Chapter 22
Credit Risk

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20.28. Suppose a 3-year corporate bond provides a coupon of 7% per year payable semiannually and has a yield of 5% (expressed with semiannual compounding). The yields for all maturities on risk-free bonds is 4% per annum (expressed with semiannual compounding). Assume that defaults can take place every 6 months (immediately before a coupon payment) and the recovery rate is 45%. Estimate the default probabilities assuming (a) that the unconditional default probabilities are the same on each possible default date and (b) that the default probabilities conditional on no earlier default are the same on each possible default date.

Solution (a) Assuming that the unconditional default probabilities are the same on each possible default date. The calculation are as follows:

Time (years)	Default probability	Recovery rate(%)	Risk-free value($)	Loss given default($)	Discount factor	PV of expected loss($)
0.5	Q	45	109.97	64.97	0.9802	63.68Q
1	Q	45	108.63	63.63	0.9608	61.14Q
1.5	Q	45	107.27	62.27	0.9418	58.64Q
2	Q	45	105.87	60.87	0.9231	56.19Q
2.5	Q	45	104.45	59.45	0.9048	53.79Q
3	Q	45	103.5	58.5	0.8869	51.88Q
Total						345.33Q

The bond pays a coupon of 3.5 every six months and has a continuously compounded yield of 5% per year. Its market price is 105.3289. The risk-free value of the bond is obtained by discounting the promised cash flows at 4%. It is 108.2837. The total loss from defaults should therefore be equated to $108.2837 - 105.3289 = 2.9548$. The value of Q implied by the bond price is therefore given by $345.33Q = 2.9548$, or $Q = 0.0086$. The implied probability of default is 1.72% per year.

(b) Assuming that the default probabilities conditional on no earlier default are the same on each possible default date.The calculation are as follows:

Time (years)	Default probability	Recovery rate(%)	Risk-free value($)	Loss given default($)	Discount factor	PV of expected loss($)
0.5	Q	45	109.97	64.97	0.9802	63.68Q
1	Q(1-Q)	45	108.63	63.63	0.9608	61.14Q(1-Q)
1.5	$Q(1\text{-}Q)^2$	45	107.27	62.27	0.9418	$58.64Q(1\text{-}Q)^2$
2	$Q(1\text{-}Q)^3$	45	105.87	60.87	0.9231	$56.19Q(1\text{-}Q)^3$
2.5	$Q(1\text{-}Q)^4$	45	104.45	59.45	0.9048	$53.79Q(1\text{-}Q)^4$
3	$Q(1\text{-}Q)^5$	45	103.5	58.5	0.8869	$51.88Q(1\text{-}Q)^5$

That is

$$63.68Q+61.14Q(1-Q)+58.64Q(1-Q)^2+56.19Q(1-Q)^3+53.79Q(1-Q)^4+51.88Q(1-Q)^5 = 2.9548$$

or $Q = 0.0097$. The implied probability of default is 1.94% per year.

20.29. A company has 1- and 2-year bonds outstanding, each providing a coupon of 8% per year payable annually. The yields on the bonds (expressed with continuous compounding) are 6.0% and 6.6%, respectively. Risk-free rates are 4.5% for all maturities. The recovery rate is 35%. Defaults can take place halfway through each year. Estimate the risk-neutral default rate each year.

Solution The yields imply that the price of the 1-year bond is

$$\frac{108}{1+6.0\%} = 101.89$$

the price of the 2-year bond is

$$\frac{8}{1+6.6\%} + \frac{108}{(1+6.6\%)^2} = 102.55$$

the price of the similar 1-year risk-free bond is

$$\frac{108}{1+4.5\%} = 103.35$$

the price of the similar 2-year risk-free bond is

$$\frac{8}{1+4.5\%} + \frac{108}{(1+4.5\%)^2} = 106.55$$

First we consider the 1-year corporate bond.

Table 1 below calculates the expected loss from default in terms of Q on the assumption that defaults can take place halfway through each year, or at time 0.5 years.

The expected value of the risk-free bond at time 0.5 years is

$$108e^{-0.045\times 0.5} = 105.60$$

Table 1: Calculation of loss from default on a bond in terms of the default probabilities per year, Q. Notional principal=$100

Time (years)	Default probability	Recovery rate(%)	Risk-free value($)	Loss given default($)	Discount factor	PV of expected loss($)
0.5	Q	35	105.60	68.64	0.9778	67.11Q

The expected loss is 67.11Q. Let

$$67.11Q = 103.35 - 101.89$$

We obtain a value for Q equal to 2.18%.

Then we consider the 5-year corporate bond.

Table 2 below calculates the expected loss from default in terms of Q on the assumption that defaults can take place halfway through each year, or at times 0.5, 1.5 years.

The expected value of the risk-free bond at time 1.5 years is

$$108e^{-0.045\times 0.5} = 105.60$$

The expected value of the risk-free bond at time 0.5 years is

$$8e^{-0.045\times 0.5} + 108e^{-0.045\times 1.5} = 108.77$$

Table 2: Calculation of loss from default on a bond in terms of the default probabilities per year, Q. Notional principal=$100

Time (years)	Default probability	Recovery rate(%)	Risk-free value($)	Loss given default($)	Discount factor	PV of expected loss($)
0.5	Q	35	105.60	68.64	0.9778	67.11Q
1.5	Q	35	108.77	70.66	0.9347	66.04Q
Total						133.15Q

The expected loss is 133.15Q. Let

$$133.15Q = 106.55 - 102.55$$

We obtain a value for Q equal to 3.00%.

Since

$$3.00\% \times 2 - 2.18\% = 3.82\%$$

Thus the risk-neutral default rate in the first year is 2.18% and in the second year is 3.82%.

22.30. Explain carefully the distinction between real-world and risk-neutral default probabilities. Which is higher? A bank enters into a credit derivative where it agrees to pay \$100 at the end of 1 year if a certain company's credit rating falls from A to Baa or lower during the year. The 1-year risk-free rate is 5%. Using Table 22.6, estimate a value for the derivative. What assumptions are you making? Do they tend to overstate or understate the value of the derivative.

Solution Real world default probabilities are the true probabilities of defaults which can be estimated from historical data. Risk-neutral default probabilities are the probabilities of defaults in a world where all market participants are risk neutral and which can be estimated from bond prices. Risk-neutral default probabilities are higher. This means that returns in the risk-neutral world are lower. From Table 22.6, the probability of a company moving from A to Baa or lower in one year is 5.92%, thus the value of the derivative is

$$0.0592 \times 100 \times e^{-0.05\times 1} = 5.6313$$

The approximation in this is that the real-world probability of a downgrade is used. To value the derivative correctly the risk-neutral probability of a downgrade should be used. Since the risk-neutral probability of a default is higher than the real-world probability, it seems likely that the same is true of a downgrade. This means that 5.63 tends to understate the value of the derivative.

22.31. The value of a company's equity is \$4 million and the volatility of its equity is 60%. The debt that will have to be repaid in 2 years is \$15 million. The risk-free interest rate is 6% per annum. Use Merton's model to estimate the expected loss from default, the probability of default, and the recovery rate in the event of default. Explain why Merton's model gives a high recovery rate. (*Hint*: The Solver function in Excel can be used for this question.)

Solution Merton's model is

$$E_0 = V_0 N(d_1) - De^{-rT} N(d_2)$$

where

$$d_1 = \frac{\ln V_0/D + (r + \sigma_V^2/2)T}{\sigma_V \sqrt{T}} \quad \text{and} \quad d_2 = d_1 - \sigma_V \sqrt{T}$$

and

$$\sigma_E E_0 = N(d_1)\sigma_V V_0$$

In this case,

$$E_0 = 4, \quad \sigma_E = 60\%, \quad r = 0.06, \quad T = 2, \quad D = 15$$

Using the Solver function in Excel, there are

$$V_0 = 17.0839, \quad \sigma_V = 0.1576, \quad d_2 = 1.0105$$

The market value of the debt is $V_0 - E_0$, or 13.0839. The present value of the promised payment on the debt therefore is

$$15e^{-0.06\times 2} = 13.3038$$

The expected loss from default is

$$\frac{13.3038 - 13.0839}{13.3038} = 1.65\%$$

And the probability of default is

$$N(-d_2) = 0.1561$$

i.e., 15.61%. But there is

$$(1 - R) \times Q = 1.65\%$$

Thus the recovery rate is

$$R = \frac{15.61 - 1.65}{15.61} = 89.43\%$$

The reason the recovery rate is so high is as follows. There is a default if the value of the assets moves from 17.08 to below 15. A value for the assets significantly below 15 is unlikely. Conditional on a default, the expected value of the assets is not a huge amount below 15.

22.32 Suppose that a bank has a total of \$10 million of exposures of a certain type. The 1-year probability of default averages 1% and the recovery rate averages 40%. The copula correlation parameter is 0.2. Estimate the 99.5% 1-year credit VaR.

Solution In this case,

$$V(0.995, 1) = N\left(\frac{N^{-1}(0.01) + \sqrt{0.2}N^{-1}(0.995))}{\sqrt{1 - 0.2}}\right) = 0.0946$$

Showing that the 99.5% worst case default rate is 9.46%. The 1-year 99.5% credit VaR is therefore $10 \times 0.0946 \times (1 - 0.4)$or \$0.57 million.