc.

References

Black, F., and Scholes, M. 1972. The valuation of option contracts and a test of market efficiency. *Journal of Finance* 27 (May): 399–417.

Black, F., and Scholes, M. 1973. The pricing of options and corporate liabilities. *Journal of Political Economy* 81 (May–June): 637–54.

Fama, E. F. 1972. Components of investment performance. *Journal of Finance* 27 (June): 551–67.

Galai, Dan. 1975. Pricing of options and the efficiency of the Chicago Board Options Exchange. Ph.D. dissertation. University of Chicago.

Galai, Dan. 1977. Test of market efficiency of the Chicago Board Options Exchange. *Journal of Business* 50 (April): 167–97.