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An Equilibrium Model of Bond Pricing and a Test of Market Efficiency

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In two previous and related papers ([3], [4]), the authors have reported the results of estimating a particular equilibrium model of bond pricing using quarterly data on Canadian government bonds for the period 1964 to 1979. This paper reports the results of applying a similar model to the pricing of U.S. government bonds for the period 1958-1979 using data from the CRSP Government Bond File. The paper also extends the previous empirical analysis by evaluating the ability of the pricing model to detect underpriced and overpriced bonds: the data reveal a strong relation between price prediction errors and subsequent bond returns.

The bond pricing model is described briefly in the following section: it relies on the assumption that there are only two independent stochastic factors which determine bond prices and, therefore, the shape and position of the yield curve at any point in time. Then, in the spirit of the Option Pricing Model ([2], [11]) and Arbitrage Pricing Theory ([13]), this assumption is shown to imply restrictions on the relative rates of return of bonds of different coupon and maturity. These restrictions take the form of a partial differential equation which must be satisfied by the equilibrium values of all default-free bonds. Solution of this equation yields bond prices, and, therefore, the term structure of interest rates or yield curve, in terms of the underlying factors or state variables: in the present paper, these unknown state variables are replaced by two interest rates, the short rate and the consol rate.

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Since the coefficients of the differential equation depend upon the parameters of the stochastic process for the underlying state variables, these parameters must be estimated before the model can be implemented. This task is undertaken in Section II which also reports the results of estimating the single, utility function dependent, market price of risk parameter which enters the model. In Section III, the ability of the model to make predictions of bond prices and yields is evaluated and evidence is found of an omitted "third factor" which affects bond prices: this finding is congruent with those of the above-mentioned Canadian study. Section IV compares the predictive performance of several different rules for forecasting bond returns conditional on forecasts of the exogenously determined state variables. The final section of the paper relates the bond pricing errors to subsequent bond returns. A strong relationship between pricing errors and bond returns is found even outside the sample period over which the pricing model was estimated. Whether or not this is interpreted as evidence of market inefficiency and a profit opportunity will depend upon one's belief in the adequacy of the underlying equilibrium model and the accuracy of the price data.

I. The Bond Pricing Model[1]

It is assumed that the prices of all default-free bonds at any moment in time may be expressed in terms of the values of two, possibly unknown, stochastic factors, u_1 and u_2, which follow continuous sample paths. Thus, the price of a bond with a continuous coupon rate c, face value of unity and maturity τ, can be written as $G(u_1,u_2,\tau,c)$.

The instantaneously riskless interest rate, the "short rate," is the yield on the currently maturing discount bond and is defined by

$$(1)\qquad r(u_1,u_2) = \lim_{\tau\to 0} \frac{-\ln G(u_1,u_2,\tau,0)}{\tau} .$$

Similarly, the "consol rate" is defined as the yield on a bond whose maturity is infinite

$$(2)\qquad \ell(u_1,u_2) = \frac{c}{G(u_1,u_2,\infty,c)} .$$

If equations (1) and (2) can be inverted and the state variables u_1 and u_2 expressed as twice differentiable functions of the potentially observable

[1]A more complete description of the model may be found in [3].

interest rates, r and ℓ, then, as pointed out by Cox, Ingersoll, and Ross [6], bond prices may be expressed as functions of the proxy state variables r and ℓ, and the value of any default-free bond written as $B(r,\ell,\tau;c)$: in what follows, it will be convenient to suppress the coupon rate argument and to write the bond value simply as $B(r,\ell,\tau)$.

The price change on a bond over any short interval of time, and, therefore, its rate of return, will depend upon the corresponding change in the state variable proxies, r and ℓ. These are assumed to follow a stochastic process of the general type

$$(3) \qquad dr = \beta_1(r,\ell,t)\,dt + \eta_1(r,\ell,t)\,dz_1$$

$$d\ell = \beta_2(r,\ell,t)\,dt + \eta_2(r,\ell,t)\,dz_2$$

where t denotes calendar time, and dz_1 and dz_2 are increments to a standard Wiener process, so that $E[dz_1] = E[dz_2] = 0$, $E[dz_1 . dz_2] = \rho dt$ and $E[dz_1^2] = E[dz_2^2] = dt$; ρ is the instantaneous correlation between the processes.

It then follows from Ito's Lemma [10] that the instantaneous return on a bond is given by

$$(4) \qquad \frac{dB + c\,dt}{B} = \mu\,dt + \frac{B_r}{B}\eta_1\,dz_1 + \frac{B_\ell}{B}\eta_2\,dz_2$$

where

$$(5) \qquad \mu = (B_r\beta_1 + B_\ell\beta_2 + 1/2\,B_{rr}\,\eta_1^2 + 1/2\,B_{\ell\ell}\,\eta_2^2 + B_{r\ell}\rho\eta_1\eta_2 - B_\tau + c)/B$$

and subscripts denote partial derivatives.

Since the instantaneous returns on all default-free bonds are by assumption from equation (4) perfect linear functions of the two stochastic increments dz_1 and dz_2, the absence of arbitrage possibilities can be shown to imply

$$(6) \qquad \mu - r = \lambda_1 \frac{B_r}{B}\eta_1 + \lambda_2 \frac{B_\ell}{B}\eta_2$$

where λ_1 and λ_2, the market prices of short-term and consol rate risk respectively, are at most functions of r, ℓ, and t. The important aspect of condition (6) is that λ_1 and λ_2 are the same for all bonds at any particular instant in time. Recognizing that μ is the expected instantaneous rate of return on the bond, it is seen that (6) expresses the risk premium as the sum of two components which depend on the sensitivity of the bond return to changes in the short and consol rates, respectively: the correspondence with Arbitrage Pricing Theory is apparent.

The value of a consol bond paying a continuous coupon at a rate of \$1 per period is $B(r,\ell,\infty;1) \equiv \ell^{-1}$, and its derivatives with respect to r and ℓ are readily computed. Substitution of these derivatives for the consol bond into (5) and (6) yields the following expression for the market price of consol rate risk:

$$\lambda_2(r,\ell,t) = \frac{-\eta_2}{\ell} + (\beta_2 - \ell^2 + r_\ell)/\eta_2. \tag{7}$$

Finally, substitution for $\mu(\cdot)$ from (5) and for $\lambda_2(\cdot)$ from (7) into the equilibrium condition (6) yields the following partial differential equation which must be satisfied by the value of all default-free bonds:

$$\begin{aligned} &1/2\ B_{rr}\ \eta_1^2 + B_{r\ell}\ \rho\eta_1\eta_2 + 1/2\ B_{\ell\ell}\ \eta_2^2 + B_r(\beta_1 - \lambda_1\eta_1) \\ &+ B_\ell(\eta_2^2/\ell + \ell^2 - r\ell) - B_\tau + c - Br = 0. \end{aligned} \tag{8}$$

The bond value must also satisfy an appropriate boundary condition determining its maturity value, $B(r,\ell,0)$. Note that it is not actually necessary to solve this equation more than once in order to value all straight default-free bonds, for if the equation is solved with $B(r,\ell,0) = 1$ and c=0, the resulting values of $B(r,\ell,\tau)$ will be the values of discount bonds with par value of unity, or discount factors which give the present value of \$1 due in τ periods when the current short and long rates are r and ℓ, respectively. By applying these discount factors to the promised payments on any straight default-free bond, its model value may be calculated directly. If the bond contains option-like features, such as call, retraction, or exchange provisions, then it also may be valued using equation (8) by appending the appropriate boundary conditions which define its payoffs. In this paper, we are concerned only with straight bonds.

II. Model Estimation

The coefficients of the partial differential equation (8) depend upon four functions which derive from the stochastic process for r and ℓ: β_1, η_1, η_2, and ρ. They also depend upon $\lambda_1(\cdot)$, the market price of short-term interest rate risk. We shall take up first the estimation of the stochastic process and then the estimation of $\lambda_1(\cdot)$.

A. The Stochastic Process

The specific form of the stochastic process (3) which was assumed for purposes of estimation was

$$
\begin{aligned}
dr &= (a_1 + b_1(\ell - r))\, dt + r\sigma_1\, dz_1 \\
d\ell &= \ell(a_2 + b_2 r + c_2 \ell)\, dt + \ell\, \sigma_2\, dz_2.
\end{aligned} \tag{9}
$$

This formulation presupposes that the scale of the unanticipated increment in each of the interest rates is proportional to the current value of that rate, an hypothesis which is tested below. The coefficient of dt in the short rate equation reflects the essence of expectations-based theories of the term structure, which is that long rates are based upon expectations about future short interest rates: if such expectations are rational, the short rate will have a tendency to regress towards the current value of the long rate so that $b_1 > 0$. The coefficient of dt in the consol rate equation was obtained by treating $\lambda_2(\cdot)$, the market price of consol rate risk, as a linear function of r and ℓ, and solving equation (7) for $\beta_2(\cdot)$; it should be observed that $\beta_2(\cdot)$ does not enter the partial differential equation and hence does not affect bond prices.[2] It may be noted also that if $a_1 < 0$, this formulation allows the possibility that r may become negative, a circumstance that is theoretically unacceptable if money exists. Despite this, the above formulation was retained in view of its empirical tractability, and it should be regarded as a linearized approximation to the true stochastic process: our interest here is not in the stochastic process per se but in the predictive ability of the bond pricing model which results from it.

For empirical purposes, the system (9) was replaced by the discrete approximation:

$$
\begin{aligned}
\frac{r_t - r_{t-1}}{r_{t-1}} &= \frac{a_1}{r_{t-1}} + b_1 \left(\frac{\ell_{t-1}}{r_{t-1}} - 1\right) + \xi_{1t} \\
\frac{\ell_t - \ell_{t-1}}{\ell_{t-1}} &= a_2 + b_2 r_{t-1} + c_2 \ell_{t-1} + \xi_{2t}.
\end{aligned} \tag{10}
$$

This system of equations was estimated using monthly data on interest rates from the CRSP Government Bond File for the period December 1958-December

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[2] This is analogous to the standard result in the option pricing literature that the value of an option does not depend on the expected rate of return on the underlying stock. See Appendix in [3].

1979. r was taken as the annualized yield to maturity (percent) on the U.S. Government Treasury Bill whose maturity was closest to 30 days on the last trading day of each month. The consol rate, ℓ, was approximated by the annualized yield to maturity on the highest-yielding U.S. Government Bond with a maturity exceeding 20 years; if no such bond was available in a particular month, then the highest-yielding bond with a maturity of more than 15 years was used instead.[3]

The system was estimated using an iterative Aitken [1] procedure which yields the maximum likelihood estimator [7]. The parameter estimates for different subperiods are reported in Table 1 along with some diagnostic statistics. We observe first that the estimated value of a_1, although predominantly negative, is small, less than 0.2 percent in absolute value. Combining this with the estimate of b_1 of about 0.1, it is seen that the change in r at $r=0$ will be positive so long as the consol rate exceeds about 2 percent. It is to be hoped, therefore, that the misspecification which allows negative values of r will have only slight consequences for bond values.

The estimated values of σ_1 and σ_2 for the two half-periods are quite similar; on the other hand, the correlation coefficient estimate exhibits much less stability. In the consol rate equation, the estimates of b_2 and c_2 are of opposite sign and approximately equal magnitude: this confirms Shiller's finding [14] that when long rates are high relative to short rates, they tend to move down in the subsequent period. The estimates of a_1 and b_1 in the short-rate equation are of potentially greater importance for the bond pricing model since they enter directly into the partial differential equation: the instability of the estimates between the two halves of the sample period, therefore, has potentially serious consequences for the model: these consequences will be explored below.

If the joint stochastic process for the two interest rates (9) was correctly specified, the disturbances in the two equations (10) would be serially independent. Table 1 reports the serial correlations of the errors from the two equations as well as Durbin's [8] h-statistic: the latter takes account of the effect of the presence of a lagged dependent variable and is normally distributed for large samples. The errors from both equations are negatively serially correlated; the correlation is statistically significant for the short-rate equation but appears to be confined to the first half of the sample

[3]The highest-yielding bond was chosen to mitigate the problem posed by "flower" bonds whose yields are distorted (bid down) on account of the privilege they offer of redemption at par for payment of estate duty.

period. The negative serial correlation found in these data contrasts with a finding of positive serial correlation of similar Canadian data [4], though the Canadian data were quarterly rather than monthly. Ignoring measurement error, serial correlation of the errors from the estimated stochastic process suggests either that the functional form of the stochastic process is mis-specified, or that the current values of r and ℓ are not sufficient statistics for the joint distribution of future values of r and ℓ, and that, therefore, if the true process is to be estimated in Markov form as is necessary for the derivation of the partial differential equation, then at least one additional state variable must be introduced.[4] On the other hand, negative serial correlation is symptomatic of measurement errors. In Section III, we shall say more about the existence of state variables in addition to r and ℓ.

The assumption that η_1 and η_2 were proportional to r and ℓ was tested using a procedure suggested by Park [12]: the logarithm of the squared error was regressed on the logarithm of the corresponding interest rate.

$$\text{(11)} \qquad \begin{aligned} \ln \hat{\xi}^2_{1t} &= \delta_1 + \gamma_1 \ln r_{t-1} \\ \ln \hat{\xi}^2_{2t} &= \delta_2 + \gamma_2 \ln \ell_{t-1}. \end{aligned}$$

Under the null hypothesis that the process is correctly specified, γ_1 and γ_2 will be equal to zero: the estimated values are shown in Table 1 and, at least for the total sample period, the null hypothesis is not rejected.

B. Estimation of $\lambda_1(\cdot)$

$\lambda_1(\cdot)$, the market price of short-term interest rate risk, was assumed to be an intertemporal constant for purposes of estimation. The details of the estimation procedure are reported elsewhere ([5], Appendix A). The principle employed was to solve the partial differential equation (8) with $c = 0$ and boundary condition $B(r,\ell,0)=1$ using a numerical procedure. The resulting present value factors were used to value the bonds represented each month on the CRSP Government Bond File, and the estimated value of λ_1 was that which minimized the price prediction errors. A generalized least squares procedure was employed to take account of serial and contemporaneous correlation of the errors. To implement this procedure, all of the bonds to be valued each month were assigned to one of ten basic portfolios according to maturity, portfolio

[4] Strictly speaking, it would still be possible to derive the partial differential equation if there were additional state variables so long as they affected only $\lambda_2(\cdot)$.

j (j=1,...,10) consisting of all bonds whose maturity fall between (j-1) and j years: within each of the basic portfolios each bond received equal weight. Bonds whose maturity exceeded ten years were ignored in the estimation because the paucity of observations for the longer maturities posed problems for the estimation of the variance-covariance matrix.

The ordinary (nonlinear) least squares estimator which minimized the squared monthly price prediction errors for the ten basic portfolios was formed and the resulting errors were then used to estimate the variance-covariance matrix for the generalized least squares estimator.

Estimates of λ_1 were obtained using two different sets of parameters from the stochastic process in the partial differential equation: those in Panel A of Table 2 were derived using the parameters of the stochastic process estimated over the whole sample period, the different estimates in this panel depending on the sample subperiod over which bond price predictions were made. The estimate of λ_1 in Panel B was obtained by pricing bonds over the first half of the sample period relying on parameter estimates of the stochastic process which were also obtained from the first half of the sample period. We shall refer to this model which was derived using only data from the first half of the sample period as the "first-half" estimator, and its performance over the second half of the sample period will be evaluated in Section IV.

It should be clearly recognized that the estimates of λ_1 presented in Table 2 were derived treating the parameters of the stochastic process as known rather than as estimated. This means, of course, that the standard errors are misstated, and it is the difference in the stochastic process parameters which accounts in substantial part for the difference between the estimates of λ_1 for the period December 1958-June 1969 reported in Panels A and B. λ_1 enters the partial differential equation (8) only as part of the term $(\beta_1 - \lambda_1\eta_1)$ and it is to be expected, therefore, that different estimates of the drift function β_1 will tend to be offset by differences in the resulting estimates of λ_1. This effect is illustrated in Table 3 which shows that, despite the large differences in the estimates of λ_1 for December 1958-June 1969 obtained using the different stochastic process estimates, the coefficients of r in the term $(\beta_1 - \lambda_1\eta_1)$ are quite similar.

III. Bond Price and Yield Predictions

The differential equation (8) was solved with c=0 and boundary condition $B(r,\ell,o)=1$ using the estimates of the stochastic process parameters and λ_1 which were obtained from the whole sample period: these are the parameters given in the first lines of Tables 1 and 2. The resulting present value factors were used to price all outstanding bonds[5] with maturities up to 20 years for the last trading day of each month of the sample period. The root mean square price prediction error is reported in the first line of Table 4 which is constructed on the assumption that each bond has a par value of $100. A predicted yield to maturity was also calculated based on the predicted bond price, and the root mean square error of this predicted yield is also reported. The balance of the table reports the prediction errors for December of each year of the sample period: it is apparent that there is considerable intertemporal variation in the predictive ability of the model, although it is encouraging to note that there is no tendency for the model error to grow systematically with time.

While the errors reported in Table 4 are from within sample predictions, Table 5 reports the errors from using the first-half estimator to predict bond prices in the second half of the sample period. As one would expect, these out-of-sample prediction errors are somewhat greater--roughly twice as great as those of the within-sample predictions.

IV. Model Error Analysis

If the assumptions of the valuation model discussed in Section I were correct and if λ_1 were truly an intertemporal constant, then the price prediction errors would be serially and cross-sectionally independent. Table 6 reports both the serial correlations and the contemporaneous correlations of the pricing errors for the ten basic portfolios over the whole sample period. Their systematic nature points either to an incorrect functional form for the model or to the omission of relevant state variables in addition to those represented by r and ℓ. The choice of an incorrect functional form for the joint stochastic process (3) or for λ_1, which was assumed to be a constant, would lead to pricing errors which were systematically related to r and ℓ. We have already adverted to the possible existence of omitted state variables in relation to the serial correlation of the errors in the estimated stochastic process of r and ℓ.

To gain further insight into the causes of the price prediction errors, they were factor analyzed. The factor analysis model assumes that the errors u_{pt}($p=1,\ldots,10$; $t=1,\ldots,253$) may be expressed as

$$u_{pt} = \Sigma_k h_{pk} f_{kt} + e_{pt} \qquad (12)$$

[5]Excluding flower bonds.

where h_{pk} is the loading of the error from portfolio p on factor k, f_{kt} is the factor score in period t, and e_{pt} is a residual error which is serially and cross-sectionally independent.

The factor analysis revealed only one important common factor which accounted for 86.7 percent of the total variance and had a serial correlation of 0.92; the second factor accounted for a further 7.8 percent of the variance. Factor loadings on the first factor for the ten basic portfolios are given in Table 7.

To assess whether the errors are due merely to an incorrect functional form of the model, the factor score was regressed on the contemporaneous values of the proxy state variables r and ℓ. While the ordinary least squares regression suggested a strong relation (Table 8), after adjustment for serial correlation the factor was found to be related only to ℓ. This is consistent with some misspecification of the functional form; however, the large unexplained component in the factor score suggests also the omission of relevant state variables.

If a state variable which has been omitted is to be relevant to bond pricing, it must be that it affects one of the elements of the model which has been assumed to be independent of any state variables except r and ℓ; the candidates are the elements of the joint stochastic process (3), β_i, η_i (i=1,2) and ρ, as well as λ_1, the market price of short-term rate risk.

It is not possible to test whether λ_1 is related to the factor score and, hence, an omitted state variable, since we have no period-by-period estimate of λ_1. However, we can test whether the drift terms of the process, β_i(i=1,2), are related to the factor by regressing the errors from the regressions (10) on the lagged factor score:

$$\text{(13)} \qquad \xi_{it} = h_{1i} + h_{2i} f_{t-1} \quad (i=1,2).$$

The results reported in Table 9 fail to reveal any significant relation, and it does not appear that we have lost anything by representing expectations about future values of r and ℓ solely in terms of their current values. However, further analysis suggests that the factor does appear to be associated with uncertainty about expected future interest rates.

To test whether the state variable is affecting interest rate uncertainty as represented by σ_1 and σ_2 which were taken as constants in the model,[6] the logarithm of the squared errors from the regressions (10) were regressed on the logarithm of the squared lagged factor score:

[6] See equation (9).

(14) $$\ln \xi_{it}^2 = k_{1i} + k_{2i} \ln f_{t-1}^2 \quad (i=1,2).$$

The results reported in Table 10 allow us to reject the null hypothesis that the variance rates are independent of the factor score. Thus, the valuation model is misspecified insofar as it assumes constant rather than stochastic variance rates. On the other hand, it should be recalled that the model was fitted over 20 years, a very long period to assume parameter stationarity, and the very high serial correlation of the factor score suggests that our two-factor model may yet be quite adequate over short periods of time.

V. Conditional Predictions of Returns

To gain insight into the ability of the valuation model to predict changes in bond prices conditional on changes in r and ℓ with and without a factor score adjustment, rates of return were calculated for each month from December 1958 to December 1979 for each of the ten basic portfolios: each portfolio ($p=1,\ldots,10$) consists of all taxable bonds with a time to maturity of between p and $(p-1)$ years at both the beginning and end of the month under consideration. These rates of return were compared with the predictions of four different rules which are based on the valuation model.

Under the first three prediction rules, the valuation model used to yield a prediction of the future value of the portfolio and the predicted rate of return is defined by

(15) $$\hat{R}_{pt} = (\hat{y}_{pt} + c_{pt} - y_{pt-1})y_{pt-1}$$

where c_{pt} is the aggregate coupon payment for the month, and y_{pt-1} is the actual value of the portfolio at the end of the previous month. The first three rules differ in the way $\hat{y}_{pt}$, the predicted value of the portfolio at the end of the current month, is calculated.

Define $B_{pt}(r_t,\ell_t)$ as the model value of portfolio p at the end of month t conditional on the values of r and ℓ. Then

Rule 1: $\hat{y}_{pt} = B_{pt}(r_t,\ell_t)$.

This rule assumes that the current price prediction error will be eliminated by the end of the month.

Rule 2: $\hat{y}_{pt} = B_{pt}(r_t,\ell_t) + \rho_p u_{pt-1}$.

This rule recognizes that the price prediction errors are serially correlated and adjusts the model prediction by the serial correlations reported in Table 6.

Rule 3: $\hat{y}_{pt} = B_{pt}(r_t, \ell_t) + h_p f_t$.

This rule adjusts the model value using the actual factor score which is treated as a state variable like r and ℓ.

Rule 4:

This is identical to Rule 1, except that y_{pt-1}, the actual value of the portfolio, is replaced by its model value, $B_{pt-1}(r_{t-1}, \ell_{t-1})$. The predicted rate of return is thus the same as the model rate of return conditional on the change in the state variables.

The Root Mean Square prediction errors for the four rules are reported in Table II. Rule 1, which assumes that the pricing error will disappear in one month, performs the worst, worse even than the naive rule that the expected rate of return is constant[7]; this is not surprising in view of the previously noted persistence of the errors. The performance of Rules 2 and 4 is approximately the same, although the former relies on assumed knowledge of the serial correlation structure of errors and the latter ignores the pricing errors altogether. Only Rule 3 which presumes knowledge of the factor score performs significantly better, and then only for portfolios with maturities longer than five years.

In summary, it appears that the factor score is of very great importance in predicting absolute bond prices--compare Rules 1 and 3. On the other hand, it is of very little importance in predicting rates of return because its serial correlation is so high--compare Rules 3 and 4.

VI. Price Prediction Errors and Bond Returns

To this point, we have examined the price prediction errors of the valuation model under the implicit assumption that they were attributable entirely to deficiencies in the valuation model itself, rather than to market inefficiencies which would imply the existence of profit opportunities, or even to errors in the price quotations which would imply the existence of apparent profit opportunities. In this section, we take the opposite viewpoint and, treating the pricing errors as possible manifestations of market inefficiency, we test whether they are related in any systematic fashion to subsequent bond returns. Since the issue of market efficiency is a vexed one, we do not wish to be dogmatic about the implications of our finding of such a systematic relation. Within the context of our equilibrium model, this finding does imply market inefficiency. This is not to say, however, that other valuation

[7]The RMSE for this rule is given by the standard deviation of returns, assuming that the mean is known.

models do not exist which would account for this anomaly, or even that the phenomenon is not due merely to the quality of the price quotations.

The discrete time approximation to the exact expression (4) for the instantaneous rate of return on a bond is

$$R_j = E[R_j] + S_{1j}\Delta Z_1 + S_{2j}\Delta Z_2 \tag{16}$$

where the subscript j denotes the particular bond, R_j is the one-period rate of return, ΔZ_1 and ΔZ_2 are the (standardized) unanticipated changes in r and ℓ, respectively, and $S_{1j} = B_r\eta_1/B$, $S_{2j} = B_\ell\eta_2/B$.

The corresponding discrete time approximation to the equilibrium condition (6) is

$$E[R_j] - r = \lambda_1 S_{1j} + \lambda_2 S_{2j} \tag{17}$$

where $E[R_j]$ is the one-period equilibrium expected return on the bond[8] and r is the one-period riskless interest rate.

Combining equations (16) and (17) we obtain

$$R_j - r = (\lambda_1 + \Delta Z_1)S_{1j} + (\lambda_2 + \Delta Z_2)S_{2j}. \tag{18}$$

Motivated by equation (18) the regression model estimated each month t was

$$R_{jt} - r_t = a_{ot} + a_{1t}S_{1jt} + a_{2t}S_{2jt} + a_{3t}E_{jt} + e_{jt} \tag{19}$$

where E_{jt} is the price prediction error for bond j at the beginning of month t under the null hypothesis of market efficiency $a_{3t}=0$. This hypothesis was tested by constructing the means of the time series of the coefficients of equation (19) and calculating the t-statistic in the manner first suggested by Fama and MacBeth [9] in a related context. The results are reported in Tables 12 to 15.

In Table 12, the valuation error E_{jt} was constructed from parameter estimates obtained from the whole sample period and the regressions (19) were estimated using all taxable bonds with maturities up to 20 years. To control for

[8] The values of λ_1 estimated in Section II presuppose that the unit of time is one year. Therefore, if the rate of return in (17) is measured on a monthly or quarterly basis, the value of λ_1 in (17) would be one-twelfth or one-quarter of the value reported above.

possible misspecification of the valuation model, the coupon rate ($COUP_{jt}$) and maturity (MAT_{jt}) of the individual bonds were included as independent variables in some of the regressions. The left half of the table reports the results obtained when bond returns were calculated on a monthly basis; the right half of the table reports results obtained using quarterly rates of return. In either case, the independent variables including the price prediction error were calculated as of the beginning of the observation interval.

The results reported in Table 12 indicate a highly significant relationship between the valuation error and the rate of return over the next time interval--approximately 15 percent of the error is corrected during the next month and 30 percent over the next quarter. The results are quite insensitive to the inclusion of the coupon and maturity variables which suggests that they are not explicable in terms of model misspecification.

Since the price prediction error used in the regressions reported in Table 12 was derived from a valuation model which was estimated over the whole sample period, the prediction errors are not true *ex ante* forecasts and it is possible that the results are attributable to testing the model on the data for which it was estimated. To investigate this issue, the preceding analysis was repeated with the difference that the price prediction error was computed using the first-half estimator which, it will be recalled, used only data from the period December 1958 to June 1969. The results obtained using the first-half estimator are reported in Table 13 which follows the same format as Table 12. It may be seen from this table that the price prediction error continues to have a highly significant effect on the subsequent return even outside the period over which the model was estimated; moreover, the coefficients of the price prediction errors are very similar to those reported in the previous table.

Although inclusion of bond coupon and maturity in the cross-sectional regressions was found to have no significant effect on the coefficient of the price prediction error, model misspecification remains the most plausible explanation of our findings. Since we have found evidence of model misspecification in the systematic behavior of the price prediction errors and related this to a possible omitted state variable which could be proxied for by the factor, it is worth exploring whether this could have accounted for our results.

Assuming that the omitted state variable can be represented by the factor extracted from the price prediction errors and that this factor affects bond prices in a linear fashion, then the expression corresponding to (16) for the rate of return on a bond is

$$(16') \qquad R_j = E[R_j] + S_{1j}\Delta Z_1 + S_{2j}\Delta Z_2 + h_j^*\Delta f$$

where h_j^* is the transformed factor loading for bond j[9] and Δf is the unanticipated change in the factor score. The corresponding equilibrium condition (17) becomes

$$(17') \qquad E[R_j] - r = \lambda_1 S_{1j} + \lambda_2 S_{2j} + \lambda_3 h_j^*.$$

Combining (16') and (17') we obtain

$$(18') \qquad R_j - r = (\lambda_1 + \Delta Z_1)S_{1j} + (\lambda_2 + \Delta Z_2)S_{2j} + (\lambda_3 + \Delta_f)h_j^*$$

and the corresponding regression equation is

$$(19') \qquad R_{jt} - r_t = a_{ot} + a_{1t}S_{1jt} + a_{2t}S_{2jt} + a_{3t}h_j^* + a_{4t}E_{jt} + e_{jt}.$$

The transformed factor loading for each bond, h_j^*, was based on its maturity and the price prediction error factor loadings reported in Table 7. Since these factor loadings were available only for maturities up to ten years, the regressions were restricted to bonds of maturity less than ten years.

Tables 14 and 15 report the regression results and correspond to Tables 12 and 13, respectively, except for the inclusion of the transformed factor loading as an independent variable and the restriction of the sample to maturities of less than ten years. Addition of the factor loading has no significant effect on the results.

It does not seem that the results can be easily explained in terms of model misspecification. If they are not due to model misspecification, then the data suggest market inefficiency, and whether true profit opportunities existed depends upon whether the bond prices used were true end-of-month transaction prices at which further transactions could have been made.

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[9] $h_j^* = h_j/B_j$ where h_j is the factor loading for the price prediction errors reported in Table 7 and B_j is the price of the bond at the beginning of the period: the transformation is required because (18') is in terms of rates of return, not prices.

TABLE 1

ESTIMATION OF THE STOCHASTIC PROCESS

(Standard Errors in Parentheses)

Period	a_1	b_1	a_2	b_2	c_2	σ_1	σ_2	ρ	$\rho(\xi_{1t}\xi_{1t-1})$	$\rho(\xi_{2t}\xi_{2t-1})$	DH_1^*	DH_2^*	γ_1	γ_2
Dec 1958 -	-.0887	.1102	.00891	.00358	-.0037	.1133	.0298	.2063	-.195	-.064	-3.52	-1.022	-.6269	.7503
Dec 1979	(.0526)	(.0301)	(.0069)	(.0017)	(.0020)								(.3710)	(.5711)
Dec 1958 -	-.1809	.1882	.0151	.00468	-.0062	.1286	.02333	.0519	-.239	-.032	-3.183	-.364	-1.9363	3.9373
June 1969	(.0754)	(.0480)	(.0200)	(.0037)	(.0067)								(.6704)	(1.701)
July 1969 -	-.0135	.0377	.0319	.00444	-.0074	.0914	.0349	.3923	-.043	-.067	-.526	-.748	-.2489	-1.1807
Dec 1979	(.0826)	(.0369)	(.0221)	(.00229)	(.0037(								(.7837)	(1.6619)
Dec 1958 -	-.4667	.3357	.2142	.00869	.0588	.1619	.0205	.2126	-.160	.018	-1.830	.146	-1.3433	-8.270
March 1964	(.1594)	(.0907)	(.0725)	(.00567)	(.0202)								(1.2774)	(6.889)
April 1964 -	-.1079	.2729	.00078	.0038	-.0022	.0688	.0248	-.1637	-.104	-.022	-1.066	-.175	-1.3955	8.656
June 1969	(.0593)	(.0796)	(.0252)	(.0069)	(.0087)								(1.9133)	(2.200)
July 1969 -	-.0350	.0551	.1233	.0120	-.0283	.1031	.0415	.3778	-.034	-.040	-.308	-.320	+.2869	-.3579
Sept 1974	(.1103)	(.0578)	(.0614)	(.0055)	(.0129)								(1.110)	(3.002)
Oct 1974 -	.0361	.0118	.1675	.0089	-.271	.0777	.0243	.4423	.026	-.032	.231	-.259	-.6625	4.348
Dec 1979	(.1375)	(.0525)	(.0713)	(.0033)	(.0109)								(1.137)	(4.286)

*Durbin's h-statistic.

TABLE 2

ESTIMATION OF THE MARKET PRICE OF SHORT-TERM INTEREST RATE RISK (λ_1)

Period	λ^*	$SE(\lambda^*)$	Number of Observations
A. Stochastic Process Parameter Estimates: December 1958 - December 1979			
Dec. 1958 - Dec. 1979	-0.450	0.028	2016
Dec. 1958 - June 1969	-0.674	0.023	960
July 1969 - Dec. 1979	-0.283	0.031	1049
Dec. 1958 - March 1964	-0.671	0.043	480
April 1964 - June 1969	-0.593	0.021	471
July 1969 - Sept. 1974	-0.322	0.068	479
Oct. 1974 - Dec. 1979	-0.216	0.013	561
B. Stochastic Process Parameter Estimates: December 1958 - June 1969			
Dec. 1958 - June 1969	-1.185	0.019	960

TABLE 3

TWO ESTIMATES OF ($\beta_1 - \lambda_1\eta_1$) FOR DECEMBER 1958-JUNE 1969

	Stochastic Process Estimate
$\beta_1 - \lambda_1\eta_1 = a_1 + b_1\ell - (b_1 + \lambda_1\sigma_1)r$	
$-0.0887 + .1102\ell - 0.0338r$	Dec. 1958 - Dec. 1979
$-0.1809 + .1882\ell - 0.0358r$	Dec. 1958 - June 1969

TABLE 4

BOND PRICE AND YIELD PREDICTIONS FOR DECEMBER 1958-DECEMBER 1979[1]

	Number of Observations	Prices[2] Mean Error[3] ($)	Prices[2] RMSE ($)	Yields Mean Error[3] (%)	Yields RMSE (%)
Full Period	11669	.18	1.58	.08	.59
1958[4]	27	-.03	1.29	.32	.38
59	33	-.50	1.74	.67	.71
60	35	-.19	.65	.04	.18
61	38	-.42	.63	.11	.23
62	40	.02	.75	-.15	.33
63	40	-.11	.60	.01	.21
64	38	-.23	.71	.12	.28
65	35	-.81	1.22	.43	.52
66	33	-.44	.77	.25	.35
67	34	-.91	1.24	.55	.71
68	33	.18	1.26	.08	.63
69	36	-2.21	2.50	1.39	1.55
70	40	.27	.43	-.27	.83
71	42	-.01	.52	-.14	.72
72	43	-1.12	1.48	.38	.54
73	45	1.70	2.40	-.38	.73
74	51	2.01	3.23	-.27	1.10
75	60	.94	1.54	-.23	.32
76	71	1.89	2.37	-.66	.72
77	81	.68	1.51	.01	.39
78	87	.20	1.59	.48	.94
79	94	.17	1.91	.67	1.31

[1]Parameter estimates from whole sample period.

[2]Per $100 par value.

[3]Actual - predicted.

[4]December.

TABLE 5

BOND PRICE AND YIELD PREDICTIONS FOR JULY 1969-DECEMBER 1979

USING FIRST-HALF ESTIMATOR

	Number of Observations	Prices		Yields	
		Mean Error[2] ($)	RMSE ($)	Mean Error[2] (%)	RMSE (%)
Full Period	7045	2.53	3.90	-.71	1.07
1969[3]	36	-1.09	1.31	.92	1.21
70	40	1.65	2.07	-.87	1.10
71	42	1.48	1.89	-.70	.93
72	43	.19	.52	-.07	.36
73	45	3.28	4.43	-.89	1.16
74	51	4.02	5.65	-1.01	1.58
75	60	3.18	4.24	-1.10	1.16
76	71	4.01	4.96	-1.44	1.49
77	81	2.87	4.06	-.82	.99
78	87	2.13	3.83	-.20	1.00
79	94	2.55	4.52	-.18	1.33

[1] Per $100 par value.

[2] Actual - predicted.

[3] December.

TABLE 6

CONTEMPORANEOUS CORRELATION OF VALUATION ERRORS FOR TEN BASIC PORTFOLIOS FOR TOTAL SAMPLE PERIOD: $\lambda_1 = -.45$

Portfolio Maturity (years)	1	2	3	4	5	6	7	8	9	10
1	1.00									
2	.87	1.00								
3	.74	.90	1.00							
4	.52	.74	.89	1.00						
5	.24	.49	.69	.86	1.00					
6	.38	.57	.67	.71	.62	1.00				
7	.31	.52	.67	.77	.69	.81	1.00			
8	-.31	-.19	-.05	.14	.45	.07	.02	1.00		
9	-.41	-.11	.10	.33	.65	.25	.37	.90	1.00	
10	-.46	-.25	-.08	.20	.47	.05	.23	.94	.94	1.00
Serial Correlation	.75	.78	.85	.88	.91	.88	.90	.97	.91	.96

TABLE 7

FACTOR LOADINGS OF PORTFOLIO ERRORS

(for total period)

Portfolio	1	2	3	4	5	6	7	8	9	10
Loading	-0.03	0.21	0.59	0.98	1.36	1.74	1.98	2.71	2.68	3.11

TABLE 8

REGRESSION OF FACTOR SCORE ON r AND ℓ

(t-ratios in parentheses)

	a	b	c	$\hat{\rho}$[1]	R^2
OLS	-2.85 (16.76)	-0.14 (3.5)	0.58 (11.6)		.52
Cochrane-Orcutt	-2.41 (4.63)	0.04 (0.80)	0.36 (3.60)	0.86	.10[2]

$$f_t = a + br_t + c\ell_t$$

[1]Serial Correlation.

[2]In terms of changes.

TABLE 9

	h_1	h_2	R^2
$r:\hat{\xi}_{1t}$	-0.003 (0.41)	-0.008 (1.19)	0.1
$\ell:\hat{\xi}_{2t}$	0.000 (0.08)	0.0007 (0.39)	0.00

$$\hat{\xi}_{it} = h_{1i} + h_{2i}\, f_{t-1}$$

TABLE 10

(t-ratios in parentheses)

	k_1	k_2	R^2
$r:\ln\hat{\xi}^2_{1t}$	-6.13 (36.06)	0.17 (1.89)	0.01
$\ell:\ln\hat{\xi}^2_{2t}$	-8.18 (48.94)	0.22 (2.20)	0.02

$$\ln\hat{\xi}^2_{it} = k_{1i} + k_{2i}\ln f^2_{t-1}$$

TABLE 11

MONTHLY RATE OF RETURN PREDICTION ERRORS FOR

TEN BASIC PORTFOLIOS DECEMBER 1958-DECEMBER 1979

Maturity (years)	Standard Deviation of Monthly Returns %	Prediction Rule 1	2	3	4
		Root Mean Square Error			
1	0.27	0.30	0.16	0.30	0.17
2	0.53	0.58	0.33	0.48	0.38
3	0.76	0.96	0.48	0.53	0.50
4	0.95	1.25	0.59	0.51	0.62
5	1.09	1.71	0.71	0.74	0.74
6	1.31	1.96	0.81	0.55	0.83
7	1.43	2.15	0.90	0.58	0.94
8	1.24	3.06	0.63	0.58	0.65
9	1.28	3.25	1.13	0.83	1.14
10	1.45	3.76	0.98	0.82	0.99

TABLE 12

BOND RETURNS AND PRICE PREDICTION ERRORS

Valuation Model Estimated over December 1958 to December 1979
All Taxable Bonds with Maturities less than 20 Years
(t-ratios in parentheses)

	Holding Period = 1 Month						Holding Period = 3 Months					
	Intercept ($\times 10^{-3}$)	S_{1jt}	S_{2jt}	$COUP_{jt}$ ($\times 10^{-3}$)	MAT_{jt} ($\times 10^{-3}$)	E_{jt}	Intercept ($\times 10^{-3}$)	S_{ijt}	S_{2jt}	$COUP_{jt}$ ($\times 10^{-3}$)	MAT_{jt} ($\times 10^{-3}$)	E_{jt}
December	.485	-.0233	.0201				.814	-.0585	.0638			
1958 to	(2.53)	(-.56)	(1.11)				(1.09)	(-.44)	(1.18)			
December	-1.22	-.114	.0352			.136	-1.44	-.139	.0675			.274
1979	(-3.72)	(-2.41)	(1.60)			(7.88)	(-1.79)	(-1.08)	(1.16)			(7.74)
	-.817	-.145	-.0448	-.104	-.0304	.146	-.933	-.129	-.0519	-.0915	-.0595	.311
	(-2.14)	(-2.08)	(-.35)	(-2.02)	(-.61)	(9.98)	(-1.02)	(-.81)	(-.18)	(-.79)	(-.53)	(8.93)
December	.385	-.0328	.0197				.147	-.125	.106			
1958 to	(1.99)	(-.51)	(.84)				(.20)	(-.67)	(1.43)			
June 1969	-.0370	-.0489	.0536			.0646	-.440	-.150	.184			.146
	(-.14)	(-.74)	(2.22)			(3.69)	(-.62)	(-.86)	(2.54)			(3.75)
	.175	-.140	-.124	-.184	-.0812	.105	.453	-.179	-.0579	-.322	-.125	.243
	(.68)	(-1.14)	(-.54)	(-3.11)	(-.95)	(6.05)	(.64)	(-.70)	(-.11)	(-2.39)	(-.65)	(5.64)
July	.586	-.0136	.0206				1.47	-.0193	.0626			
1969 to	(1.76)	(-.27)	(.73)				(1.12)	(-.13)	(.78)			
December	-2.43	-.180	.0166			.209	-2.11	-.125	.0117			.386
1979	(-4.11)	(-2.65)	(.45)			(7.30)	(-1.47)	(-.79)	(.12)			(5.80)
	-1.83	-.150	.0359	-.0229	.0211	.188	-.799	-.108	.00243	-.0931	.0105	.310
	(-2.56)	(-2.25)	(.33)	(-.27)	(.42)	(8.13)	(-.41)	(-.69)	(.01)	(-.42)	(.10)	(6.21)

TABLE 13

BOND RETURNS AND PRICE PREDICTION ERRORS

Valuation Model Estimated over December 1958 to June 1969
All Taxable Bonds with Maturities less than 20 Years
(t-ratios in parentheses)

	Holding Period = 1 Month						Holding Period = 3 Months					
	Intercept (x 10^{-3})	S_{1jt}	S_{2jt}	$COUP_{jt}$ (x 10^{-3})	MAT_{jt} (x 10^{-3})	E_{jt}	Intercept 10^{-3})	S_{1jt}	S_{2jt}	$COUP_{jt}$ (x 10^{-3})	MAT_{jt} (x 10^{-3})	E_{jt}
December	.441	-.0346	.0220				.757	-.0958	.0704			
1958 to	(1.97)	(-.67)	(1.04)				(.78)	(-.53)	(1.07)			
December	-2.40	-.312	-.0863			.152	-3.51	-.521	-.145			.283
1979	(-5.73)	(-4.82)	(-2.77)			(8.25)	(-3.26)	(-2.75)	(-1.74)			(7.81)
	-1.95	-.387	-.320	-.113	-.0767	.173	-2.83	-.565	-.600	-.0619	-.159	.341
	(-4.31)	(-5.11)	(-2.40)	(-2.17)	(-1.70)	(10.87)	(-2.52)	(-2.89)	(-1.80)	(-.53)	(-1.49)	(9.29)
December	.385	-.0457	.0218				-.0453	-.203	.117			
1958 to	(1.69)	(-.56)	(.81)				(-.05)	(-.76)	(1.32)			
June 1969	-.336	-.157	.0206			.0657	-1.35	-.440	.119			.146
	(-.98)	(-1.69)	(.72)			(3.46)	(-1.31)	(-1.62)	(1.31)			(3.37)
	-.350	-.364	-.363	-.200	-.128	.122	-.804	-.657	-.549	-.286	-.226	.258
	(-1.11)	(-2.91)	(-1.53)	(-3.28)	(-1.64)	(6.39)	(-.90)	(-2.16)	(-.95)	(-2.11)	(-1.24)	(5.61)
July	.499	-.0234	.0221				1.37	-.0387	.0693			
1969 to	(1.28)	(-.37)	(.68)				(.82)	(-.19)	(.73)			
December	-4.50	-.469	-.195			.239	-5.45	-.562	-.368			.436
1979	(-6.21)	(-5.30)	(-3.62)			(8.04)	(-2.85)	(-2.47)	(-2.84)			(6.39)
	-3.58	-.412	-.277	-.0234	.0250	.225	-3.19	.462	.450	-.0977	-.0453	.361
	(-4.30)	(-4.80)	(-2.32)	(-.28)	(-.56)	(9.07)	(-1.32)	(-2.02)	(-1.65)	(-.43)	(-.48)	(6.81)

TABLE 14

BOND RETURNS AND PRICE PREDICTION ERRORS

Valuation Model Estimated over December 1958 to December 1979
All Taxable Bonds with Maturities less than 10 Years
(t-ratios in parentheses)

	Holding Period = 1 Month							Holding Period = 3 Months						
	Intercept (x 10^{-3})	S_{1jt}	S_{2jt}	$COUP_{jt}$ (x 10^{-3})	MAT_{jt} (x 10^{-3})	h^*_{jt}	E_{jt}	Intercept (x 10^{-3})	S_{ijt}	S_{2jt}	$COUP_{jt}$ (x 10^{-3})	MAT_{jt} (x 10^{-3})	h^*_{jt}	E_{jt}
December	.516	-.0135	.0171					.753	-.0597	.0628				
1958 to	(2.97)	(-.37)	(.87)					(1.07)	(-.48)	(1.14)				
December	.548	-.00646	-.00538			-.0267		.813	-.0702	-.186			.141	
1979	(3.24)	(-.18)	(-.12)			(-.46)		(1.19)	(-.57)	(2.36)			(1.73)	
	-1.23	-.126	-.0429				.423	-1.59	-.139	.0579				.300
	(-3.98)	(-2.87)	(1.72)				(7.66)	(-2.15)	(-1.13)	(.94)				(7.58)
	-1.28	-.134	.0638			.0352	.143	-1.67	-.162	.180			.166	.303
	(-4.28)	(-3.11)	(1.92)			(.93)	(7.93)	(-2.29)	(-1.32)	(2.61)			(2.57)	(7.69)
	-.872	-.176	-.107	-.0770	-.0532		.140	-1.23	-.190	-.213	-.0347	-.113		.302
	(-2.36)	(-2.35)	(-.69)	(-1.64)	(-.87)		(9.46)	(-1.42)	(-1.12)	(-.62)	(-.34)	(-.87)		(9.25)
	-.974	-.195	-.0989	-.0709	-.0654	.0551	.142	-1.34	-.211	-.148	-.0246	-.126	.125	.309
	(-2.72)	(-2.65)	(-.63)	(-1.55)	(-1.07)	(2.07)	(9.80)	(-1.58)	(-1.26)	(-.43)	(-.25)	(-.97)	(2.77)	(9.44)
December	.449	-.00985	.0129					.302	-.0912	.0952				
1958 to	(2.64)	(-.18)	(.50)					(.44)	(-.52)	(1.28)				
June 1969	.413	-.00174	-.0513			-.0777		.356	-.114	.282			.197	
	(2.48)	(-.03)	(-1.11)			(-1.68)		(.54)	(-.67)	(2.91)			(2.27)	
	-.164	-.0718	.0702				.0714	-.597	-.174	.208				.158
	(-.70)	(-1.25)	(2.61)				(4.00)	(-.88)	(-1.06)	(2.90)				(4.11)
	-.209	-.0742	.0690			-.00532	.0751	-.552	-.184	.328			.125	.159
	(-.89)	(-1.27)	(1.92)			(-.14)	(4.20)	(-.83)	(-1.14)	(3.50)			(1.80)	(4.09)
	.122	-.210	-.273	-.165	-.135		.104	.439	-.281	-.377	-.261	-.235		.277
	(.50)	(-1.58)	(-.97)	(-3.06)	(-1.31)		(5.99)	(.65)	(-1.01)	(-.62)	(-2.29)	(-1.07)		(5.69)
	.112	-.220	-.284	-.168	-.144	.00854	.107	.462	-.296	-.326	-.259	-.251	.0753	.230
	(.46)	(-1.68)	(-1.02)	(-3.15)	(-1.40)	(.32)	(6.06)	(.69)	(-1.08)	(-.52)	(-2.28)	(-1.14)	(1.46)	(5.69)

TABLE 14 (continued)

BOND RETURNS AND PRICE PREDICTION ERRORS

Valuation Model Estimated over December 1958 to December 1979
All Taxable Bonds with Maturities less than 10 Years
(t-ratios in parentheses)

	Holding Period = 1 Month							Holding Period = 3 Months						
	Intercept (x 10^{-3})	S_{1jt}	S_{2jt}	$COUP_{jt}$ (x 10^{-3})	MAT_{jt} (x 10^{-3})	h^*_{jt}	E_{jt}	Intercept (x 10^{-3})	S_{1jt}	S_{2jt}	$COUP_{jt}$ (x 10^{-3})	MAT_{jt} (x 10^{-3})	h^*_{jt}	E_{jt}
July	.584	-.0173	.0213					1.26	-.0330	.0682				
1969 to	(1.91)	(-.36)	(.72)					(1.07)	(-.23)	(.82)				
December	.685	-.0112	.0412			.0250		1.43	-.00278	-.0318			-.167	
1979	(2.31)	(-.24)	(.51)			(.23)		(1.13)	(-.02)	(-.12)			(-.49)	
	-2.32	-.182	.0152				.213	-2.45	-.156	.0202				.402
	(-4.12)	(-2.73)	(.36)				(6.78)	(-1.76)	(-1.04)	(.20)				(5.47)
	-2.37	-.194	.0585			.0763	.212	-2.61	-.197	.157			.244	.394
	(-4.40)	(-3.09)	(1.04)			(1.16)	(6.99)	(-1.94)	(-1.33)	(1.05)			(1.38)	(5.72)
	-1.88	-.141	.0612	.0125	.0298		.176	-1.58	-.130	.135	-.0150	.0599		.280
	(-2.73)	(-2.10)	(.47)	(.16)	(.46)		(7.45)	(-.82)	(-.83)	(.45)	(-.08)	(.40)		(5.51)
	-2.08	-.170	.0889	.0275	.0139	.102	.179	-2.11	-.175	.231	.0327	.0434	.217	.287
	(-3.13)	(-2.58)	(.66)	(.37)	(.21)	(2.23)	(7.83)	(-1.15)	(-1.13)	(.80)	(.19)	(.29)	(2.26)	(5.82)

TABLE 15

BOND RETURNS AND PRICE PREDICTION ERRORS

Valuation Model Estimated over December 1958 to June 1969
All Taxable Bonds with Maturities less than 10 Years
(t-ratios in parentheses)

	Holding Period = 1 Month							Holding Period = 3 Months						
	Intercept ($\times 10^{-3}$)	S_{1jt}	S_{2jt}	$COUP_{jt}$ ($\times 10^{-3}$)	MAT_{jt} ($\times 10^{-3}$)	h^*_{jt}	E_{jt}	Intercept ($\times 10^{-3}$)	S_{1jt}	S_{2jt}	$COUP_{jt}$ ($\times 10^{-3}$)	MAT_{jt} ($\times 10^{-3}$)	h^*_{jt}	E_{jt}
December 1958 to December 1979	.474	-.0211	.0185					.709	-.0860	.0675				
	(2.39)	(-.49)	(.82)					(.82)	(-.53)	(1.00)				
	.334	-.0440	-.0114			-.0336		.427	-.125	-.0511			-.0740	
	(2.12)	(-1.23)	(.44)			(-.77)		(.59)	(-.85)	(.76)			(-.74)	
	-2.34	-.308	-.0907				.158	-3.38	-.465	-.205				.314
	(-5.67)	(-5.10)	(-2.54)				(7.96)	(-3.02)	(-2.66)	(-2.24)				(7.64)
	-2.20	-.288	-.0872			-.0145	.154	-3.02	-.417	-.198			.0103	.309
	(-5.41)	(-5.12)	(-2.41)			(-.43)	(7.73)	(-3.38)	(-2.60)	(-2.25)			(.15)	(7.64)
	-1.92	-.385	-.336	-.0839	-.0842		.164	-2.81	-.535	-.663	-.0217	-.180		.331
	(-4.31)	(-5.00)	(-2.11)	(-1.76)	(-1.54)		(10.16)	(-2.62)	(-2.81)	(-1.78)	(-.21)	(-1.49)		(9.47)
	-1.98	-.470	-.642	-.0454	-.188	-.0760	.164	-2.71	-.663	-.0126	-.0418	-.385	-.151	.326
	(-4.40)	(-.538)	(-3.40)	(-1.01)	(-2.91)	(-2.27)	(10.24)	(-2.50)	(-3.10)	(-2.89)	(.41)	(-2.65)	(-2.31)	(9.35)
December 1958 to June 1969	.451	-.0148	.0143					.241	-.133	.102				
	(2.29)	(-.23)	(.47)					(.28)	(-.57)	(1.14)				
	.291	-.0550	-.00438			-.0447		-.103	-.211	.0989			.0480	
	(1.84)	(-1.16)	(.14)			(-1.01)		(-.15)	(-.97)	(1.15)			(-.45)	
	-.421	-.165	.0294				.0704	-.134	-.421	.124				.154
	(-1.35)	(-2.10)	(.96)				(3.66)	(-1.46)	(-1.74)	(1.37)				(3.63)
	-.413	-.153	.0298			-.00726	.0703	-1.22	-.411	.131			.00170	.149
	(-1.32)	(-2.17)	(.98)			(-.18)	(3.64)	(-1.49)	(-1.82)	(1.58)			(.02)	(3.87)
	-.341	-.378	-.434	-.189	-.149		.118	-.574	-.606	-.652	-.264	-.254		.242
	(-1.11)	(-2.94)	(-1.53)	(-3.35)	(-1.61)		(6.14)	(-.67)	(-2.02)	(-1.00)	(-2.21)	(-1.25)		(5.63)
	-.445	-.542	-.957	-.135	-.314	-.108	.121	-.706	-.862	-.0168	-.156	-.588	-.225	.246
	(-1.49)	(-3.38)	(-2.84)	(-2.47)	(-2.96)	(-2.84)	(6.31)	(-.83)	(-2.43)	(-2.30)	(-1.27)	(-2.69)	(-3.07)	(5.67)

TABLE 15 (continued)

BOND RETURNS AND PRICE PREDICTION ERRORS

Valuation Models Estimated over December 1958 to June 1969
All Taxable Bonds with Maturities less than 10 Years
(t-ratios in parentheses)

	Holding Period = 1 Month							Holding Period = 3 Months						
	Intercept $(\times 10^{-3})$	S_{1jt}	S_{2jt}	$COUP_{jt}$ $(\times 10^{-3})$	MAT_{jt} $(\times 10^{-3})$	h^*_{jt}	E_{jt}	Intercept $(\times 10^{-3})$	S_{1jt}	S_{2jt}	$COUP_{jt}$ $(\times 10^{-3})$	MAT_{jt} $(\times 10^{-3})$	h^*_{jt}	E_{jt}
July	.497	-.0275	.0229					1.09	-.0569	.0751				
1969 to	(1.44)	(-.47)	(.67)					(.76)	(-.31)	(.77)				
December	.379	-.0328	.0186			.0222		-3.15	-.145	-.0632			-.0822	
1979	(1.37)	(-.61)	(.44)			(-.29)		(-.22)	(-.77)	(.57)			(-.37)	
	-4.28	-.454	-.213				.246	-5.50	-.568	-.383				.449
	(-5.88)	(-5.00)	(-3.36)				(7.45)	(-2.88)	(-2.59)	(-2.76)				(5.89)
	-4.01	-.426	-.206			-.0219	.240	-5.15	-.554	-.339			.0512	.406
	(-5.56)	(-4.92)	(-3.19)			(-.40)	(7.15)	(-2.95)	(-2.73)	(-2.53)			(-.38)	(5.55)
	-3.51	-.393	-.236	.0233	-.0188		.211	-3.67	-.461	-.263	-.00402	.00603		.316
	(-4.30)	(-4.61)	(-1.68)	(.31)	(-.32)		(8.29)	(-1.50)	(-2.07)	(-.82)	(-.02)	(-.05)		(5.65)
	-3.55	-.397	-.322	.0459	-.0608	-.0429	.209	-3.70	-.491	-.577	.0461	-.139	-.132	.315
	(-4.25)	(-4.84)	(-1.96)	(.64)	(-.84)	(-.78)	(8.23)	(-1.47)	(-2.32)	(-1.42)	(.26)	(-.77)	(-1.07)	(5.85)

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