Chapter 23
Credit Derivatives

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23.25. Suppose that the risk-free zero curve is flat at 6% per annum with continuous compounding and that defaults can occur at times 0.25 years, 0.75 years, 1.25 years, and 1.75 years in a 2-year plain vanilla credit default swap with semiannual payments. Suppose that the recovery rate is 20% and the unconditional probabilities of default (as seen at time zero) are 1% at times 0.25 years and 0.75 years, and 1.5% at times 1.25 years and 1.75 years. What is the credit default swap spread? What would the credit default spread be if the instrument were a binary credit default swap?

Solution The analysis is as below (Table 1–Table 4):

Table 1 Unconditional default probabilities and survival probabilities

Time (year)	*Default probability*	*Survival probability*
0.25	0.0100	0.9900
0.75	0.0100	0.9800
1.25	0.0150	0.9650
1.75	0.0150	0.9500

Table 2 Calculation of the present value of expected payments. Payment = s per annum.

Time (year)	*Probability of survival*	*Expected payment*	*Discount factor*	*PV of expected payment*
0.5	0.9900	$0.9900s$	0.9704	$0.9607s$
1.0	0.9800	$0.9800s$	0.9418	$0.9229s$
1.5	0.9650	$0.9650s$	0.9139	$0.8819s$
2.0	0.9500	$0.9500s$	0.8869	$0.8426s$
Total				$3.6082s$

Therefore the PV of expected payments is $3.6082s$, the PV of the expected payoff is

Table 3 Calculation of the present value of expected payoff.
Notional principal = $1.

Time (year)	*Probability of default*	*Recovery rate*	*Expected payoff ($)*	*Discount factor*	*PV of expected payoff ($)*
0.25	0.0100	0.2	0.0080	0.9851	0.0079
0.75	0.0100	0.2	0.0080	0.9560	0.0076
1.25	0.0150	0.2	0.0120	0.9277	0.0111
1.75	0.0150	0.2	0.0120	0.9003	0.0108
Total					0.0375

Table 4 Calculation of the present value of accrual payment.

Time (year)	*Probability of default*	*Expected accrual payment*	*Discount factor*	*PV of expected accrual payment*
0.25	0.0100	$0.0050s$	0.9851	$0.0049s$
0.75	0.0100	$0.0050s$	0.9560	$0.0048s$
1.25	0.0150	$0.0075s$	0.9277	$0.0070s$
1.75	0.0150	$0.0075s$	0.9003	$0.0068s$
Total				$0.0234s$

0.0375, and the PV of the expected accrual payment is $0.0234s$.

$$3.6082s + 0.0234s = 0.0375$$

and

$$s = 0.0103$$

i.e., the credit default swap spread is 103 basis points.

If the CDS is a binary CDS, Table 3 becomes:

Table 5 Calculation of the present value of expected payoff.
from a binary credit default swap. Principal = $1.

Time (year)	*Probability of default*	*Expected payoff ($)*	*Discount factor*	*PV of expected payoff ($)*
0.25	0.0100	0.0100	0.9851	0.0099
0.75	0.0100	0.0100	0.9560	0.0096
1.25	0.0150	0.0150	0.9277	0.0139
1.75	0.0150	0.0150	0.9003	0.0135
Total				0.0468

In this case, the CDS spread is $s = 0.0468/(3.6082 + 0.0234) = 0.0129$, or 129 basis points.

23.26. Assume that the default probability for a company in a year, conditional on no earlier defaults is λ and the recovery rate is R. The risk-free interest rate is 5% per annum. Default always occurs halfway through a year. The spread for a 5-year plain vanilla CDS where payments are made annually is 120 basis points and the spread for a 5-year binary CDS where payments are made annually is 160 basis points. Estimate R and λ.

Solution The spread for a plain vanilla CDS should be $1 - R$ times the spread for a similar binary CDS, where R is the recovery rate. Thus the recovery rate is

$$R = 1 - \frac{0.0120}{0.0160} = 0.25$$

i.e., 25%. Using the same worksheet that estimates Problem 21.14, the Solver tool in Excel gives λ as 1.55%.

23.27. Explain how you would expect the yields offered on the various trenches in a CDO to change when the correlation between the bonds in the portfolio increases.

Solution When the correlation between the bonds in the portfolio increases, the volatility of the portfolio will increase, and the yields offered on the tranche 1 in Figure 23.3 tends to increase, so does the yields offered on the tranche 2 and tranche 3. The yields offered on the final tranche therefore decrease.

23.28. Suppose that:

(a) The yield on a 5-year risk-free bond is 7%.

(b) The yield on a 5-year corporate bond issued by company X is 9.5%.

(c) A 5-year credit default swap providing insurance against company X defaulting costs 150 basis paints per year.

What arbitrage opportunity is there in this situation? What arbitrage opportunity would there be if the credit default spread were 300 basis points instead of 150 basis points? Give two reasons why arbitrage opportunities such as those you identify are less than perfect.

Solution The position of a buyer of a credit default swap is similar to the position of someone who is long a risk-free bond and short a corporate bond. In this case, an arbitrager could short the corporate bond, long the risk-free bond, and sell the CDS to gain a profit of 100 basis points. If the credit default spread were 300 basis points, the arbitrager should take an opposite transaction to gain a profit of 50 basis points. The reasons why arbitrage opportunities are less than perfect are: (a) the risk-free bond will not be always risk-free, i.e., LIBOR might higher or lower than 7%; (b) the default of company X should also make this arbitrage imperfect.

23.29. In the ABS CDO structure in Figure 23.4, suppose that there is a 20% loss on each portfolio. What is the percentage loss experience by each of the six tranches shown?

Solution The ABS equity tranche is wiped out. There are no losses to the senior ABS tranche. The ABS mezzanine tranche loses 15/20=75% 0f the principal.

Total losses on the ABS CDO are 75%. The ABS CDO equity and mezzanine tranches are wiped out. The ABS CDO senior tranche loses 50/75=66.67% of the principal.