Chapter 18

Volatility Smiles

Implied Volatilities

- Implied volatilities are predictions on future volatilities derived from market option prices

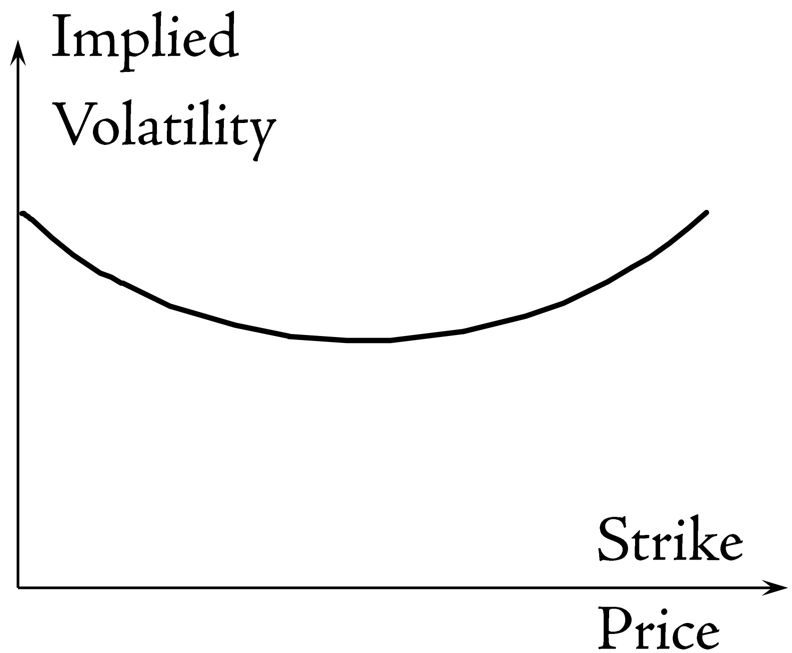
- In theory, the implied volatilities should be equal
  - for calls and puts with the same X and T-t
  - for different strike prices

  and is possible to be different for different maturities.

Volatility Smile

- It is the relationship between implied volatility and strike price for options with a certain maturity.
- A volatility smile shows the variation of the implied volatility with the strike price

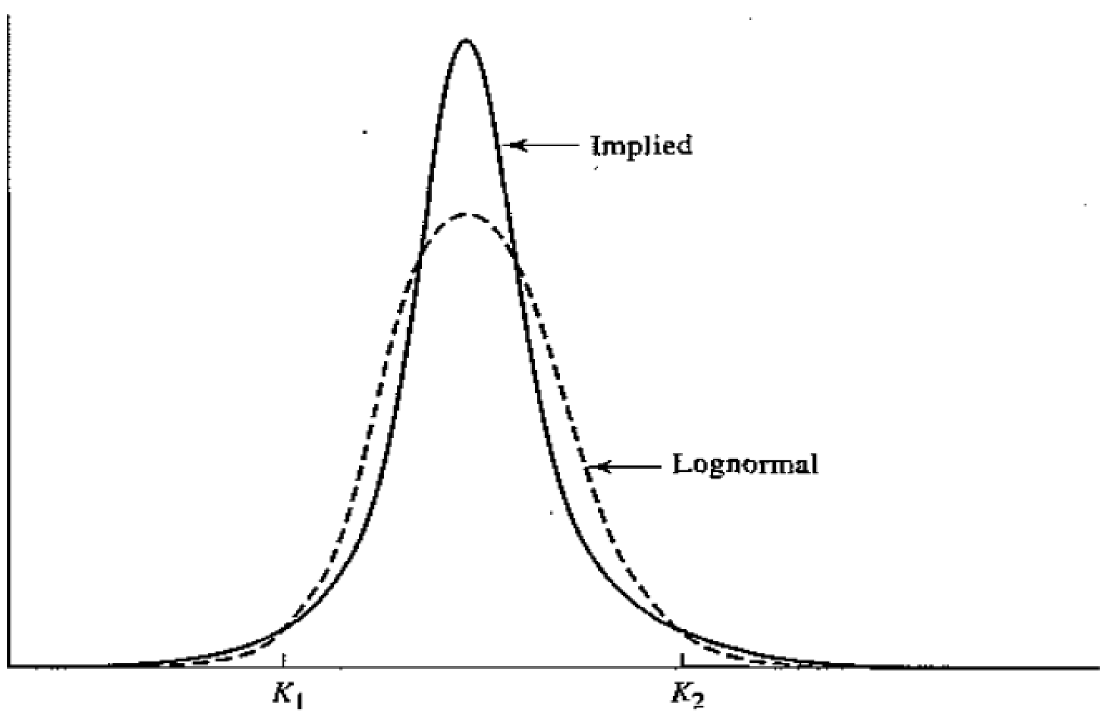
The Volatility Smile for Foreign Currency Options



Implied Distribution for Foreign Currency Options

- Both tails are heavier than the lognormal distribution
- It is also more peaked than the lognormal distribution

Figure 16.2 Implied and lognormal distribution for foreign currency options.



Historical Analysis of Exchange Rate Changes

	Real World (%)	Normal Model (%)
>1 SD	25.04	31.73
>2SD	5.27	4.55
>3SD	1.34	0.27
>4SD	0.29	0.01
>5SD	0.08	0.00
>6SD	0.03	0.00

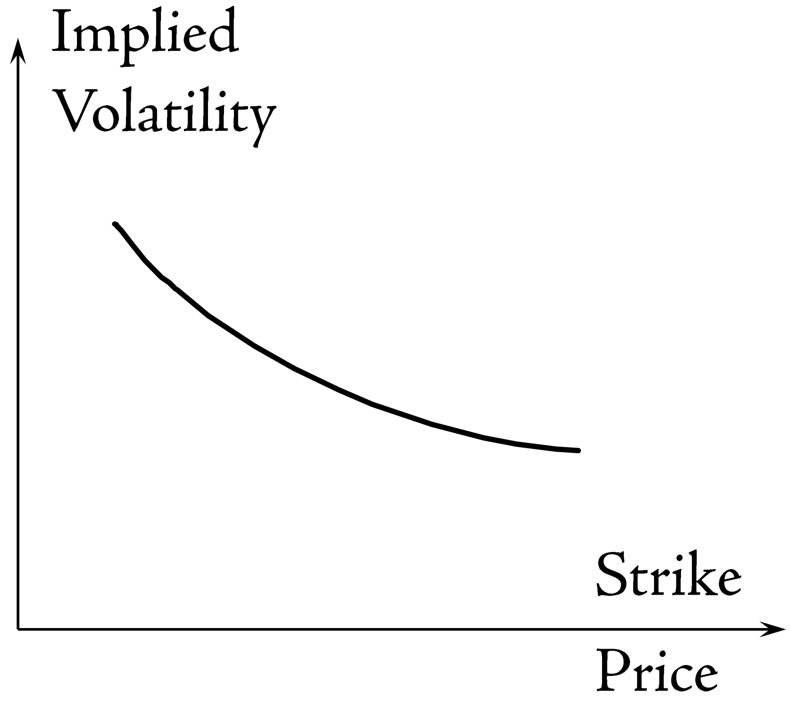
How to Make Money from VS?

- Business Snapshot 18.1

Possible Causes of Volatility Smile for Foreign Currencies

- Exchange rate exhibits jumps rather than continuous changes
- Volatility of exchange rate is stochastic

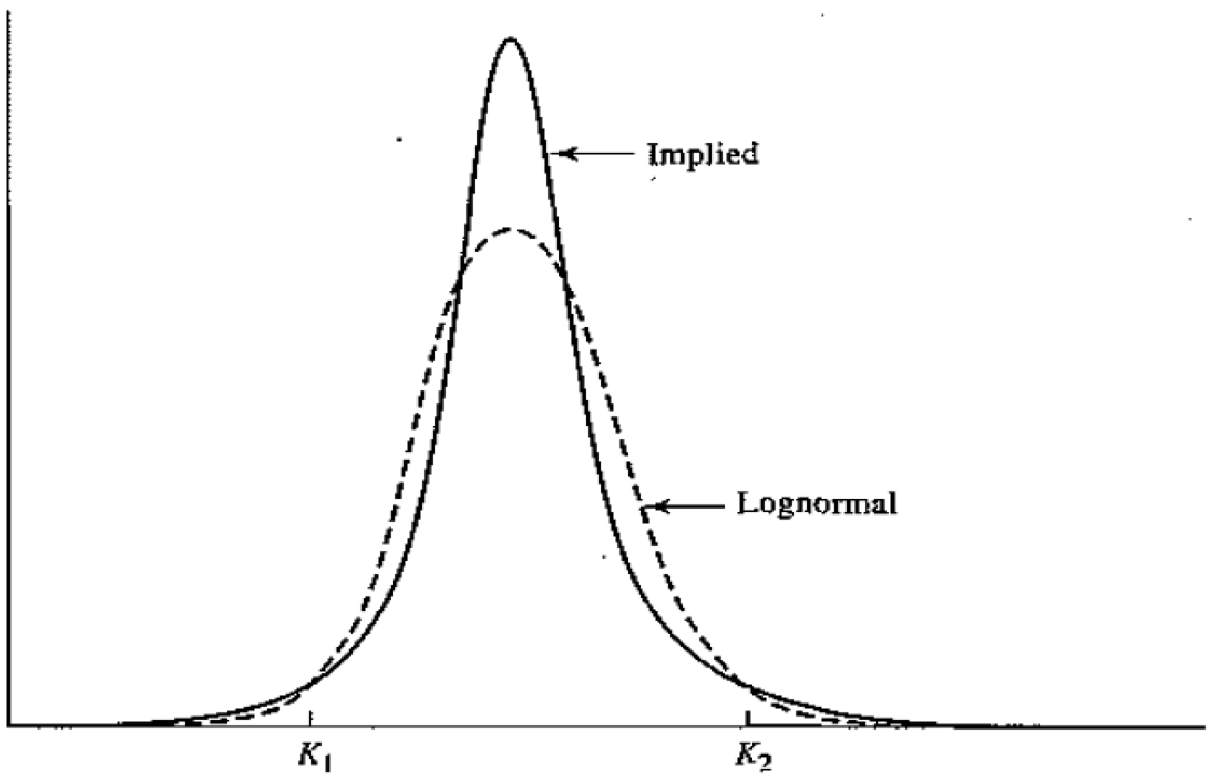
The Volatility Skew for Equity Options



Implied Distribution for Equity Options

- The left tail is heavier than the lognormal distribution
- The right tail is less heavy than the lognormal distribution

Figure 16.2 Implied and lognormal distribution for foreign currency options.



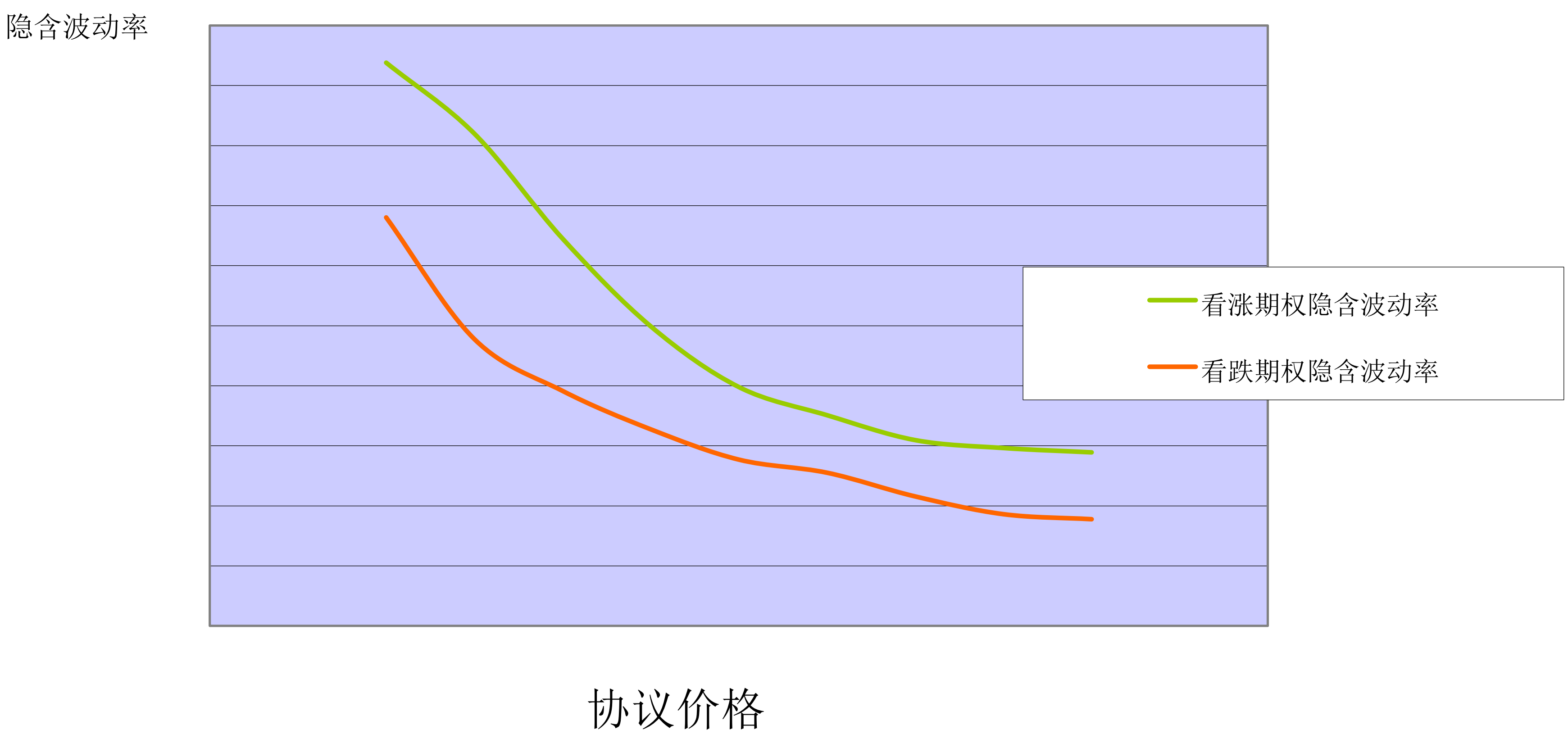
Reasons for Smile in Equity Options

- Leverage
- Crashophobia

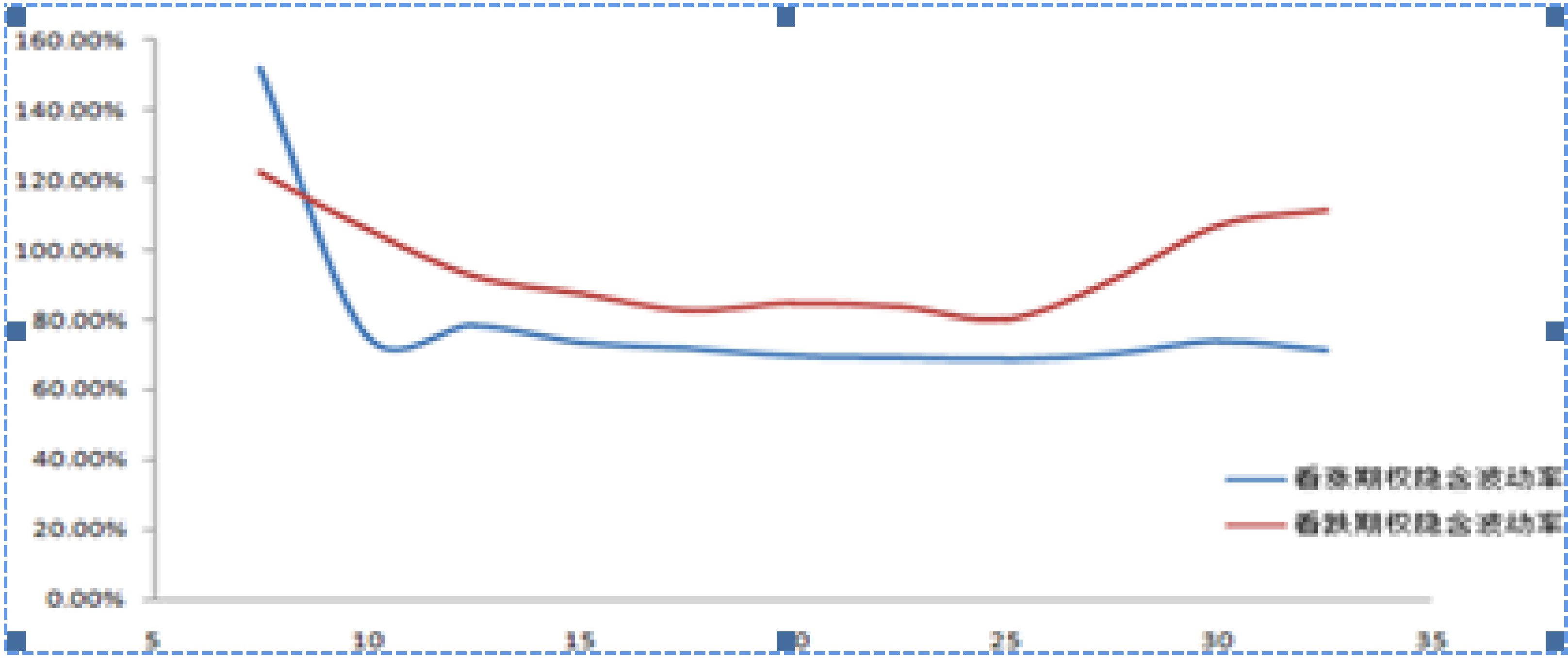
Other Volatility Smiles?

- What is the volatility smile if
  - True distribution has a less heavy left tail and heavier right tail
  - True distribution has both a less heavy left tail and a less heavy right tail

Volatility Skew for Yahoo Stock Options



Volatility Skew for Citi Stock Options



协议价格

图 10　2008 年 10 月 20 日花旗银行股票的波动率微笑

注：当天股价等于 15.09 美元。

资料来源：作者计算。

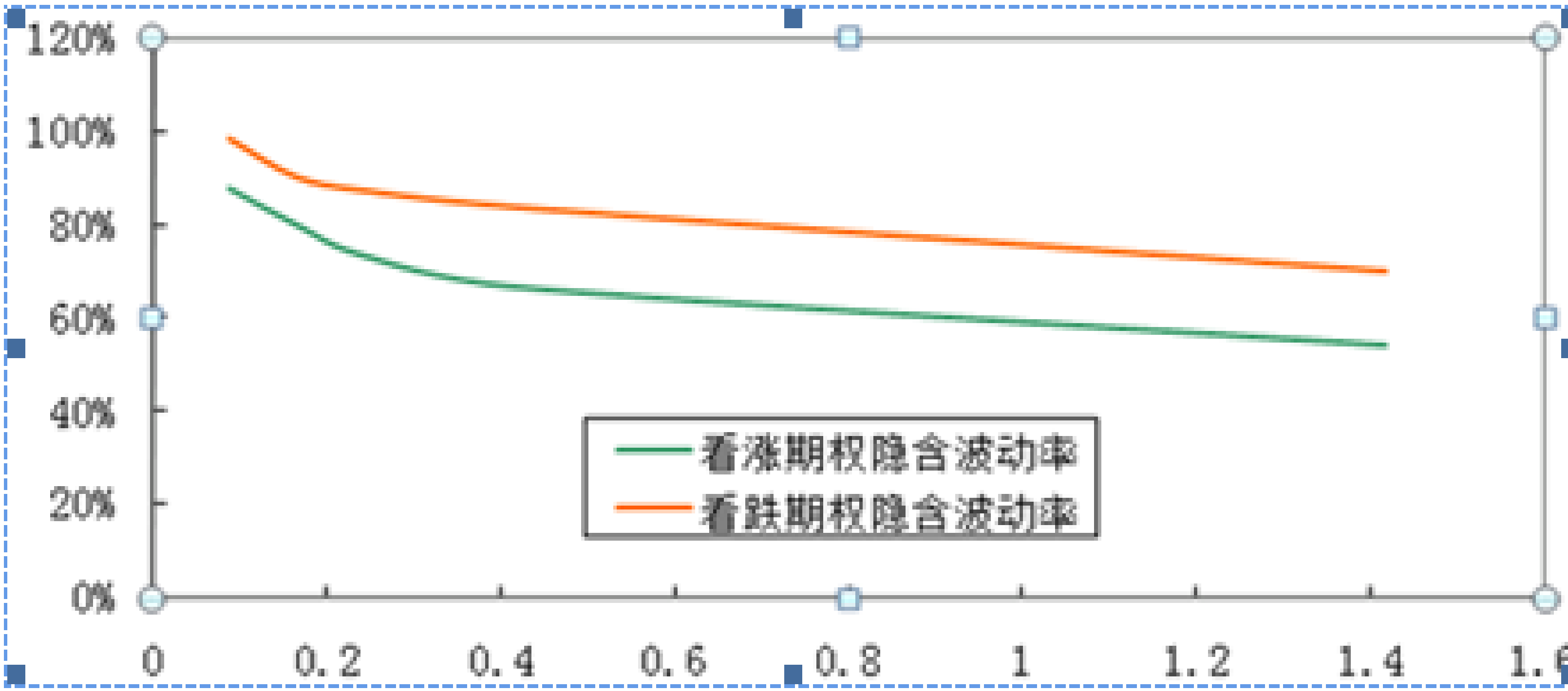
Ways of Characterizing the Volatility Smiles

- Plot implied volatility against K/S_0
- Plot implied volatility against K/F_0
  - Note: traders frequently define an option as at-the-money when K equals the forward price, F_0, not when it equals the spot price S_0
- Plot implied volatility against delta of the option
  - Note: traders sometimes define at-the money as a call with a delta of 0.5 or a put with a delta of –0.5. These are referred to as "50-delta options"

Volatility Term Structure

- In addition to calculating a volatility smile, traders also calculate a volatility term structure
- This shows the variation of implied volatility with the time to maturity of the option
- The volatility term structure tends to be downward sloping when volatility is high and upward sloping when it is low

Volatility Term Structure for Citi Stock Options



期限（年）

图 11　花旗银行股票波动率的期限结构（2008 年 10 月 20 日）

注：协议价格为 15 美元，当天股价为 15.09 美元。

Volatility Surface

- The implied volatility as a function of the strike price and time to maturity is known as a volatility surface

Example of a Volatility Surface

	Strike Price				
	0.90	0.95	1.00	1.05	1.10
1 mnth	14.2	13.0	12.0	13.1	14.5
3 mnth	14.0	13.0	12.0	13.1	14.2
6 mnth	14.1	13.3	12.5	13.4	14.3
1 year	14.7	14.0	13.5	14.0	14.8
2 year	15.0	14.4	14.0	14.5	15.1
5 year	14.8	14.6	14.4	14.7	15.0

Volatility Smiles When a Large Jump is Expected

- At the money implied volatilities are higher than that in-the-money or out-of-the-money options (so that the smile is a frown!)

Determining the Implied Distribution

$$c = e^{-rT} \int_{S_T = K}^{\infty} (S_T - K) g(S_T) dS_T$$

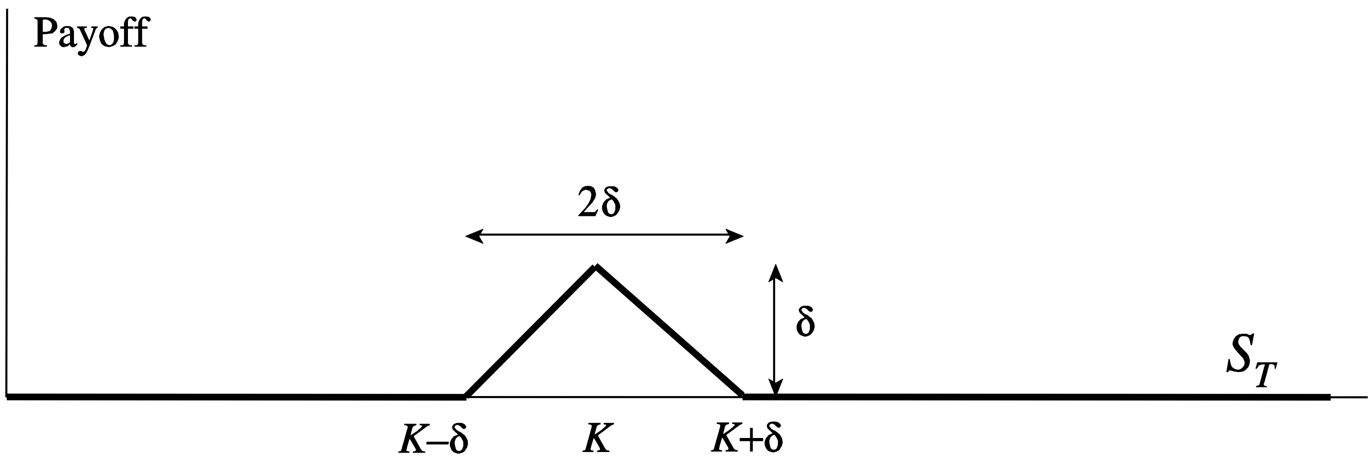
$$\frac{\partial^2 c}{\partial K^2} = e^{-rT} g(K)$$

If $c_1, c_2,$ and c_3 are call prices for strikes

$K - \delta, K,$ and $K + \delta$ then

$$g(K) = e^{rT} \frac{c_1 + c_3 - 2c_2}{\delta^2}$$

A Geometric Interpretation



Assuming that density is $g(K)$ from $K-\delta$ to $K+\delta$, $c_1 + c_3 - c_2 = e^{-rT} \delta^2 g(K)$

Model-free Implied Volatilities

- 无模型隐含波动率：
  - Demeterfi，Derman，Kamal and Zou（1999）
  - Britten-Jones and Neuberger（2000）
  - VIX(2003)
  - Jiang andTian(2007)：上述的等价性

Model-free Implied Volatilities

- Demeterfi， Derman， Kamal and Zou（1999）

$$E^F(Var)=\frac{2}{T}\left\{rT-[\frac{S_0}{S_*}e^{rT}-1]-\ln\frac{S_*}{S_0}+e^{rT}\int_0^{S_*}\frac{1}{K^2}P(K)dK+e^{rT}\int_{S_*}^{\infty}\frac{1}{K^2}C(K)dK\right\}$$

- Britten-Jones and Neuberger（2000）

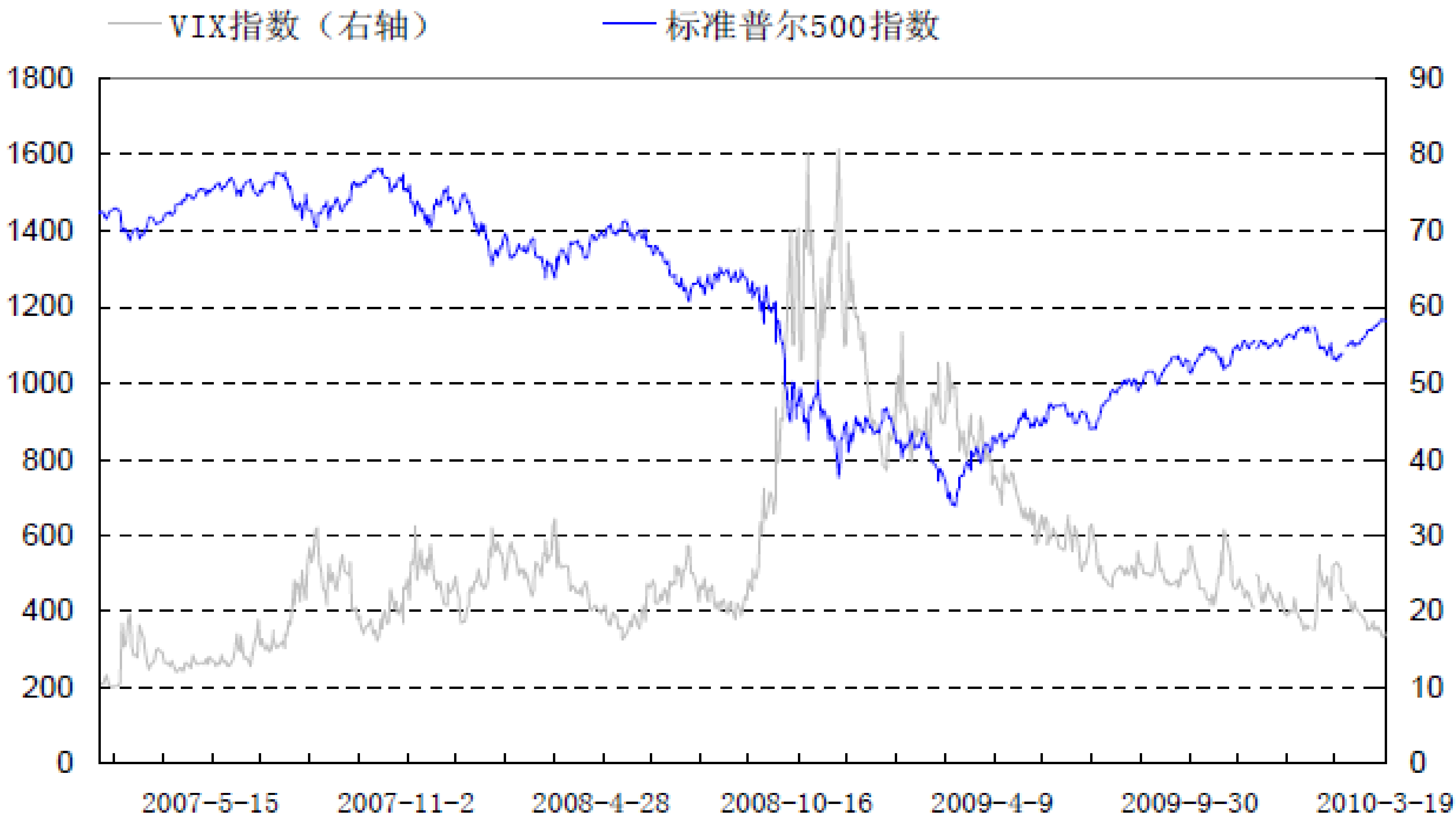
$$E^F(Var)=\frac{2e^{rT}}{T}\left[\int_0^{F_0}\frac{P(T,K)}{K^2}dK+\int_{F_0}^{\infty}\frac{C(T,K)}{K^2}dK\right]$$

- VIX(2003)

$$VIX=\frac{2}{T}\sum_i\frac{\Delta K_i}{K_i^2}e^{rT}Q(T,K_i)-\frac{1}{T}\left(\frac{F_0}{K_0}-1\right)^2$$

- Jiang andTian(2007)

标准普尔500指数和VIX指数



Implied Volatility and Future Volatility

隐含波动率可以看作是在国债风险中性世界中市场对未来波动率的预测

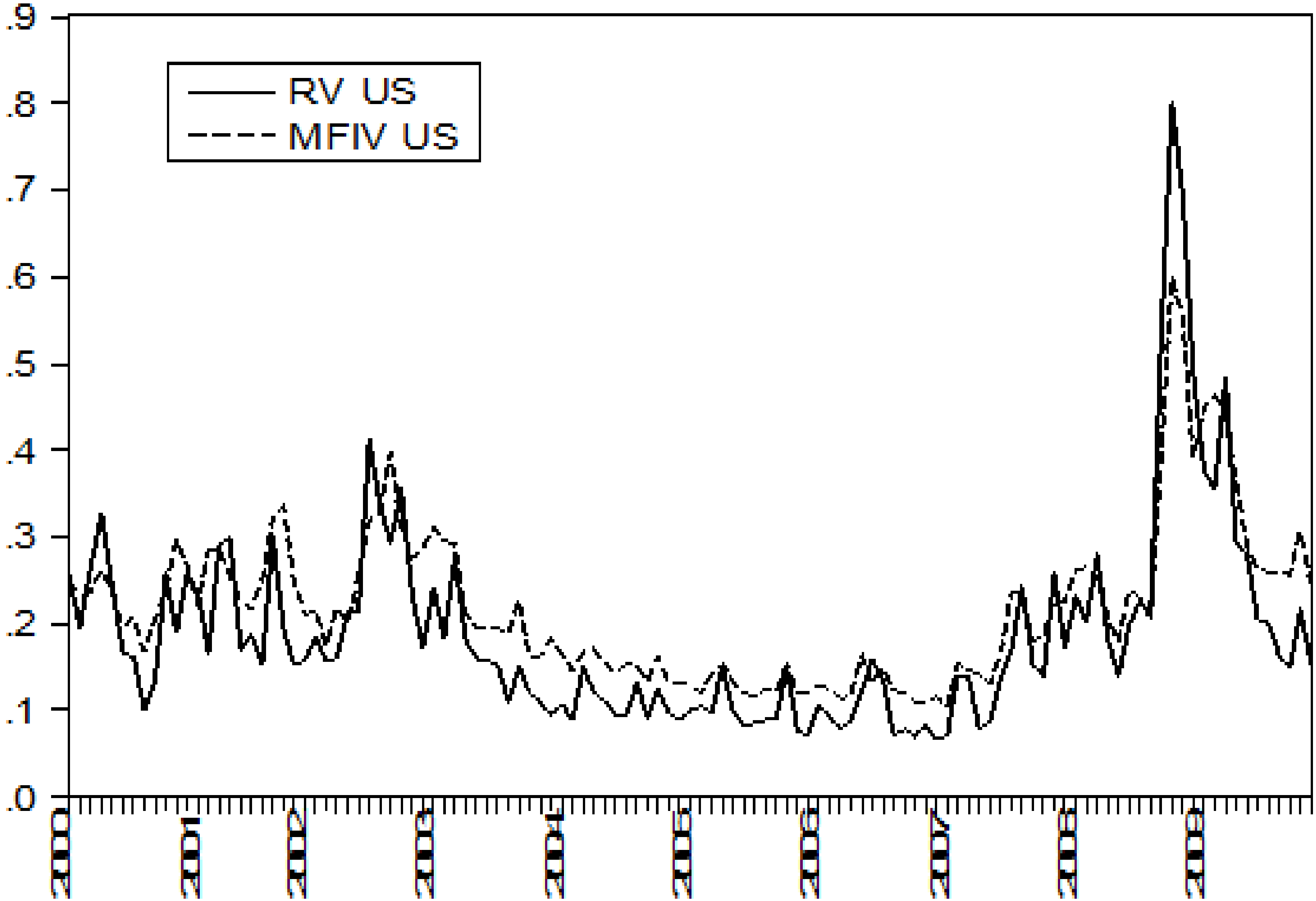
$$\tilde{\sigma}(t,T) = E_t^T[\sigma(t,T)] \quad (6)$$

其中，t表示当前时刻，T表示期权到期时刻，$\tilde{\sigma}(t,T)$ 表示t至T时刻的隐含波动率，$E_t^T(\cdot)$ 则表示以贴现国债 $B(t,T)$ 为记账单位的风险中性世界的期望值。而国债风险中性世界中市场对未来波动率的预期又等于现实世界中市场对未来波动率的预期 $E_t[\sigma(t,T)]$ 乘上利率风险溢酬因子 ρ_t^σ ：

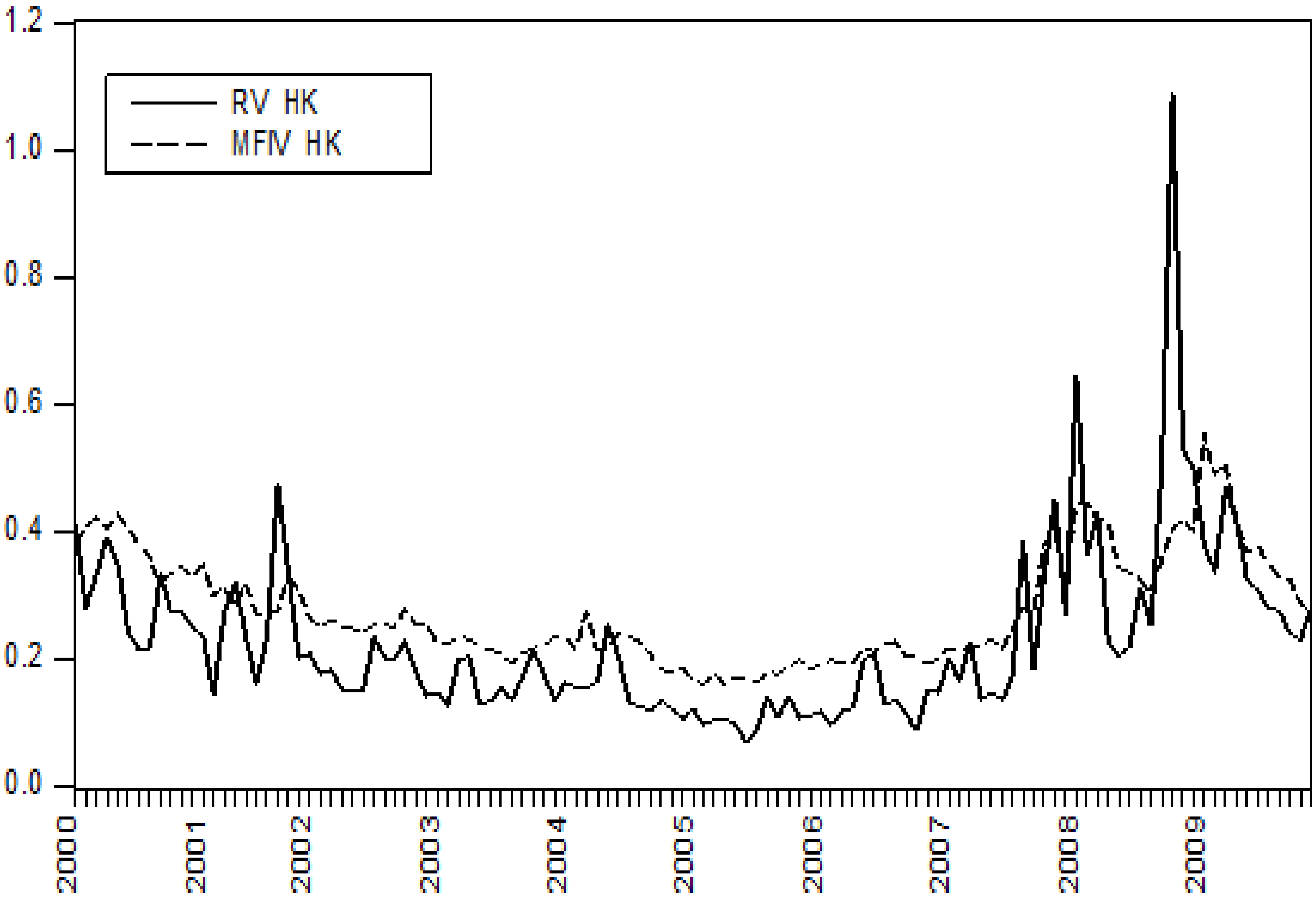
$$E_t^T[\sigma(t,T)] = E_t[\sigma(t,T)]\rho_t^\sigma \quad (7)$$

风险溢酬与预期

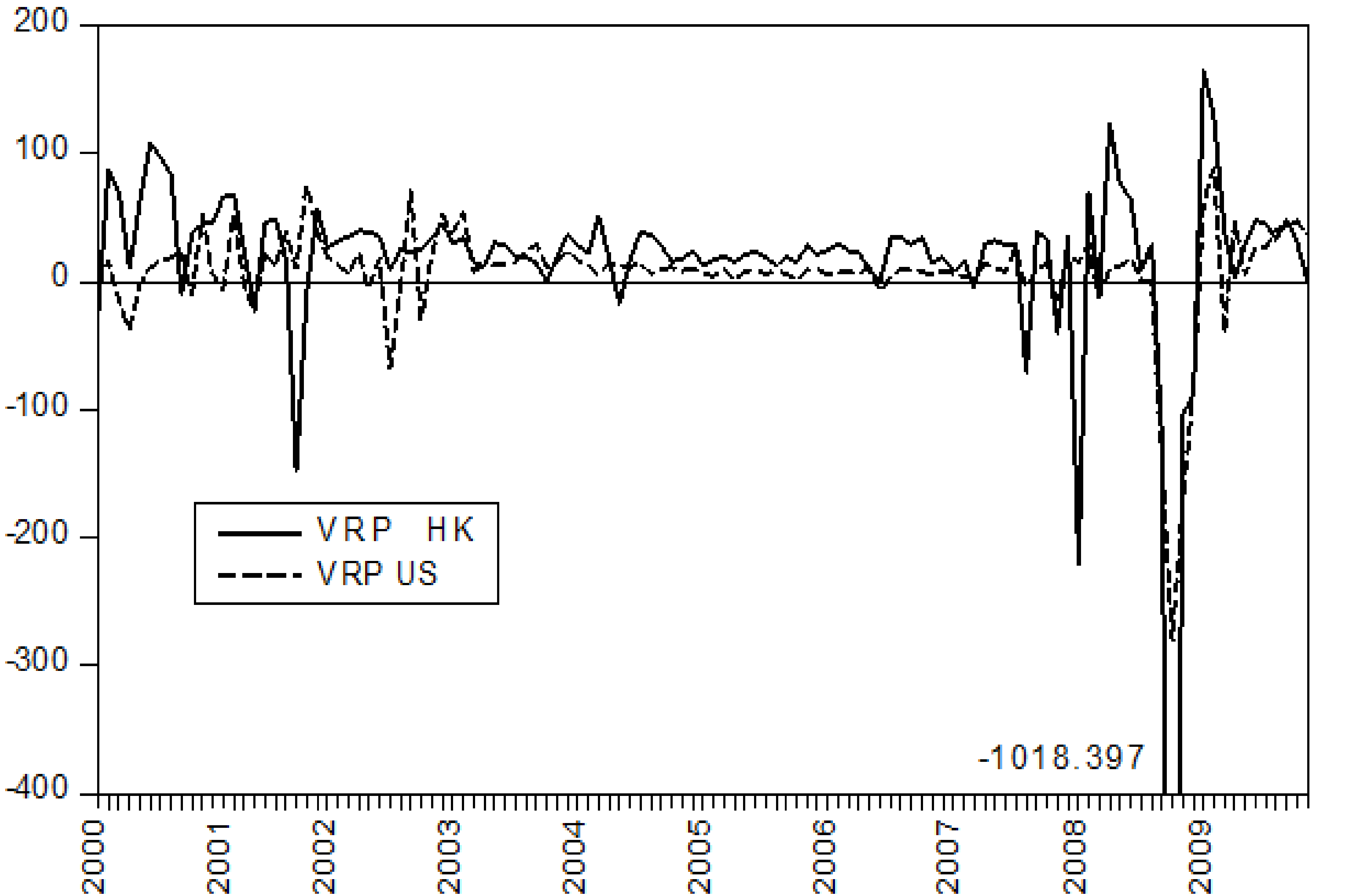
- 如果假定波动率风险溢酬是常数，或者能够从其它途径获取波动率的风险溢酬，我们就可以得到现实世界中市场对未来波动率的预期。通过波动率预期与过去波动率对比的时间序列数据，我们可以研究波动率预期的形成机制和特点；通过波动率预期与未来已实现波动率对比的时间序列数据，我们可以研究波动率预期的准确性。而如果能够从其它途径（如调研等）获得现实世界中市场对未来波动率的预期，我们就可以得到波动率风险溢酬，并研究其时间序列特征，特别是在金融危机情形下的特征。



图二：美国市场的已实现波动率（RV US）和无模型隐含波动率(MFIV US)



图一：香港市场的已实现波动率（RV HK）和无模型隐含波动率(MFIV HK)



图四：香港市场的波动率风险溢酬（VRP HK）和美国市场的波动率风险溢酬 （VRP US）