CHAPTER 22
Credit Risk

Problem 22.1.
The spread between the yield on a three-year corporate bond and the yield on a similar risk-free bond is 50 basis points. The recovery rate is 30%. Estimate the average default intensity per year over the three-year period.

From equation (22.2) the average default intensity over the three years is \(0.0050/(1-0.3) = 0.0071\) or 0.71% per year.

Problem 22.2.
Suppose that in Problem 22.1 the spread between the yield on a five-year bond issued by the same company and the yield on a similar risk-free bond is 60 basis points. Assume the same recovery rate of 30%. Estimate the average default intensity per year over the five-year period. What do your results indicate about the average default intensity in years 4 and 5?

From equation (22.2) the average default intensity over the five years is \(0.0060/(1-0.3) = 0.0086\) or 0.86% per year. Using the results in the previous question, the default intensity is 0.71% per year for the first three years and

\[
\frac{0.0086 \times 5 - 0.0071 \times 3}{2} = 0.0107
\]

or 1.07% per year in years 4 and 5.

Problem 22.3.
Should researchers use real-world or risk-neutral default probabilities for a) calculating credit value at risk and b) adjusting the price of a derivative for defaults?

Real-world probabilities of default should be used for calculating credit value at risk. Risk-neutral probabilities of default should be used for adjusting the price of a derivative for default.

Problem 22.4.
How are recovery rates usually defined?

The recovery rate for a bond is the value of the bond immediately after the issuer defaults as a percent of its face value.

Problem 22.5.
Explain the difference between an unconditional default probability density and a default intensity.
The default intensity, \( h(t) \) at time \( t \) is defined so that \( h(t) \Delta t \) is the probability of default between times \( t \) and \( t + \Delta t \) conditional on no default prior to time \( t \). The unconditional default probability density \( q(t) \) is defined so that \( q(t) \Delta t \) is the probability of default between times \( t \) and \( t + \Delta t \) as seen at time zero.

**Problem 22.6.**
Verify a) that the numbers in the second column of Table 22.4 are consistent with the numbers in Table 22.1 and b) that the numbers in the fourth column of Table 22.5 are consistent with the numbers in Table 22.4 and a recovery rate of 40%.

The first number in the second column of Table 22.4 is calculated as

\[
\frac{1}{7} \ln(1 - 0.00251) = 0.000359
\]

or 0.04% per year. Other numbers in the column are calculated similarly. The numbers in the fourth column of Table 22.5 are the numbers in the second column of Table 22.4 multiplied by one minus the expected recovery rate. In this case the expected recovery rate is 0.4.

**Problem 22.7.**
Describe how netting works. A bank already has one transaction with a counterparty on its books. Explain why a new transaction by a bank with a counterparty can have the effect of increasing or reducing the bank's credit exposure to the counterparty.

Suppose company A goes bankrupt when it has a number of outstanding contracts with company B. Netting means that the contracts with a positive value to A are netted against those with a negative value in order to determine how much, if anything, company A owes company B. Company A is not allowed to "cherry pick" by keeping the positive-value contracts and defaulting on the negative-value contracts.

The new transaction will increase the bank's exposure to the counterparty if the contract tends to have a positive value whenever the existing contract has a positive value and a negative value whenever the existing contract has a negative value. However, if the new transaction tends to offset the existing transaction, it is likely to have the incremental effect of reducing credit risk.

**Problem 22.8.**
Suppose that the measure \( \beta_{AB}(T) \) in equation (22.9) is the same in the real world and the risk-neutral world. Is the same true of the Gaussian copula measure, \( \rho_{AB} \)?

Equation (22.14) gives the relationship between \( \beta_{AB}(T) \) and \( \rho_{AB} \). This involves \( Q_A(T) \) and \( Q_B(T) \). These change as we move from the real world to the risk-neutral world. It follows that the relationship between \( \beta_{AB}(T) \) and \( \rho_{AB} \) in the real world is not the same as in the risk-neutral world. If \( \beta_{AB}(T) \) is the same in the two worlds, \( \rho_{AB} \) is not.
Problem 22.9.

What is meant by a haircut in a collateralization agreement. A company offers to post its own equity as collateral. How would you respond?

When securities are pledged as collateral the haircut is the discount applied to their market value for margin calculations. A company's own equity would not be good collateral. When the company defaults on its contracts its equity is likely to be worth very little.

Problem 22.10.

Explain the difference between the Gaussian copula model for the time to default and CreditMetrics as far as the following are concerned: a) the definition of a credit loss and b) the way in which default correlation is modeled.

(a) In the Gaussian copula model for time to default a credit loss is recognized only when a default occurs. In CreditMetrics it is recognized when there is a credit downgrade as well as when there is a default.

(b) In the Gaussian copula model of time to default, the default correlation arises because the value of the factor \( M \). This defines the default environment or average default rate in the economy. In CreditMetrics a copula model is applied to credit ratings migration and this determines the joint probability of particular changes in the credit ratings of two companies.

Problem 22.11.

Suppose that the probability of company A defaulting during a two year period is 0.2 and the probability of company B defaulting during this period is 0.15. If the Gaussian copula measure of default correlation is 0.3, what is the binomial correlation measure?

In equation (22.14), \( Q_A(2) = 0.2 \), \( Q_B(2) = 0.15 \), and \( \rho_{AB} = 0.3 \). Also

\[
x_A(2) = N^{-1}(0.2) = -0.84162
\]

\[
x_B(2) = N^{-1}(0.15) = -1.03643
\]

\[
M(-0.84162, -1.03643, 0.3) = 0.0522
\]

\[
\beta_{AB}(2) = \frac{0.0522 - 0.2 \times 0.15}{\sqrt{(0.2 - 0.2^2)(0.15 - 0.15^2)}} = 0.156
\]

Problem 22.12.

Suppose that the LIBOR/swap curve is flat at 6% with continuous compounding and a five-year bond with a coupon of 5% (paid semiannually) sells for 90.00. How would an asset swap on the bond be structured? What is the asset swap spread that would be calculated in this situation?

Suppose that the principal is $100. The asset swap is structured so that the $10 is paid initially. After that $2.50 is paid every six months. In return LIBOR plus a spread is received on the principal of $100. The present value of the fixed payments is

\[
10 + 2.5e^{-0.06 \times 0.5} + 2.5e^{-0.06 \times 1} + \ldots + 2.5e^{-0.06 \times 5} + 100e^{-0.06 \times 5} = 105.3579
\]
The spread over LIBOR must therefore have a present value of 5.3579. The present value of $1 received every six months for five years is 8.5105. The spread received every six months must therefore be \( \frac{5.3579}{8.5105} = 0.6296 \). The asset swap spread is therefore \( 2 \times 0.6296 = 1.2592\% \) per annum.

**Problem 22.13.**

Show that the value of a coupon-bearing corporate bond is the sum of the values of its constituent zero-coupon bonds when the amount claimed in the event of default is the no-default value of the bond, but that this is not so when the claim amount is the face value of the bond plus accrued interest.

When the claim amount is the no-default value, the loss for a corporate bond arising from a default at time \( t \) is

\[
v(t)(1 - \hat{R})B^*
\]

where \( v(t) \) is the discount factor for time \( t \) and \( B^* \) is the no-default value of the bond at time \( t \). Suppose that the zero-coupon bonds comprising the corporate bond have no-default values at time \( t \) of \( Z_1, Z_2, \ldots, Z_n \), respectively. The loss from the \( i \)th zero-coupon bond arising from a default at time \( t \) is

\[
v(t)(1 - \hat{R})Z_i
\]

The total loss from all the zero-coupon bonds is

\[
v(t)(1 - \hat{R}) \sum_{i=1}^{n} Z_i = v(t)(1 - \hat{R})B^*
\]

This shows that the loss arising from a default at time \( t \) is the same for the corporate bond as for the portfolio of its constituent zero-coupon bonds. It follows that the value of the corporate bond is the same as the value of its constituent zero-coupon bonds.

When the claim amount is the face value plus accrued interest, the loss for a corporate bond arising from a default at time \( t \) is

\[
v(t)B^* - v(t)\hat{R}[L + a(t)]
\]

where \( L \) is the face value and \( a(t) \) is the accrued interest at time \( t \). In general this is not the same as the loss from the sum of the losses on the constituent zero-coupon bonds.

**Problem 22.14.**

A four-year corporate bond provides a coupon of 4% per year payable semiannually and has a yield of 5% expressed with continuous compounding. The risk-free yield curve is flat at 3% with continuous compounding. Assume that defaults can take place at the end of each year (immediately before a coupon or principal payment and the recovery rate is 30%. Estimate the risk-neutral default probability on the assumption that it is the same each year.
Define $Q$ as the risk-free rate. The calculations are as follows:

<table>
<thead>
<tr>
<th>Time (yrs)</th>
<th>Def. Prob.</th>
<th>Recovery Amount ($)</th>
<th>Risk-free Value ($)</th>
<th>Loss Given Default ($)</th>
<th>Discount Factor</th>
<th>PV of Expected Loss ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>$Q$</td>
<td>30</td>
<td>104.78</td>
<td>74.78</td>
<td>0.9704</td>
<td>72.57$Q$</td>
</tr>
<tr>
<td>2.0</td>
<td>$Q$</td>
<td>30</td>
<td>103.88</td>
<td>73.88</td>
<td>0.9418</td>
<td>69.58$Q$</td>
</tr>
<tr>
<td>3.0</td>
<td>$Q$</td>
<td>30</td>
<td>102.96</td>
<td>72.96</td>
<td>0.9139</td>
<td>66.68$Q$</td>
</tr>
<tr>
<td>4.0</td>
<td>$Q$</td>
<td>30</td>
<td>102.00</td>
<td>72.00</td>
<td>0.8869</td>
<td>63.86$Q$</td>
</tr>
</tbody>
</table>

Total: $272.69Q$

The bond pays a coupon of 2 every six months and has a continuously compounded yield of 5% per year. Its market price is 96.19. The risk-free value of the bond is obtained by discounting the promised cash flows at 3%. It is 103.66. The total loss from defaults should therefore be equated to $103.66 - 96.19 = 7.46$. The value of $Q$ implied by the bond price is therefore given by $272.69Q = 7.46$. or $Q = 0.0274$. The implied probability of default is 2.74% per year.

**Problem 22.15.**

A company has issued 3- and 5-year bonds with a coupon of 4% per annum payable annually. The yields on the bonds (expressed with continuous compounding) are 4.5% and 4.75%, respectively. Risk-free rates are 3.5% with continuous compounding for all maturities. The recovery rate is 40%. Defaults can take place half way through each year. The risk-neutral default rates per year are $Q_1$ for years 1 to 3 and $Q_2$ for years 4 and 5. Estimate $Q_1$ and $Q_2$.

The table for the first bond is:

<table>
<thead>
<tr>
<th>Time (yrs)</th>
<th>Def. Prob.</th>
<th>Recovery Amount ($)</th>
<th>Risk-free Value ($)</th>
<th>Loss Given Default ($)</th>
<th>Discount Factor</th>
<th>PV of Expected Loss ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>$Q_1$</td>
<td>40</td>
<td>103.01</td>
<td>63.01</td>
<td>0.9827</td>
<td>61.92$Q_1$</td>
</tr>
<tr>
<td>1.5</td>
<td>$Q_1$</td>
<td>40</td>
<td>102.61</td>
<td>62.61</td>
<td>0.9489</td>
<td>59.41$Q_1$</td>
</tr>
<tr>
<td>2.5</td>
<td>$Q_1$</td>
<td>40</td>
<td>102.20</td>
<td>62.20</td>
<td>0.9162</td>
<td>56.98$Q_1$</td>
</tr>
</tbody>
</table>

Total: $178.31Q_1$

The market price of the bond is 98.35 and the risk-free value is 101.23. It follows that $Q_1$ is given by

$$178.31Q_1 = 101.23 - 98.35$$

so that $Q_1 = 0.0161$.

The table for the second bond is
<table>
<thead>
<tr>
<th>Time (yrs)</th>
<th>Def. Prob.</th>
<th>Recovery Amount ($)</th>
<th>Risk-free Value ($)</th>
<th>Loss Given Default ($)</th>
<th>Discount Factor</th>
<th>PV of Expected Loss ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>$Q_1$</td>
<td>40</td>
<td>103.77</td>
<td>63.77</td>
<td>0.9827</td>
<td>62.67$Q_1$</td>
</tr>
<tr>
<td>1.5</td>
<td>$Q_1$</td>
<td>40</td>
<td>103.40</td>
<td>63.40</td>
<td>0.9489</td>
<td>60.16$Q_1$</td>
</tr>
<tr>
<td>2.5</td>
<td>$Q_1$</td>
<td>40</td>
<td>103.01</td>
<td>63.01</td>
<td>0.9162</td>
<td>57.73$Q_1$</td>
</tr>
<tr>
<td>3.5</td>
<td>$Q_2$</td>
<td>40</td>
<td>102.61</td>
<td>62.61</td>
<td>0.8847</td>
<td>55.39$Q_2$</td>
</tr>
<tr>
<td>4.5</td>
<td>$Q_2$</td>
<td>40</td>
<td>102.20</td>
<td>62.20</td>
<td>0.8543</td>
<td>53.13$Q_2$</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td><strong>180.56Q_1 + 108.53Q_2</strong></td>
</tr>
</tbody>
</table>

The market price of the bond is 96.24 is and the risk-free value is 101.97. It follows that

$$180.56Q_1 + 108.53Q_2 = 101.97 - 96.24$$

From which we get $Q_2 = 0.0260$ The bond prices therefore imply a probability of default of 1.61% per year for the first three years and 2.60% for the next two years.

**Problem 22.16.**

Suppose that a financial institution has entered into a swap dependent on the sterling interest rate with counterparty X and an exactly offsetting swap with counterparty Y. Which of the following statements are true and which are false.

(a) The total present value of the cost of defaults is the sum of the present value of the cost of defaults on the contract with X plus the present value of the cost of defaults on the contract with Y.

(b) The expected exposure in one year on both contracts is the sum of the expected exposure on the contract with X and the expected exposure on the contract with Y.

(c) The 95% upper confidence limit for the exposure in one year on both contracts is the sum of the 95% upper confidence limit for the exposure in one year on the contract with X and the 95% upper confidence limit for the exposure in one year on the contract with Y.

Explain your answers.

The statements in (a) and (b) are true. The statement in (c) is not. Suppose that $v_X$ and $v_Y$ are the exposures to X and Y. The expected value of $v_X + v_Y$ is the expected value of $v_X$ plus the expected value of $v_Y$. The same is not true of 95% confidence limits.

**Problem 22.17.**

A company enters into a one-year forward contract to sell $100 for AUD150. The contract is initially at the money. In other words, the forward exchange rate is 1.50. The one-year dollar risk-free rate of interest is 5% per annum. The one-year dollar rate of interest at which the counterparty can borrow is 6% per annum. The exchange rate volatility is 12% per annum. Estimate the present value of the cost of defaults on the contract? Assume that defaults are recognized only at the end of the life of the contract.
The cost of defaults is \( uv \) where \( u \) is percentage loss from defaults during the life of the contract and \( v \) is the value of an option that pays off \( \max(150S_T - 100, 0) \) in one year and \( S_T \) is the value in dollars of one AUD. The value of \( u \) is

\[
\begin{align*}
  u &= 1 - e^{-(0.06 - 0.05) \times 1} = 0.009950
\end{align*}
\]

The variable \( v \) is 150 times a call option to buy one AUD for 0.6667. The formula for the call option in terms of forward prices is

\[
[FN(d_1) - KN(d_2)]e^{-rT}
\]

where

\[
\begin{align*}
  d_1 &= \frac{\log(F/K) + \sigma^2T/2}{\sigma\sqrt{T}} \\
  d_2 &= d_1 - \sigma\sqrt{T}
\end{align*}
\]

In this case \( F = 0.6667, K = 0.6667, \sigma = 0.12, T = 1, \) and \( r = 0.05 \) so that \( d_1 = 0.06, d_2 = -0.06 \) and the value of the call option is 0.0303. It follows that \( v = 150 \times 0.0303 = 4.545 \) so that the cost of defaults is

\[
4.545 \times 0.009950 = 0.04522
\]

**Problem 22.18.**

"Suppose that in Problem 22.17, the six-month forward rate is also 1.50 and the six-month dollar risk-free interest rate is 5% per annum. Suppose further that the six-month dollar rate of interest at which the counterparty can borrow is 5.5% per annum. Estimate the present value of the cost of defaults assuming that defaults can occur either at the six-month point or at the one-year point? (If a default occurs at the six-month point, the company’s potential loss is the market value of the contract.)"

In this case the costs of defaults is \( u_1v_1 + u_2v_2 \) where

\[
\begin{align*}
  u_1 &= 1 - e^{-(0.055 - 0.05) \times 0.5} = 0.002497 \\
  u_2 &= e^{-(0.055 - 0.05) \times 0.5} - e^{-(0.06 - 0.05) \times 1} = 0.007453
\end{align*}
\]

\( v_1 \) is the value of an option that pays off \( \max(150S_T - 100, 0) \) in six months and \( v_2 \) is the value of a option that pays off \( \max(150S_T - 100, 0) \) in one year. The calculations in Problem 22.17 shows that \( v_2 = 4.545 \). Similarly \( v_1 = 3.300 \) so that the cost of defaults is

\[
0.002497 \times 3.300 + 0.007453 \times 4.545 = 0.04211
\]

**Problem 22.19.**

"A long forward contract subject to credit risk is a combination of a short position in a no-default put and a long position in a call subject to credit risk." Explain this statement.
Assume that defaults happen only at the end of the life of the forward contract. In a default-free world the forward contract is the combination of a long European call and a short European put where the strike price of the options equals the delivery price and the maturity of the options equals the maturity of the forward contract. If the no-default value of the contract is positive at maturity, the call has a positive value and the put is worth zero. The impact of defaults on the forward contract is the same as that on the call. If the no-default value of the contract is negative at maturity, the call has a zero value and the put has a positive value. In this case defaults have no effect. Again the impact of defaults on the forward contract is the same as that on the call. It follows that the contract has a value equal to a long position in a call that is subject to default risk and short position in a default-free put.

Problem 22.20.

Explain why the credit exposure on a matched pair of forward contracts resembles a straddle.

Suppose that the forward contract provides a payoff at time $T$. With our usual notation, the value of a long forward contract is $S_T - Ke^{-rT}$. The credit exposure on a long forward contract is therefore $\max(S_T - Ke^{-rT}, 0)$; that is, it is a call on the asset price with strike price $Ke^{-rT}$. Similarly, the credit exposure on a short forward contract is $\max(Ke^{-rT} - S_T, 0)$; that is, it is a put on the asset price with strike price $Ke^{-rT}$. The total credit exposure is, therefore, a straddle with strike price $Ke^{-rT}$.

Problem 22.21.

Explain why the impact of credit risk on a matched pair of interest rate swaps tends to be less than that on a matched pair of currency swaps.

The credit risk on a matched pair of interest rate swaps is $|B_{\text{fixed}} - B_{\text{floating}}|$. As maturity is approached all bond prices tend to par and this tends to zero. The credit risk on a matched pair of currency swaps is $|SB_{\text{foreign}} - B_{\text{fixed}}|$ where $S$ is the exchange rate. The expected value of this tends to increase as the swap maturity is approached because of the uncertainty in $S$.

Problem 22.22.

"When a bank is negotiating currency swaps, it should try to ensure that it is receiving the lower interest rate currency from a company with a low credit risk." Explain.

As time passes there is a tendency for the currency which has the lower interest rate to strengthen. This means that a swap where we are receiving this currency will tend to move in the money (i.e., have a positive value). Similarly, a swap where we are paying the currency will tend to move out of the money (i.e., have a negative value). From this it follows that our expected exposure on the swap where we are receiving the low-interest currency is much greater than our expected exposure on the swap where we are receiving the high-interest currency. We should therefore look for counterparties with a low credit risk on the side of the swap where we are receiving the low-interest currency. On the other side of the swap we are far less concerned about the creditworthiness of the counterparty.
Problem 22.23.

Does put–call parity hold when there is default risk? Explain your answer.

No, put–call parity does not hold when there is default risk. Suppose \(c^*\) and \(p^*\) are the no-default prices of a European call and put with strike price \(K\) and maturity \(T\) on a non-dividend-paying stock whose price is \(S\), and that \(c\) and \(p\) are the corresponding values when there is default risk. The text shows that when we make the independence assumption (that is, we assume that the variables determining the no-default value of the option are independent of the variables determining default probabilities and recovery rates),

\[
c^* + Ke^{-\psi(T) - \psi'(T)T} = p^* + S
\]

which holds in a no-default world therefore becomes

\[
c + Ke^{-\psi(T)T} = p + Se^{-\psi(T) - \psi'(T)T}
\]

when there is default risk. This is not the same as regular put–call parity. What is more, the relationship depends on the independence assumption and cannot be deduced from the same sort of simple no-arbitrage arguments that we used in Chapter 9 for the put–call parity relationship in a no-default world.

Problem 22.24.

Suppose that in an asset swap \(B\) is the market price of the bond per dollar of principal, \(B^*\) is the default-free value of the bond per dollar of principal, and \(V\) is the present value of the asset swap spread per dollar of principal. Show that \(V = B^* - B\).

We can assume that the principal is paid and received at the end of the life of the swap without changing the swap's value. If the spread were zero the present value of the floating payments per dollar of principal would be 1. The payment of LIBOR plus the spread therefore has a present value of \(1 + V\). The payment of the bond cash flows has a present value per dollar of principal of \(B^*\). The initial payment required from the payer of the bond cash flows per dollar of principal is \(1 - B\). (This may be negative; an initial amount of \(B - 1\) is then paid by the payer of the floating rate). Because the asset swap is initially worth zero we have

\[
1 + V = B^* + 1 - B
\]

so that

\[
V = B^* - B
\]

Problem 22.25.

Show that under Merton's model in Section 22.6 the credit spread on a \(T\)-year zero-coupon bond is

\[
-\ln[N(d_2) + N(-d_1)/L]/T \text{ where } L = De^{-rT}/V_0.
\]

The value of the debt in Merton's model is \(V_0 - E_0\) or

\[
De^{-rT}N(d_2) - V_0N(d_1) + V_0 = De^{-rT}N(d_2) + V_0N(-d_1)
\]

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If the credit spread is \( s \) this should equal \( D e^{-(r+s)T} \) so that

\[
D e^{-(r+s)T} = D e^{-rT} N(d_2) + V_0 N(-d_1)
\]

Substituting \( D e^{-rT} = LV_0 \)

\[
LV_0 e^{-sT} = LV_0 N(d_2) + V_0 N(-d_1)
\]

or

\[
Le^{-sT} = LN(d_2) + N(-d_1)
\]

so that

\[
s = -\ln[N(d_2) + N(-d_1)/L]/T
\]

**Problem 22.26.**

Suppose that the spread between the yield on a 3-year zero-coupon riskless bond and a 3-year zero-coupon bond issued by a corporation is 1%. By how much does Black–Scholes overstate the value of a 3-year European option sold by the corporation.

When the default risk of the seller of the option is taken into account the option value is the Black–Scholes price multiplied by \( e^{-0.01 \times 3} = 0.9704 \). Black–Scholes overprices the option by about 3%.

**Problem 22.27**

Give an example of a) right-way risk and b) wrong-way risk.

(a) Right way risk describes the situation when a default by the counterparty is most likely to occur when the contract has a positive value to the counterparty. A example of right way risk would be when a counterparty's future depends on the price of a commodity and it enters into a contract to partially hedging that exposure.

(b) Wrong way risk describes the situation when a default by the counterparty is most likely to occur when the contract has a negative value to the counterparty. A example of right way risk would be when a counterparty is a speculator and the contract has the same exposure as the rest of the counterparty's portfolio.