Chapter 17

The Greek Letters

Example

- A bank has sold for $300,000 a European call option on 100,000 shares of a nondividend paying stock
- $S_0 = 49$, $X = 50$, $r = 5\%$, $\sigma = 20\%$,
  $T = 20$ weeks, $\mu = 13\%$
- The Black-Scholes value of the option is $240,000
- How does the bank hedge its risk to lock in a $60,000 profit?

Naked & Covered Positions

Naked position

Take no action

Covered position

Buy 100,000 shares today

Both strategies leave the bank exposed to significant risk

Stop-Loss Strategy

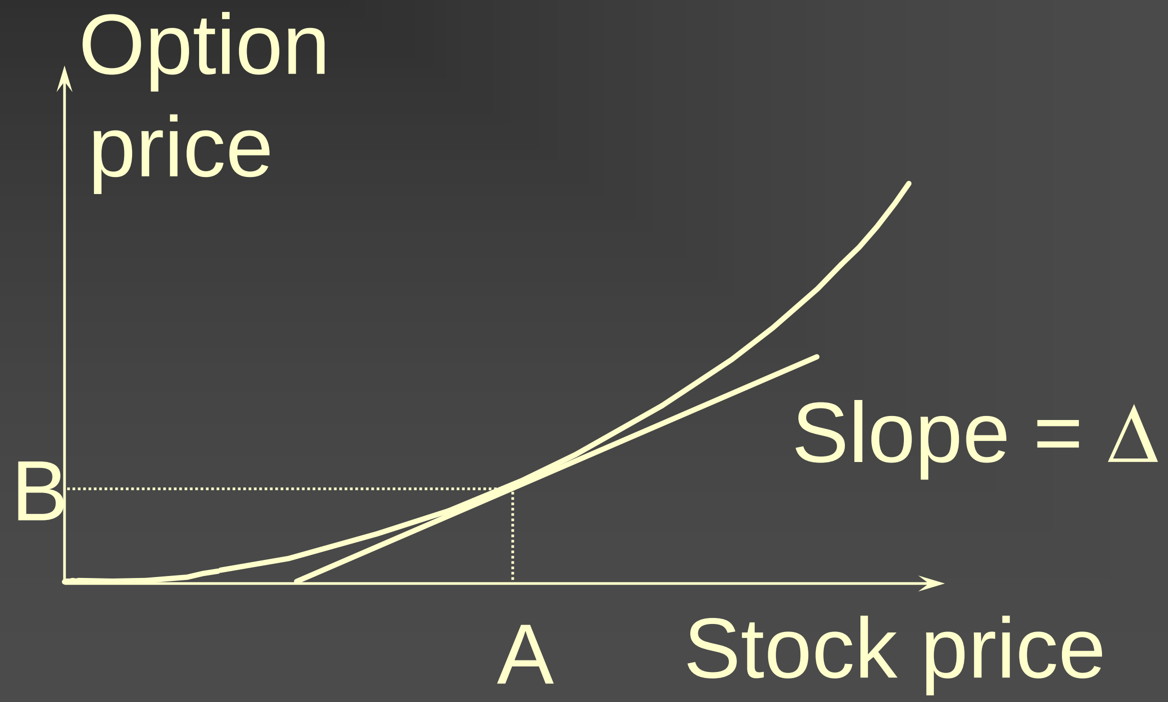
This involves:

- Buying 100,000 shares as soon as price reaches $50
- Selling 100,000 shares as soon as price falls below $50

This deceptively simple hedging strategy does not work well

Delta

- Delta (Δ) is the rate of change of the option price with respect to the underlying



Delta Hedging

- This involves maintaining a delta neutral portfolio
- The delta of a European call on a stock paying dividends at rate q is $N(d_1)e^{-qT}$
- The delta of a European put is

$$e^{-qT}[N(d_1)-1]$$

Delta Hedging continued

- The hedge position must be frequently rebalanced
- Delta hedging a written option involves a “buy high, sell low” trading rule
- See Tables 17.2 (page 356) and 17.3 (page 357) for examples of delta hedging

Using Futures for Delta Hedging

- The delta of a futures contract is $e^{(r-q)T}$ times the delta of a spot
- The position required in futures for delta hedging is therefore $e^{-(r-q)T}$ times the position required in the corresponding spot

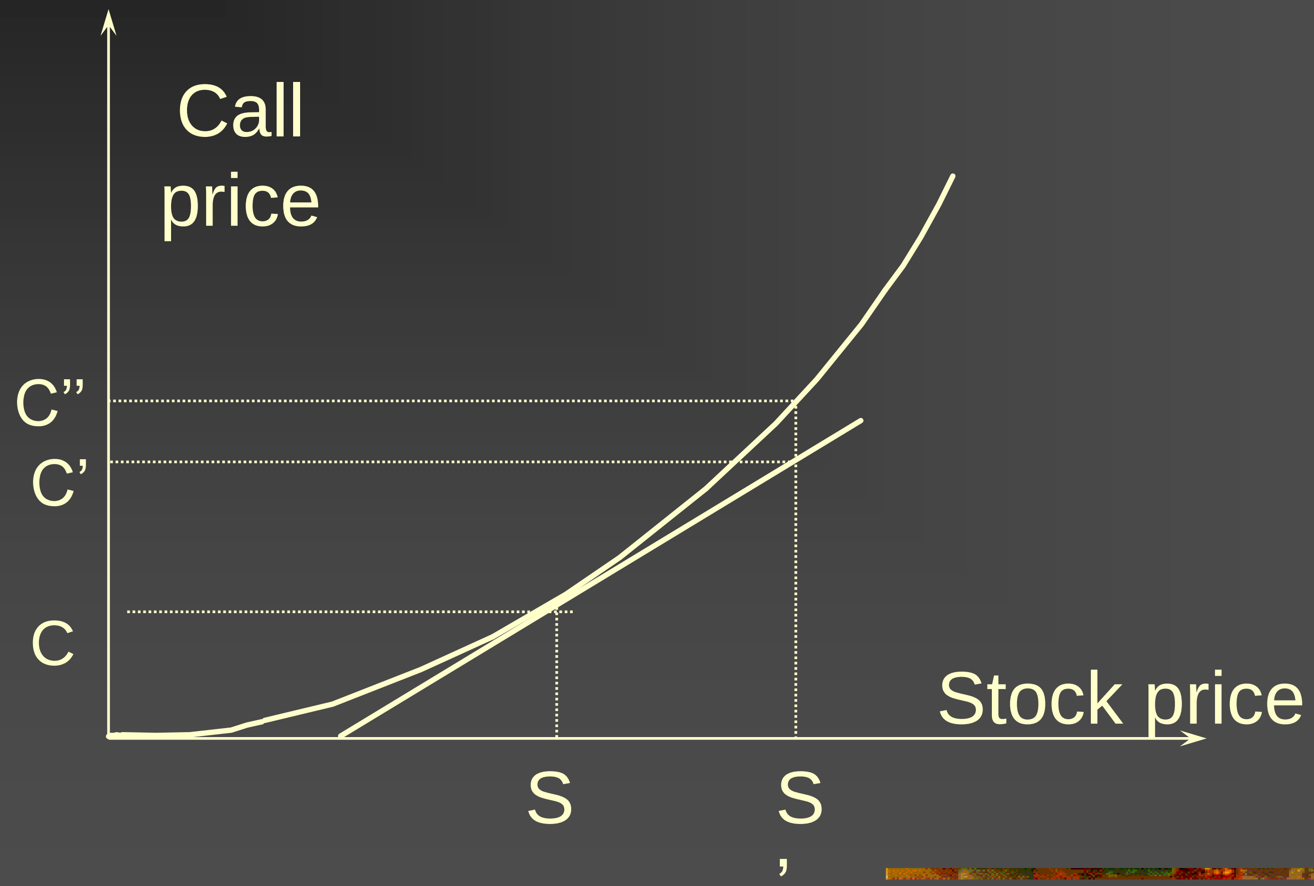
Theta

- Theta (Θ) of a derivative (or portfolio of derivatives) is the rate of change of the value with respect to the passage of time
- See Figure 15.5 for the variation of Θ with respect to the stock price for a European call

Gamma

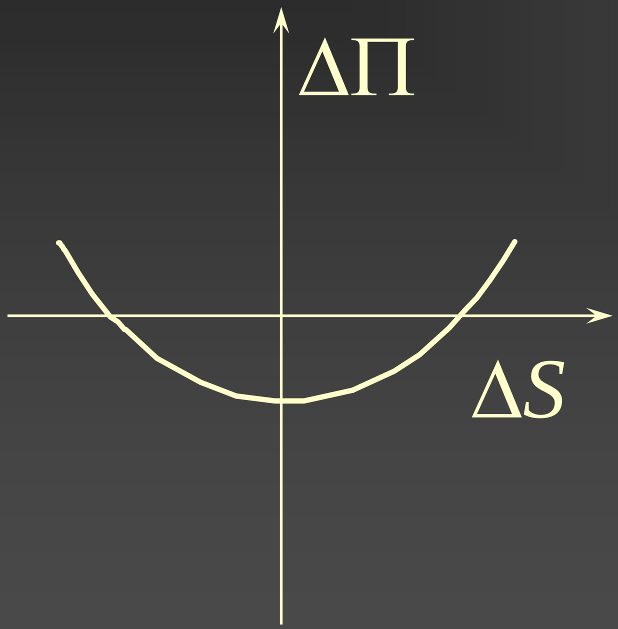
- Gamma (Γ) is the rate of change of delta (Δ) with respect to the price of the underlying asset
- Gamma is greatest for options that are close to the money (see Figure 17.9, page 364)

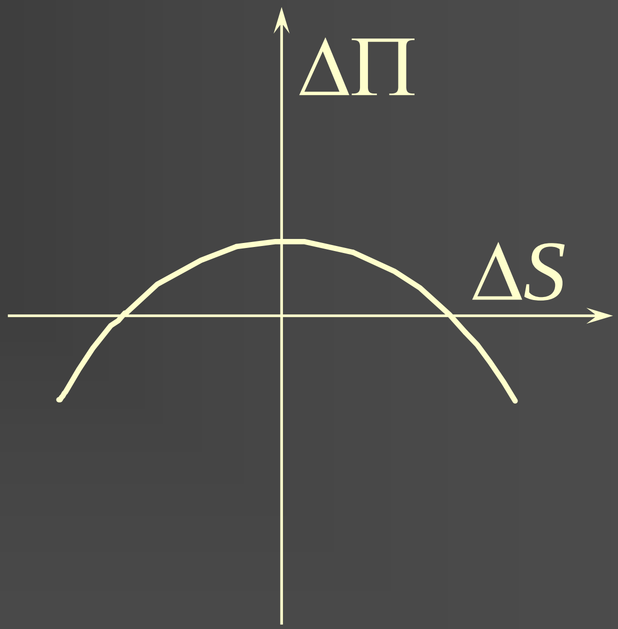
Gamma Addresses Delta Hedging Errors Caused By Curvature



Interpretation of Gamma

For a delta neutral portfolio, $\Delta\Pi \approx \Theta\,\Delta t + \frac{1}{2}\Gamma\Delta S^2$





Relationship Among Delta, Gamma, and Theta

For a portfolio of derivatives on a stock paying a continuous dividend yield at rate q

$$\Theta + (r - q)S\Delta + \frac{1}{2}\sigma^2 S^2 \Gamma = r\Pi$$

Vega

- Vega (ν) is the rate of change of the value of a derivatives portfolio with respect to volatility
- Vega tends to be greatest for options that are close to the money (See Figure 17.11, page 366)

Managing Delta, Gamma, & Vega

- Δ can be changed by taking a position in the underlying
- To adjust Γ & ν it is necessary to take a position in an option or other derivative

Rho

- Rho is the rate of change of the value of a derivative with respect to the interest rate
- For currency options there are 2 rhos

Hedging in Practice

- Traders usually ensure that their portfolios are delta-neutral at least once a day
- Whenever the opportunity arises, they improve gamma and vega
- As portfolio becomes larger hedging becomes less expensive

Scenario Analysis

A scenario analysis involves testing the effect on the value of a portfolio of different assumptions concerning asset prices and their volatilities

Greek Letters for Options on an Asset that Provides a Dividend Yield at Rate q

- See Table 17.6 on page 370

Hedging vs Creation of an Option Synthetically

- When we are hedging we take positions that offset Δ, Γ, ν, etc.
- When we create an option synthetically we take positions that match Δ, Γ, & ν

Portfolio Insurance

- In October of 1987 many portfolio managers attempted to create a put option on a portfolio synthetically
- This involves initially selling enough of the portfolio (or of index futures) to match the Δ of the put option

Portfolio Insurance continued

- As the value of the portfolio increases, the Δ of the put becomes less negative & some of the original portfolio is repurchased
- As the value of the portfolio decreases, the Δ of the put becomes more negative & more of the portfolio must be sold

Portfolio Insurance continued

The strategy did not work well on October 19, 1987...