## Assignment 1 for Stochastic Calculus

This an exercise in the use of binomial tree methods. The solution to this exercise should be handed in on or before Thursday February 3. You may either provide a printed version of your calculations or an Excel file which does the calculations (via e-mail, peterb@fee.uva.nl).

1. Obtain, from the newspaper or the internet, the price of a (European) put option $P_0$ on the AEX index, as well as the level $S_0$ of the AEX index for that same day. Note the strike price $K$ for the option (choose an option that is in the money, $K > S_0$), and calculate the time to expiration $T$, measured in years. Assume that the continuously compounded interest rate is $r = \ln(1 + EURIBOR_0)$, where $EURIBOR_0$ is the 1-year EURO InterBank Offered Rate for the same day. Finally, the volatility $\sigma$ may be taken as 0.18, which is about the average historical volatility of the AEX index since its start in 1983.

2. Set up a one-period binomial branch model, to obtain a possible value of the put option (to be compared with its market value $P_0$). That is, obtain (from the historical volatility) the factors $u$ and $d = 1/u$, and calculate the risk-neutral probability $q$. What is the value of the option?

3. Now extend this to a binomial tree with 4 periods, i.e., $t = 0, \delta, 2\delta, 3\delta, 4\delta = T$. Adjust $d$, $u$, and $q$ to this new situation. Set up the binomial tree for the stock price, and derive from that the tree for the option value.

4. Determine, at each time period $i = 1, 2, 3, 4$, the appropriate portfolio weights $(\phi_i, \psi_i)$, of the AEX index and a bank account paying $r$, respectively, such that the resulting portfolio replicates the payoff of the option.

5. All options on the AEX index are European-type options. Suppose, however, that we wish to value an American put option on the AEX index with the same strike price and time to expiration as before. Use the same 4-period binomial tree to value this put option.

6. Use the Black-Scholes formula to find the value of the (European) option. You may find that this value, and also the values found under 2. and 3., differ substantially from the market value $P_0$. One of the most important reasons for this is that the volatility over the period $[0, T]$ is not equal to the historical volatility. Calculate the *implied volatility*, i.e., that value of $\sigma$ for which the Black-Scholes formula coincides with the market value $P_0$ (this may be done e.g. using the solver in Excel, or simply by trial and error).

7. Answer questions 3 and 5 again using the implied volatility instead of the historical volatility.