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THE PRICING OF OPTIONS ON DEBT SECURITIES

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I. Introduction

In this paper we present a method for valuing American and European put and call options on debt securities. Although no exhange-traded options of this type currently exist in the United States, the Chicago Board Options Exchange plans to introduce option contracts on several government bonds, and the Chicago Board of Trade petitioned the Commodities Futures Trading Commission to allow the trading of options on the Ginny Mae futures contract. In addition to pricing put and call options, the model developed here can be applied to the valuation of other securities such as callable bonds and bank loan commitments.

In their seminal paper on option pricing, Black and Scholes [3] laid the groundwork for a general theory of contingent claim pricing which has since been applied to many problems in finance. The Black-Scholes model depends upon the assumption that the returns of the underlying security follow a stationary random walk through time. However, when the underlying security is a default-free bond, the price of the bond is not likely to follow any type of well-defined probability distribution due to the coupon and maturity effects specific to each instrument. As a result, the random returns of the bond will not, except in some special cases,[1] be stationary, thereby creating a need for an alternative option pricing methodology.

By assuming that the value of the instantaneous interest rate through time is stochastic and described by geometric Brownian motion, Brennan and Schwartz [4] were able to derive a differential equation for the pricing of interest-rate-related securities and options. They solved this equation numerically to

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[1] The only specific case that readily comes to mind is the default-free consol. It might be possible to force a configuration of bond payments and/or time path of interest rates so that the returns of the bond follow a stationary random walk. However, these cases are of little interest.

determine the values of pure discount bonds, savings bonds, retractable bonds, and callable bonds. The latter three securities can be interpreted as having option components.

In this paper we employ the two-state pricing methodology of Rendleman and Bartter [9], Rubinstein [10], and Sharpe [11]. Under certain conditions, the two-state and Brennan-Schwartz models are asymptotically equivalent. However, the generality of the two-state model lends itself to the valuation of nearly any type of contingent claim security. Moreover, the model does not require the formulation or solution of differential equations, and is quite simple to understand, even for the student or practitioner who has not been exposed to other applications of option pricing theory.

II. The Model

A. Assumptions

We make the usual perfect and efficient market assumptions. There are no transaction costs, taxes, or restrictions on short sales involved with the purchase or sale of any security. Information is costless, and the securities market is efficient in the sense that it equates the expected returns of all securities of equivalent risk.

B. Riskless Hedging

As in the Black-Scholes model, the two-state model is based upon the idea that riskless hedges involving an option and its underlying security should be priced to yield the riskless interest rate. Consider a bond on which the option contract is issued. We assume that the price of the bond can take on only one of two values at time t, given its price at time t-1. Let $H_t^+|S(t-1)$ and $H_t^-|S(t-1)$ represent the returns per dollar invested in the bond from time t-1 to time t, given the state of the world at time t-1. The + represents the state at time t, given S(t-1) for which the price of the bond is the higher. Similarly, let $V_t^+|S(t-1)$ and $V_t^-|S(t-1)$ represent the value of the option contract at time t, given the corresponding return per dollar invested in the bond at time t. In the remaining analysis we will omit all state designations when we are describing general pricing relationships.

Rendleman and Bartter [9] have shown that it is always possible to form a riskless hedge between the bond and its option over the period t-1 to t by investing \$1 in the bond and purchasing $\frac{H_t^- - H_t^+}{V_t^+ - V_t^-}$ units of the option. If this term is negative, a short sale of the option (or a writing position) is implied.

C. Pricing the Option

Since the return on the hedged portfolios is certain, the option should be priced so that the hedge yields the riskless rate of interest from time t-1 to t. The price of the option at t-1 that meets this condition is given by:

(1) $$P_{t-1} = \frac{V_t^+(1+R_{t-1}/N-H_t^-) + V_t^-(H_t^+-1-R_{t-1}/N)}{(H_t^+ - H_t^-)(1+R_{t-1}/N)} .$$

R_{t-1} is the annual rate of interest prevailing at time t-1 associated with a default-free bond maturing at time t, and $\frac{1}{N}$ is the fraction of the year represented by the time interval t-1 to t. Thus, R_{t-1}/N is the actual rate of interest earned on the default-free bond from time t-1 to t.

Note that a distinction is made between values (V) and prices (P). If it is worthwhile to exercise the option at time t rather than hold until t+1, then $V_t = VEXER_t$, where $VEXER_t$ is the exercisable value of the option at time t. Otherwise, the option will not be exercised, and the value of owning the option at time t is simply its price.

Assuming that investors are rational and will exercise the option when it is in their best interests to do so, the value of the option at time t becomes:

(2) $$V_t = MAX[VEXER_t, P_t].$$

The distinguishing feature between American and European options is that the American option can be exercised at any time, whereas the European option can only be exercised on its maturity date, which we designate as time T. Letting D_t represent the value of the underlying debt instrument at time t and X_t represent the option's exercise price, which does not have to be assumed constant, the following expressions define the exercisable values for American and European puts and calls:

American Call:

(3A) $$VEXER_t = D_t - X_t \quad \text{for all t.}$$

American Put:

(3B) $$VEXER_t = X_t - D_t \quad \text{for all t.}$$

European Call:

(3C) $$VEXER_t = D_t - X_t \quad \text{for } t = T,$$
$$VEXER_t = 0 \quad \text{for } t < T.$$

European Put:

(3D) $$VEXER_t = X_t - D_t \quad \text{for } t = T.$$
$$VEXER_t = 0 \quad \text{for } t < T.$$

By defining the terminal price of the option as

$$P_T = 0, \quad (4)$$

we can represent the entire option pricing model by equations 1-4. These equations represent recursive relationships which can be applied at any time t-1 to determine the price of the option as a function of its value at time t. Thus, through repeated application of the recursive relationship, one can begin at the option's maturity date and eventually solve for the option's current price.

Unlike the Black-Scholes and Brennan-Schwartz models, our model is probability free. The same option price will obtain independently of the probabilities associated with the + and - states. Of course, it is always possible to choose a time-differencing interval and a set of probabilities which with the H^+ and H^- values will make our model asymptotically equivalent to the continuous time approach. However, the generality of the model makes it appropriate for other types of return-generating processes as well. All that is necessary is that the distribution of returns of the underlying debt instrument be well described by a two-state process over some particular time-differencing interval. Finally, it should be noted that the option price does not depend upon preferences. This feature of the model is consistent with most other modern models of option pricing.

III. Bond Pricing under Uncertainty

A. Interest Rate Uncertainty

Before examining option prices obtained from 1-4, it is necessary to develop a theory of bond pricing under conditions of uncertainty regarding the stochastic movements in the interest rate. Several recent papers have addressed the issue of pricing bonds and related contingent claims as a function of interest rate risk. In modeling the behavior of interest rates, Brennan and Schwartz [4] and Dothan [6] have assumed that the instantaneous riskless interest rate follows a lognormal distribution. Cox, Ingersoll and Ross [5] and Vasicek [12] have assumed that the instantaneous riskless interest rate follows a diffusion process with a reverting mean. We assume that the annual rate of interest for pure discount bonds maturing one period into the future can take on only one of two values at time t, given the rate of t-1. To simplify the exposition, we assume that the ratio of the interest rate at time t to that at time t-1, Z^+, is the same for all "up" states (where "up" refers to the price of the underlying bond) at each point in time. Similarly, the ratio for the "down" states, Z^-, is also assumed to be constant. We do not assume that the down ratio is the reciprocal of the up ratio.

By selecting the parameters of the distribution in the appropriate manner, the two-state, or log-binomial, distribution can be used as a numerical procedure for the lognormal distribution with stationary mean and variance, the mean reverting diffusion process, the Poisson distribution and others. Parkinson [8] has employed a similar procedure for describing stock prices in order to obtain a numerical solution for the price of an American put option when the returns of the underlying stock are assumed to follow a lognormal distribution. Parkinson's model and our model are not equivalent, however, since a stable lognormal (or log-binomial) distribution for the short-term interest rate does not imply the same distribution of returns for the bond that serves as the underlying asset for the option we price.

Unlike the option, the price of the bond will not be preference-free if interest rates are uncertain. A particular time path of interest rates will not imply a preference-independent set of bond prices across all maturities. This is due to the fact that one needs to know the prices of two securities and their state-dependent payoffs in order to determine the market's implicit state prices.[2] In the case of bonds, we know only the short-term interest rate and are unable to rely on an arbitrage mechanism in deriving bond prices. Therefore, it is necessary to make some assumption about investors' preferences.

B. Pricing the Bond

As in Brennan and Schwartz's analysis, we assume that the expectations hypothesis of the term structure of interest rates holds. Under the expectations hypothesis, prices are set so that the expected returns of bonds of all maturities and coupons will be the same over any interval of time t-1 to t.[3]

Consider a bond with a face value of \$F which pays a coupon at a rate of \$C per year and matures at date T^* ($T^* \geq T$). With a time-differencing interval of N times per year, we assume that a coupon of \$C/N is paid at each time t.

At any time t, the value of owning the bond consists of both the coupon and price of the bond. Under the expectations hypothesis, the bond should be priced at each point in time so that its expected return over the next interval of time is the same as that of a default-free pure discount bond maturing in

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[2]In general, if there are M states of the world, one must know the prices and state-dependent payoffs of M securities in order to specify the implicit state prices.

[3]Although bonds are priced as if investors are risk neutral under the expectations hypothesis, the assumption of risk neutrality is not implied. In fact, Mark Rubinstein has pointed out to us that our model implies that investors have logarithmic utility functions.

the next period. Therefore, we obtain the following pricing equation for the bond at any time t-1 as a function of its expected price and coupon at time t:

$$D_{t-1} = \frac{E[D_t] + C/N}{1 + R_{t-1}/N} . \tag{5}$$

If we begin at time T^*-1, the price of the bond at time T^* is known. It is simply the bond's face value. Therefore, the price of the bond at time T^*-1, given the state of the world at time T^*-1 becomes:

$$D_{T^*-1}|S(T-1) = \frac{(F + C/N)}{(1+R_{T*-1}/N}|S(T-1) . \tag{6}$$

Equation (6) implies that there is no uncertainty surrounding the bond's end-of-period value as of time T^*-1. However, the price of the bond at time T^*-1 will depend upon the prevailing interest rate. If the interest rate falls (denoted as the "up" state since bond prices will rise) from time T^*-2 to T^*-1, the price of the bond will be

$$D^+_{T*-1}|S(T^*-2) = \frac{F + C/N}{(1+R_{T*-2}Z^+/N)}|S(T^*-2) , \tag{7}$$

and if the interest rate rises,

$$D^-_{T*-1}|S(T^*-2) = \frac{F + C/N}{(1+R_{T*-2}Z^-/N)}|S(T^*-2) . \tag{8}$$

With the expectations hypothesis, the expected return from T^*-2 to T^*-1 should equal the riskless single-period rate of interest prevailing at T^*-2. Letting θ represent the probability that the "up" state will occur and $(1-\theta)$ the probability associated with the occurrence of the down state, (to simplify the analysis, θ is assumed to be constant across all dates and states), the bond's price at T^*-2 becomes:

$$D_{T*-2} = \frac{E[D_{T*-1}]}{1+R_{T*-2}/N}$$

$$= \frac{(F+C/N) \left(\frac{\theta}{1+R_{T*-2}Z^+/N} + \frac{(1-\theta)}{1+R_{T*-2}Z^-/N}\right)}{1 + R_{T*-2}/N} \tag{9}$$

This procedure can be carried out for T^* periods to eventually determine all of the possible intertemporal prices of the bond as well as its present price. These prices can then be employed to determine the H^+ and H^- values entering (1) (i.e., $H_t^+ = \frac{D_t^+ + C/N}{D_{t-1}}$) along with the exercisable values of the option.

C. Specifying the Interest Rate Parameters

As noted earlier, one must specify the ratios of the interest rates prevailing from one period to the next for the up and down states. The magnitude of these ratios together with the probabilities associated with the occurrence of the two states implies a particular mean and variance for the ratio. For many applications of the model one might wish to start with the mean, variance, and probabilities in order to obtain the implied Z^+ and Z^- values.

If one assumes that the probabilities associated with the up and down states remain stable through time, then the logarithm of the interest rate ratio will follow a binomial distribution with an annual mean of

$$\mu = N[z^+\theta + z^-(1-\theta)] = N[(z^+ - z^-)\theta + z^-] \tag{10}$$

and annual variance

$$\sigma^2 = N(z^+ - z^-)^2\, \theta(1-\theta), \tag{11}$$

where

$$z^+ = \ln(Z^+)$$
$$z^- = \ln(Z^-).$$

This distribution will approximate a lognormal distribution with the same mean and variance. As N becomes large, the two distributions will be asymptotically equivalent. By specifying values for μ, σ^2, and θ, one can simultaneously solve (10) and (11) for the implied values of z^+ and z^-. This solution is given below:[4]

$$z^- = \mu/N - \frac{\sigma\theta}{\sqrt{N\theta(1-\theta)}}, \tag{12}$$

$$z^+ = \mu/N + \frac{\sigma(1-\theta)}{\sqrt{N\theta(1-\theta)}}. \tag{13}$$

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[4] Rendleman and Bartter [9] used a similar procedure in pricing stock options for determining implied H^+ and H^- values for the returns of the underlying stock when the binomial distribution was used as an approximation for a lognormal distribution with a particular variance. The difference between their European put and call prices and those of the Black-Scholes model for options with one year to maturity was generally less than 6/10 of 1 percent when $\theta = .5$ and $N = 52$.

IV. An Illustration of the Model

In this section we present the prices of various European and American options on debt securities. Each option is assumed to mature in one year and each gives its owner the right to either buy or sell a six percent, 10-year bond with a face value of $100. Although the bond initially has a 10-year maturity, it will have a 9-year maturity when the option matures. The exercise prices of the options are varied from $80 to $120 in increments of $10. The annual standard deviation of the logarithm of the interest rate ratio is varied from .10 to .30 in increments of .05. The expected value of the logarithm of the ratio is varied from -.1 to .1 per year in increments of .1. In each case the differencing interval is 10 times per year, the initial short-term interest rate is 6 percent, and the probability of the interest rate rising is 50 percent.

An examination of Tables 1-4 reveals that both the option and bond prices are functions of both the mean (μ) and standard deviation (σ) of the logarithm of the interest rate ratio. As one would expect, bond prices vary inversely with the expected direction of the level of interest rates. In addition, bond prices vary inversely with the standard deviation.[5] For example, with $\mu = 0$ and $\sigma = .2$, the bond price is $98.28. With $\mu = .1$ and .1 the prices are $115.18 and $75.94, respectively, for the same standard deviation. Holding the mean equal to 0 and varying the standard deviation from .1 to .3, the prices range from $99.99 to $95.84.

The theory of stock option pricing suggests that American puts will always sell for more than their European counterparts and that American calls will sell for more than European calls only if the underlying stock pays a dividend. A coupon-paying bond is analogous to a dividend-paying stock. Therefore, as expected, the tables show the American option prices to be at least as great as the European prices for both puts and calls.

It is well known that the prices of stock options vary directly with volatility of stock returns. Therefore, one might conclude that the prices of bond options would be increasing functions of the standard deviation of interest rate changes. Although bond prices are more volatile at higher standard deviations, the bond prices are also lower. Consequently, there is no systematic relationship between the standard deviation of interest rate changes and bond option prices. For example, for the European calls in Table 1, for $\mu = 0$, and an

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[5]Brennan and Schwartz [4] and Dothan [6] have found that the price of the bond varies directly with the standard deviation when the arithmetic drift is held constant. If the arithmetic drift ($\mu + 1/2\sigma^2$) is held constant while the standard deviation increases, the geometric drift (μ) must fall. In this sense, our results are consistent.

TABLE 1

PRICES OF EUROPEAN CALL OPTIONS ON 6%,
10-YEAR BOND WITH $100 FACE VALUE

Exercise Price	Standard Deviation (σ) .10	.15	.20	.25	.30
			$\mu = -.1$		
80	35.37	34.73	33.86	32.76	31.46
90	25.92	25.29	24.42	23.32	22.09
100	16.48	15.85	15.01	14.07	13.06
110	7.05	6.60	6.25	5.82	5.65
120	0.34	0.59	0.83	1.00	1.12
Bond Price	116.72	116.08	115.18	114.06	112.74
			$\mu = 0$		
80	18.84	18.12	17.21	16.29	15.31
90	9.43	8.91	8.51	8.06	7.83
100	1.53	1.98	2.35	2.61	2.80
110	0.00	0.06	0.19	.31	.45
120	0.00	0.00	0.00	0.00	0.03
Bond Price	99.99	99.26	98.28	97.13	95.84
			$\mu = .1$		
80	0.47	0.95	1.47	1.97	2.40
90	0.00	0.03	0.15	0.32	0.46
100	0.00	0.00	0.00	0.02	0.05
110	0.00	0.00	0.00	0.00	0.00
120	0.00	0.00	0.00	0.00	0.00
Bond Price	76.87	76.45	75.94	75.38	74.83

TABLE 2

PRICES OF AMERICAN CALL OPTIONS ON 6%, 10-YEAR BOND WITH $100 FACE VALUE

Exercise Price	Standard Deviation (σ) .10	.15	.20	.25	.30
			$\mu = -.1$		
80	36.72*	36.08*	35.18*	34.06*	32.74*
90	26.72*	26.08*	25.18*	24.06*	22.83
100	16.76	16.17	15.36	14.46	13.48
110	7.12	6.71	6.41	6.01	5.84
120	0.34	0.62	0.87	1.06	1.19
Bond Price	116.72	116.08	115.18	114.06	112.74
			$\mu = 0$		
80	19.99*	19.26*	18.28*	17.17	16.12
90	9.99*	9.35	8.88	8.44	8.14
100	1.60	2.06	2.43	2.71	2.90
110	0.00	0.06	0.20	0.32	0.48
120	0.00	0.00	0.00	0.00	0.03
Bond Price	99.99	99.26	98.28	97.13	95.84
			$\mu = .1$		
80	0.50	1.01	1.57	2.09	2.53
90	0.00	0.03	0.16	0.33	0.49
100	0.00	0.00	0.00	0.02	0.05
110	0.00	0.00	0.00	0.00	0.00
120	0.00	0.00	0.00	0.00	0.00
Bond Price	76.87	76.45	75.94	75.38	74.83

*Option price is equal to its current exercisable value. Therefore, the option should be exercised immediately.

TABLE 3

PRICES OF EUROPEAN PUT OPTIONS ON 6%,
10-YEAR BOND WITH 100 FACE VALUE

Exercise Price	Standard Deviation (σ) .10	.15	.20	.25	.30
			μ = -.1		
80	0.00	0.00	0.00	0.00	0.00
90	0.00	0.00	0.00	0.01	0.07
100	0.00	0.00	0.03	0.19	0.47
110	0.14	0.19	0.71	1.37	2.50
120	2.74	3.63	4.73	6.00	7.40
Bond Price	116.72	116.08	115.18	114.06	112.74
			μ = 0		
80	0.00	0.00	0.05	0.26	0.54
90	0.01	0.21	0176	1.45	2.46
100	1.53	2.70	4.01	5.41	6.85
110	9.42	10.19	11.27	12.51	13.91
120	18.86	19.55	20.50	21.62	22.89
Bond Price	99.99	99.26	98.28	97.13	95.84
			μ = .1		
80	4.54	5.43	6.44	7.47	8.43
90	13.46	13.89	14.51	15.20	15.87
100	22.85	23.26	23.75	24.28	24.84
110	32.24	32.65	33.13	33.65	34.17
120	41.64	42.04	42.53	43.04	43.56
Bond Price	76.87	76.45	75.94	75.38	74.83

TABLE 4

PRICES OF AMERICAN PUT OPTIONS ON 6%,
10-YEAR BOND WITH $100 FACE VALUE

Exercise Price	Standard Deviation (σ) .10	.15	.20	.25	.30
			$\mu = -.1$		
80	0.00	0.00	0.00	0.00	0.00
90	0.00	0.00	0.00	0.01	0.07
100	0.00	0.00	0.03	0.21	0.51
110	0.01	0.20	0.76	1.50	2.68
120	3.28	4.13	5.25	6.58	8.07
Bond Price	116.72	116.08	115.18	114.06	112.74
			$\mu = 0$		
80	0.00	0.00	0.05	0.27	0.57
90	0.01	0.21	0.78	1.49	2.57
100	1.60	2.82	4.19	5.64	7.14
110	10.01*	10.74*	11.86	13.15	14.59
120	20.01*	20.74*	21.72*	22.87*	24.16*
Bond Price	99.99	99.26	98.28	97.13	95.84
			$\mu = .1$		
80	4.54	5.44	6.49	7.56	8.56
90	13.52	14.01	14.69	15.44	16.18
100	23.17	23.63	24.19	24.80	25.44
110	33.13*	33.55*	34.06*	34.62*	35.20
120	43.13*	43.55*	44.06*	44.62*	45.17*
Bond Price	76.87	76.45	75.94	75.38	74.83

*Option price is equal to its current exercisable value. Therefore, the option should be exercised immediately.

exercise price of $80, as the standard deviation increases from .1 to .3 the option price decreases from $18.84 to $15.31. On the other hand, for an exercise price of $110, the option price increases from $0.00 to $.45 over the same range.

If we examine bond option premiums instead of prices, we find that premiums are an increasing function of the standard deviation for all exercise prices.[6] For the example above, the premium on the call option with an exercise price of $80 increases from $18.84 - ($99.99-$80) = -$1.15 to $15.31 - ($95.84-$80) = -$.53 and likewise, with an exercise price of $110 increases from $0.00 - ($99.99-$110) to $.45 - ($95.84-$110) = $14.61. This relationship also holds for American calls, as well as European and American puts.

One major problem with this approach to option pricing is that the price of the underlying bond is an output of the model rather than an input. In most situations one would know the price of the bond prior to determining the value of the option. However, with knowledge of the bond price, one could at least eliminate one of the subjective inputs (μ or σ) to the model since, given the other parameter, the observed bond price would imply the other.

V. Further Applications of the Model

In addition to pricing puts and calls, the model has application in the pricing of interest-related contingent claims such as callable bonds, savings bonds, and retractable bonds that have previously been valued by Brennan and Schwartz, as well as bank loan commitments. Bartter and Rendleman [1, 2] have applied this methodology to both fee and balance based pricing of fixed-rate bank loan commitments. As Hong and Greenbaum [7] first recognized, a bank loan commitment is similar to a put option on a pure discount bond in which the bank customer has the right to sell the bank a loan at a specified borrowing rate within a specified time period. Unlike the American options valued in this paper, the loan commitment allows the bank customer to sell a loan with a fixed life, rather than a fixed maturity date. In effect, the underlying loan is different at each point in time throughout the life of the commitment. Although this feature makes the valuation problem more complex, it can be easily handled using the two-state methodology.

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[6]The premium on a call option is the call price less the difference between the bond price and the exercise price. The premium on a put option is the put price less the difference between the exercise price and the bond price.

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