## *Chapter 15-16*

# Options on Stock Indices, Currencies, and Futures

# *Index Options*

- The most popular underlying indices in the U.S. are
  - The Dow Jones Index times 0.01 (DJX)
  - The Nasdaq 100 Index (NDX)
  - The S&P 100 Index (OEX and XEO)
  - The S&P 500 Index (SPX)

- Exchange-traded contracts are on 100 times index; they are settled in cash

- OEX is American and the rest are European.

# *LEAPS (Long-term equity anticipation securities)*

- Leaps are options on stock indices that last up to 3 years
- They have December expiration dates
- They are on 10 times the index
- Leaps also trade on some individual stocks

# *Index Option Example*

- Consider a call option on an index with a strike price of 880
- Suppose 1 contract is exercised when the index level is 900
- What is the payoff?

# *Using Index Options for Portfolio Insurance*

- Suppose the value of the index is $S_0$ and the strike price is $X$
- If a portfolio has a β of 1.0, the portfolio insurance is obtained by buying 1 put option contract on the index for each $100S_0$ dollars held
- If the β is not 1.0, the portfolio manager buys β put options for each $100S_0$ dollars held
- In both cases, $X$ is chosen to give the appropriate insurance level

# Example 1

- Portfolio has a beta of 1.0
- It is currently worth $500,000
- The index currently stands at 1000
- What trade is necessary to provide insurance against the portfolio value falling below $450,000?

# *Example 2*

- Portfolio has a beta of 2.0
- It is currently worth $500,000 and index stands at 1000
- The risk-free rate is 12% per annum with quarterly compounding
- The dividend yield on both the portfolio and the index is 4% with quarterly compounding
- How many put option contracts should be purchased for portfolio insurance in 3 months?

# *Calculating Relation Between Index Level and Portfolio Value in 3 months*

$$r_i = r_f + \beta(r_m - r_f)$$

- If index rises to 1040, it provides a 40/1000 or 4% return in 3 months
- Total return (incl. dividends)=5%
- Excess return over risk-free rate=2%
- Excess return for portfolio=4%
- Increase in Portfolio Value=4+3-1=6%
- Portfolio value=$530,000

# Determining the Strike Price

| Value of Index in 3 months | Expected Portfolio Value in 3 months ($ millions) |
|---|---|
| 1,080 | 570,000 |
| 1,040 | 530,000 |
| 1,000 | 490,000 |
| 960 | 450,000 |
| 920 | 410,000 |
| 880 | 370,000 |

An option with a strike price of 960 will provide protection against a 10% decline in the portfolio value

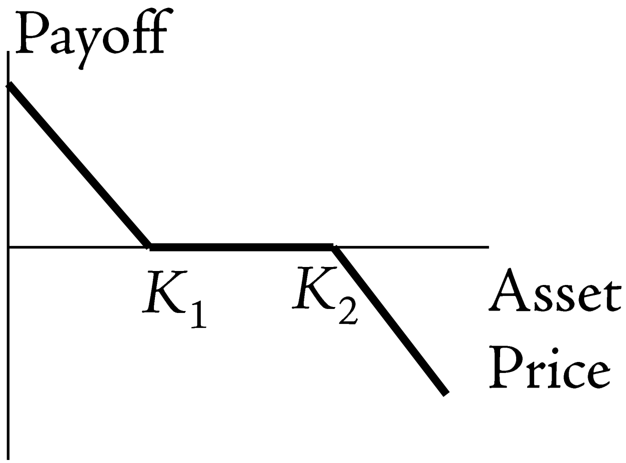
- Why the cost of hedging increase as the beta increases?
  - More puts are required
  - The strike price is higher

# Currency Options

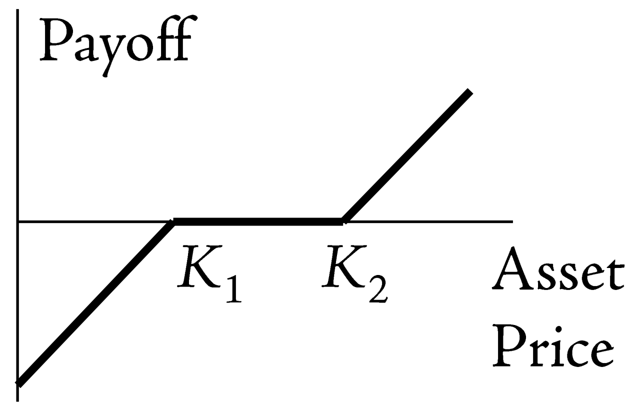
- Currency options trade on the Philadelphia Exchange (PHLX)
- There also exists an active over-the-counter (OTC) market
- Currency options are used by corporations to buy insurance when they have an FX exposure

# Range Forward Contracts

- Have the effect of ensuring that the exchange rate paid or received will lie within a certain range
- When currency is to be paid it involves selling a put with strike $K_1$ and buying a call with strike $K_2$ (with $K_2 > K_1$)
- When currency is to be received it involves buying a put with strike $K_1$ and selling a call with strike $K_2$
- Normally the price of the put equals the price of the call




Short Position




Long Position

# Assets Providing a Known Yield q

- Index options:
  - average dividend yield expected during the life of the option
- Currency options:
  - the foreign interest rate $r_f$
    - When a U.S. company buys one unit of the foreign currency it has an investment of $S_0$ dollars
    - A unit foreign currency will become $e^{r_f T}$ units
    - This shows that the foreign currency provides a "dividend yield" at rate $r_f$

# European Options on Assets Providing a Known Yield

We get the same probability distribution for the asset price at time T in each of the following cases:

1. The asset starts at price $S_0$ and provides a yield = $q$
2. The asset starts at price $S_0e^{-qT}$ and provides no income

**We can value European options by reducing the asset price to $S_0e^{-qT}$ and then behaving as though there is no income.**

- Put-call parity

$$c + Xe^{-rT} = p + S_0 e^{-qT} = p + F_0 e^{-rT}$$

- Option prices

$$c = S_0 e^{-qT} N(d_1) - Xe^{-rT} N(d_2)$$

$$p = Xe^{-rT} N(-d_2) - S_0 e^{-qT} N(-d_1)$$

where $$d_1 = \frac{\ln\left(\frac{S_0}{X}\right) + \left(r - q + \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}}$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

$$c = e^{-rT}\left[F_0 N(d_1) - XN(d_2)\right]$$

$$p = e^{-rT}\left[XN(-d_2) - F_0 N(-d_1)\right]$$

$$d_1 = \frac{\ln\left(\frac{F_0}{X}\right) + \frac{1}{2}\sigma^2 T}{\sigma\sqrt{T}}$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

$$F_0 = S_0 e^{(r-q)T}$$

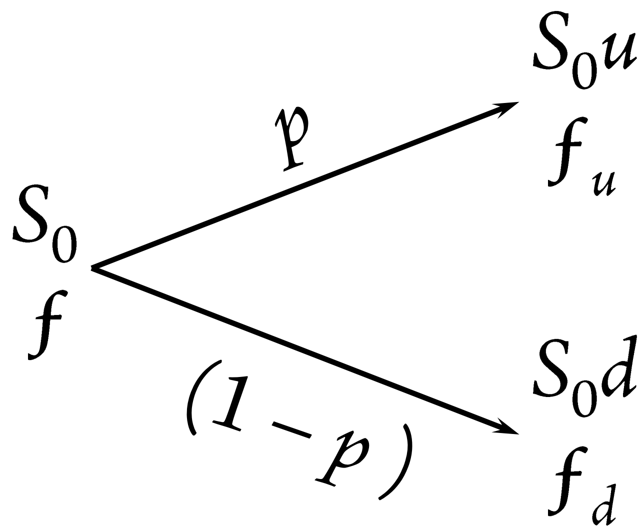
# Implied Forward Prices and Dividend Yields

- From European calls and puts with the same strike price and time to maturity

$$F_0 = X + (c - p)e^{rT} \qquad q = -\frac{1}{T}\ln\frac{c - p + Xe^{-rT}}{S_0}$$

- These formulas allow term structures of forward prices and dividend yields to be estimated
- OTC European options are typically valued using the forward prices (Estimates of $q$ are not then required)
- American options require the dividend yield term structure

# The Binomial Model




$$f = e^{-rT}[pf_u + (1-p)f_d]$$

- In a risk-neutral world the asset price grows at $r-q$ rather than at $r$ when there is a dividend yield at rate $q$
- The probability, $p$, of an up movement must therefore satisfy

$$pS_0u+(1-p)S_0d = S_0e^{(r-q)T}$$

so that

$$p = \frac{e^{(r-q)T} - d}{u-d}$$

# *Options on Futures*

- Referred to by the maturity month of the underlying futures

- The option is American and usually expires on or a few days before the earliest delivery date of the underlying futures contract

# Mechanics of Call Futures Options

- When a call futures option is exercised the holder acquires
  - A long position in the futures
  - A cash amount equal to the excess of the futures price at the time of the most recent settlement over the strike price

# Mechanics of Put Futures Option

- When a put futures option is exercised the holder acquires
  - A short position in the futures
  - A cash amount equal to the excess of the strike price over the futures price at the time of the most recent settlement

# Example 1

- Dec. call option contract on copper futures has a strike of 240 cents per pound. It is exercised when futures price is 251 cents and most recent settlement is 250. One contract is on 25,000 pounds
- Trader receives
  - Long Dec. futures contract on copper
  - $2,500 in cash

# Example 2

- Dec put option contract on corn futures has a strike price of 400 cents per bushel. It is exercised when the futures price is 380 cents per bushel and the most recent settlement price is 379 cents per bushel. One contract is on 5,000 bushels
- Trader receives
  - Short Dec futures contract on corn
  - $1,050 in cash

# The Payoffs

If the futures position is closed out immediately:

Payoff from call = $F_t$-$X$

Payoff from put = $X$-$F_t$

where $F_t$ is futures price at time of exercise

# Potential Advantages of Futures Options over Spot Options

- Futures contracts may be easier to trade and more liquid than the underlying asset
- Exercise of option does not lead to delivery of underlying asset
- Futures options and futures usually trade on same exchange
- Futures options may entail lower transactions costs

# European Futures Options

- European futures options and spot options are equivalent when futures contract matures at the same time as the option
- It is common to regard European spot options as European futures options when they are valued in the over-the-counter markets

# *Put-Call Parity for Futures Option*

Consider the following two portfolios:

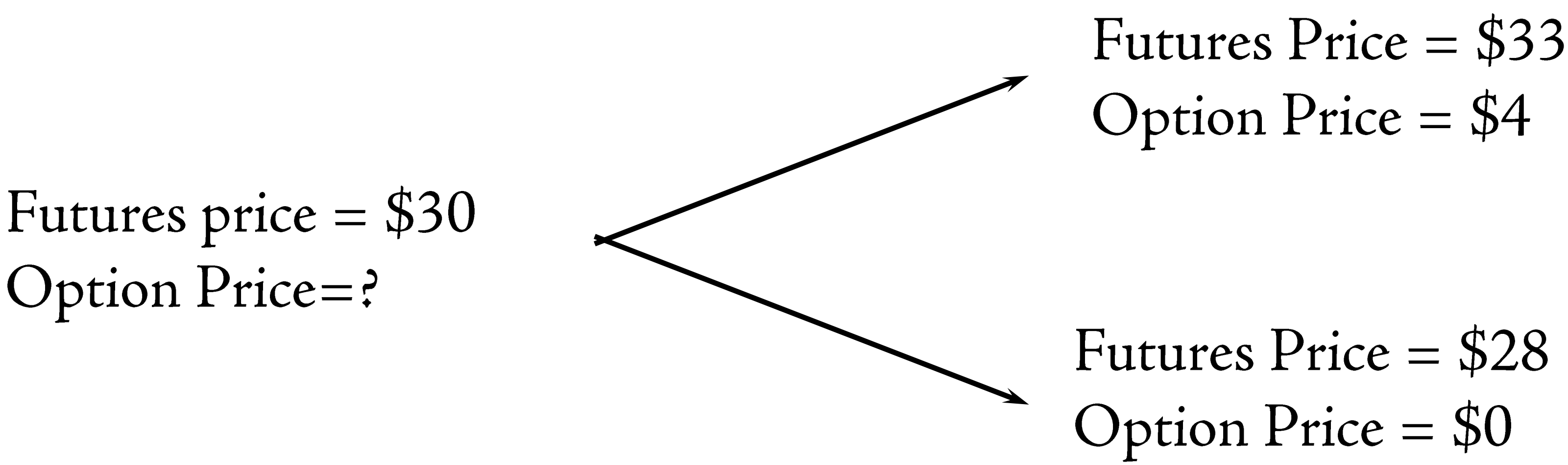
1. European call plus $Xe^{-rT}$ of cash
2. European put plus long futures plus cash equal to $F_0e^{-rT}$

They must be worth the same at time $T$ so that

$$c+Xe^{-rT}=p+F_0\,e^{-rT}$$

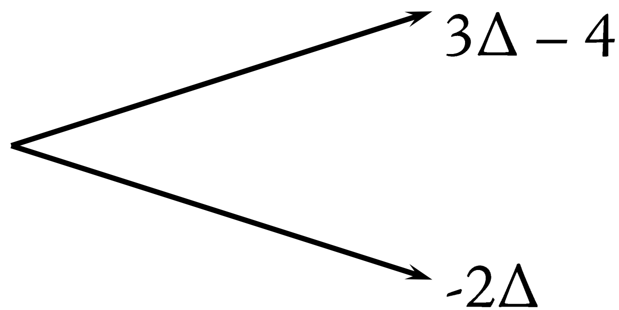
# *Binomial Tree Example*

A 1-month  call option on futures has a strike price of 29. Risk-Free Rate is 6% .



# *Setting Up a Riskless Portfolio*

- Consider the Portfolio: long $\Delta$ futures
  short 1 call option




- Portfolio is riskless when $3\Delta - 4 = -2\Delta$ or $\Delta = 0.8$

# *Valuing the Portfolio*

- The riskless portfolio is:

  long 0.8 futures

  short 1 call option

- The value of the portfolio in 1 month is

  -1.6

- The value of the portfolio today is

  $-1.6e^{-0.06/12} = -1.592$

# *Valuing the Option*

- The portfolio that is

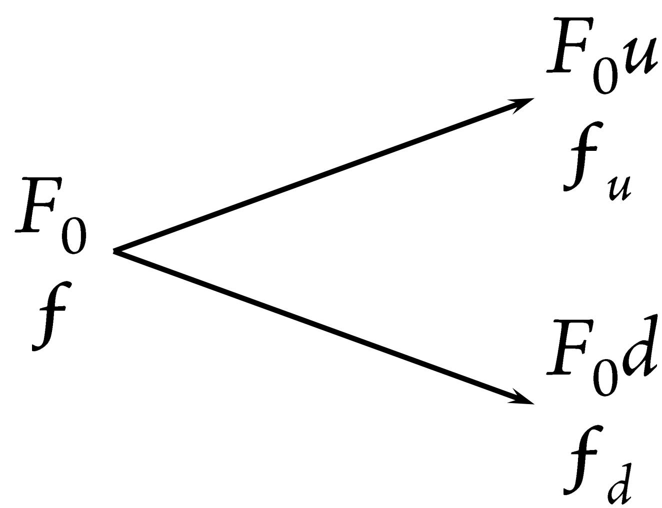
  long 0.8 futures

  short 1 option

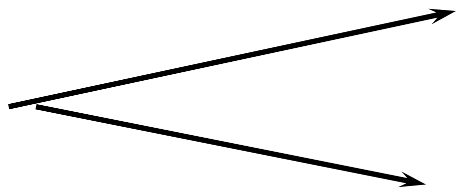
  is worth -1.592
- The value of the futures is zero
- The value of the option must therefore be 1.592

# Generalization of Binomial Tree Example

- A derivative lasts for time $T$ & is dependent on a futures



- Consider the portfolio that is long $\Delta$ futures and short 1 derivative



$$F_0 u\Delta - F_0\Delta - f_u$$

$$F_0 d\Delta - F_0\Delta - f_d$$

- The portfolio is riskless when

$$\Delta = \frac{f_u - f_d}{F_0 u - F_0 d}$$

- Value of the portfolio at time $T$ is $F_0 u\Delta - F_0\Delta - f_u$
- Value of portfolio *today* is $-f$
- Hence $f = -[F_0 u\Delta - F_0\Delta - f_u]e^{-rT}$

- Substituting for D we obtain

$$f = [\, pf_u + (1 - p)f_d \,]e^{-rT}$$

where

$$p = \frac{1 - d}{u - d}$$

# Growth Rates For Futures Prices

- A futures contract requires no initial investment
- In a risk-neutral world the expected return should be zero
- The expected growth rate of the futures price is therefore zero
- The futures price can therefore be treated like a stock paying a dividend yield of $r$

# Futures price vs. expected future spot ptice

- In a risk-neutral world, the expected growth rate of the futures price is zero,so

$$F_0 = \hat{E}(F_T)$$

Because $F_T = S_T$

$$F_0 = \hat{E}(S_T)$$

# Valuing European Futures Options

- We can use the formula for an option on a stock paying a continuous dividend yield

  Set $S_0$ = current futures price ($F_0$)

  Set $q$ = domestic risk-free rate ($r$)

- Setting $q = r$ ensures that the expected growth of $F$ in a risk-neutral world is zero

# Black's Formula

- The formulas for European options on futures are known as Black's formula

$$c = e^{-rT}\left[F_0 N(d_1) - XN(d_2)\right]$$

$$p = e^{-rT}\left[XN(-d_2) - F_0 N(-d_1)\right]$$

$$\text{where} \quad d_1 = \frac{\ln\left(\frac{F_0}{X}\right) + \frac{1}{2}\sigma^2 T}{\sigma\sqrt{T}}$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

# Using Black's Model Instead of Black-Scholes-Merton

- Consider a 6-month European call option on spot gold
- 6-month futures price is 1,240, 6-month risk-free rate is 5%, strike price is 1,200, and volatility of futures price is 20%
- Value of option is given by Black's model with $F_0 = 1{,}240$, $K=1{,}200$, $r = 0.05$, $T=0.5$, and s = 0.2
- It is 88.37

# European Futures Option Prices vs Spot Option Prices

- If the European futures option matures at the same time as the futures contract, then the two options are in theory equivalent.
- If the European call future option matures before the futures contract, it is worth more than the corresponding spot option in a normal market, and less in an inverted market.

# American Futures Option Prices vs Spot Option Prices

- There is always some chance that it will be optimal to exercise an American futures option early. So, if futures prices are higher than spot prices (normal market), an American call on futures is worth more than a similar American call on spot. An American put on futures is worth less than a similar American put on spot
- When futures prices are lower than spot prices (inverted market) the reverse is true

# Futures Style Options

- A futures-style option is a futures contract on the option payoff
- Some exchanges trade these in preference to regular futures options
- A call futures-style option has value

$$F_0N(d_1) - XN(d_2)$$

- A put futures style option has value

$$XN(-d_2) - F_0N(-d_1)$$

# Put-Call Parity Results

Non-Dividend-Paying Stock

$$c + Xe^{-rT} = p + S$$

Indices:

$$c + Xe^{-rT} = p + S_0\ e^{-qT}$$

Foreign exchange:

$$c + Xe^{-rT} = p + S_0 e^{-r_f T}$$

Futures:

$$c + Xe^{-rT} = p + F_0 e^{-rT}$$

# *Summary of Key Results*

- We can treat stock indices, currencies, & futures like a stock paying a continuous dividend yield of $q$
  - For stock indices, $q$ = average dividend yield on the index over the option life
  - For currencies, $q = r_f$
  - For futures, $q = r$