

(c) Use the boundary condition $P_T = 1$ to find a strong solution for P_t in terms of $(r_s, B_s, t \leq s \leq T)$.

(d) The yield-to-maturity is defined as

$$R_t = \frac{-\log P_t}{T-t}.$$

Find the SDE for R_t . Explain the sign (plus or minus) of the volatility, as well as its magnitude as a function of $(T-t)$.

(e) Explain how you would generate an Euler approximation for P_t , given values for $(r_{t_i}, B_{t_i}, i = 0, 1, \dots, n)$.

(f) Find the SDE for the discounted bond price $Z_t = P_t/\beta_t$, where $\beta_t = \exp\left(\int_0^t r_s ds\right)$, and show that it is a martingale.