Assessing Credit Risk in a Financial Institution's Off-Balance Sheet Commitments

John Hull*

Abstract

The first part of this paper presents a general approach to valuing a financial institution's contracts when there is credit risk. The approach uses contingent claims pricing theory and is particularly appropriate for an off-balance sheet contract, such as a swap, that can have either a positive or a negative value to the counterparty. The second part of the paper extends the analysis by considering the problem, faced by bank supervisory authorities, of determining capital requirements for off-balance sheet contracts.

I. Introduction

During the 1980s, there has been a rapid increase in the off-balance sheet commitments of major banks. Among the contracts that have led to this increase are swaps, forward rate agreements, currency options, interest rate caps, bankers' acceptances, note issuance facilities, and revolving underwriting facilities. Assessing the risks posed by these contracts has now become critically important to stock market analysts, to bank supervisory authorities, and to the banks themselves.

Two types of risk can be distinguished in off-balance sheet commitments: credit risk and market risk. Credit risk arises from the possibility of default by the counterparty. Market risk arises from the possibility of adverse movements in market variables such as interest rates and foreign exchange rates. Market risks can be hedged. Credit risks, by contrast, cannot usually be hedged. It is credit risks that are the main concern of this paper.

Traditionally, the capital adequacy of a bank has been measured using balance sheet ratios such as equity: total assets. The growth of off-balance sheet commitments has made these ratios less relevant. As a result, most bank supervisory authorities are now moving towards a credit risk weighting scheme for monitoring capital adequacy.[1] This scheme involves assigning to each on- and off-balance sheet item a weight reflecting its relative credit risk. Minimum levels are then set for the ratio of capital to total risk-weighted exposure.

* Faculty of Management, University of Toronto, Ontario M5S 1V4, Canada. The author would like to thank David Pyle, Alan White, and Anthony Santomero, a *JFQA* referee, for making several comments that have led to significant improvements in this paper.

[1] See Bank for International Settlements (1988) and Federal Reserve Board and Bank of England (1987).

The first part of this paper discusses how credit risk can be taken into account in the valuation of financial contracts. Other research has considered this question with reference to particular contracts. For example, Kane (1980) analyzes the effect of credit risk on forward and futures contracts; Johnson and Stulz (1987) show how options can be valued when there is credit risk; Whittaker (1987) considers the effect of credit risk on the valuation of interest rate swaps. The approach taken in this paper is more general and can, in principle, be applied to any off-balance sheet (or on-balance sheet) contract.

The second part of the paper discusses how capital requirements should be set so that they reflect the credit risk in off-balance sheet items. Santomero and Watson (1977) provide an excellent discussion of the objectives of bank regulators in setting capital requirements. They argue that regulators wish to reduce the probability of bankruptcy to an acceptable level and are faced with a trade-off between the costs associated with bank failure and the costs arising from forced overcapitalization. The discussion in the paper builds upon the work of these authors and upon the work of authors, such as Arak et al. (1987), Belton (1987), Cooper and Watson (1987), and Wall and Fung (1987), who have considered the credit risk in swaps and other off-balance sheet contracts.

The rest of the paper is organized as follows. Section II shows how an off-balance sheet contract can be valued when there is credit risk. Section III applies the ideas in Section II to currency swaps. Section IV presents a set of assumptions necessary to justify credit risk weighting. These assumptions are discussed in Section V. Conclusions are in Section VI.

II. Valuation of Contracts when There is Credit Risk

The option of a borrower to default on a debt contract has been widely discussed in the finance literature. It is generally assumed that, when the value of a company's liabilities exceeds the value of its assets, the company goes bankrupt and defaults on its liabilities.

Some off-balance sheet contracts, such as swaps and forward rate agreements, differ from debt contracts in that they can have either positive or negative values to the counterparty at any given time. In other words, they can be either assets or liabilities to the counterparty. We can reasonably assume that default will take place on these contracts only when the following two conditions are satisfied:

1. the counterparty is bankrupt, and
2. the value of the contract to the counterparty (assuming that it chooses not to default) is negative.

Note that Condition 2 is not redundant. It is possible for a counterparty to experience financial distress even though it has positive value in one particular contract. Our assumption is that, if a counterparty goes bankrupt when the contract has a positive value to the counterparty, the counterparty is able to sell the contract to a third party, or rearrange its affairs in some way, so that its positive value in the contract is not lost.[2]

[2] Often, contracts can be sold, providing the credit risk of the new counterparty is satisfactory.

The default option in an off-balance sheet contract can be viewed as a contingent claim. There is now a well-established literature on the valuation of contingent claims. The risk-neutral valuation arguments of Cox and Ross (1976) show that claims contingent on the prices of traded securities can be valued on the assumption that the world is risk neutral. These arguments have been extended by Cox, Ingersoll, and Ross (1985), who show that any claim can be valued by reducing the proportional drift rate of each underlying variable by the product of its market price of risk and its volatility, and then assuming that the world is risk neutral.[3] This is true for history-dependent contingent claims as well as for contingent claims that are dependent only on the current value of the underlying variables.

Consider an off-balance sheet contract dependent on a number of state variables and time, t. Define:

$V(t)$ = value of off-balance sheet contract to bank at time t, if there has been no bankruptcy by the counterparty up to and including time t,

$f(t)\Delta t$ = probability of bankruptcy by the counterparty between times t and $t + \Delta t$,

$U(t)$ = value of contract to bank at time t, assuming no default options,

$W(t)$ = value of future default options to the counterparty at time t, and

T = life of the contract.

Both V and U can be positive or negative. The value of the contract to the counterparty is the reverse of its value to the bank. Thus, with no default possibilities, its value to the counterparty is $-U$ and, when default possibilities are taken into account, its value to the counterparty is $-V$. For ease of exposition, we assume no recoveries are made in the event of a default.[4] We also assume that the state variables affecting the probability of bankruptcy have zero market price of risk and are independent of the state variables affecting U.

A. Formulation 1

In our first formulation of the problem, we make the following assumptions:

1. there is no possibility of default by the bank, and
2. if the counterparty goes bankrupt, it has the option of selling the contract to another counterparty whose default risk is zero.

The second assumption implies that, if a counterparty goes bankrupt and sells the contract, there will be no further defaults and the contract is worth $-U$ to the new counterparty. It follows that, when bankruptcy occurs, the original counter-

Clauses such as "consent to assignment to a third party will not reasonably be withheld" are common. Ideally, the bank would like to prohibit assignment to a third party whenever it feels that this is being done because of an impending bankruptcy. In practice, this is likely to be difficult. An issue related to the treatment of contracts in the event of bankruptcy is known as netting. Some banks have argued that when they have several contracts with the same counterparty, only the net exposure is relevant for determining capital requirements because a default on one contract will trigger defaults on the other contracts.

[3] For a discussion of this result, see Hull ((1989), Chapter 7).

[4] The analysis can easily be modified to allow for partial recoveries.

party can choose between (a) defaulting on the contract and (b) realizing an amount $-U$ for the contract. Hence, default will take place if $U > 0$. The loss to the bank arising from bankruptcy by the counterparty is, therefore, max(U, 0).[5] This loss can be regarded as the payoff from a contingent claim. Using the Cox, Ingersoll, and Ross extension of the risk-neutral valuation argument, $W(0)$ is the discounted expected value of the default option in a world in which all state variables (i.e., those affecting U and those affecting bankruptcy probabilities) have risk-adjusted drift rates. The variables affecting bankruptcy probabilities are assumed to have zero market price of risk so that no risk adjustments are necessary to f. Since the variables affecting U are independent of the variables affecting bankruptcy probabilities, we can integrate over the former first to obtain

$$W(0) = \int_0^T C(t) f(t) dt , \tag{1}$$

where $C(t)$ is the value of a contingent claim that pays off max(U, 0) at time t. (This contingent claim is a European call option on U with exercise price zero and maturity date, t.) The value of the contract at time 0, $V(0)$, can be calculated from (1) and the identity

$$V(0) = U(0) - W(0) . \tag{2}$$

B. Formulation 2

In our second formulation, we assume

1. there is no possibility of default by the bank, and
2. if the counterparty goes bankrupt at time τ, it has the option of selling the contract to a new counterparty that has a probability $f(t)\Delta t$ of going bankrupt between times t and $t + \Delta t$ ($t \geqslant \tau$).

In this case, if bankruptcy occurs and the contract is sold, it is worth $-V$ to the new counterparty. It follows that, in the event of bankruptcy, the counterparty can choose between (a) defaulting on the contract, and (b) realizing an amount $-V$ for the contract. Hence, default will take place when $V > 0$. The default costs the bank U.[6] If $V < 0$, there is, in this formulation, a chance that there will be future defaults. Hence, when a bankruptcy occurs, $W = U$ if $V > 0$, and $W = U - V$ if $V < 0$, i.e.,

$$W = U + \max [-V, 0] . \tag{3}$$

The value of $W(0)$ is given by Equation (1) with $C(t)$ being defined as a contingent claim that pays off $U(t) + \max[-V(t), 0]$ at time t. Since $V(t) = U(t) - W(t)$, the situation is more complicated than before. As will be illustrated in the next section, the lattice approach of Cox, Ross, and Rubinstein (1979) can be adapted to value the default option numerically.

[5] Note that the loss is not max(V,0). In essence, we are using U as our benchmark for determining the value of the contract to the bank, and we are considering different ways in which positive U's can be lost by the bank.

[6] Note that the cost of the default is U, not V (see footnote 5).

III. Application to Currency Swaps

Here, we show how the models developed in Section II can be used to value the default option in currency swaps.

A plain deal currency swap is a contract in which principal and fixed-rate interest payments on a loan in one currency are exchanged for principal and fixed-rate interest payments on a similar loan in another currency. Suppose that a bank is making payments in U.S. dollars and receiving payments in the foreign currency. The bank is long a foreign-denominated bond and short a U.S. dollar-denominated bond. The value of the swap, assuming no defaults, is given by

$$U = SB_F - B_D , \tag{4}$$

where B_F is the default-free value, measured in the foreign currency, of the foreign-denominated bond; B_D is the default-free value, measured in U.S. dollars, of the dollar-denominated bond; and S is the exchange rate (i.e., the value, measured in the domestic currency, of one unit of the foreign currency).

To simplify matters, we assume that the term structure of interest rates in both currencies is flat and that interest rates are constant. We also assume that payments are made under the swap every 6 months and that the payments made equal the risk-free rates of interest in the two currencies. We also assume that the probability of bankruptcy by the counterparty in time Δt, conditional on no earlier bankruptcy, is $\lambda \Delta t$ where λ is a constant.

Consider first Formulation 1 in which the probability of default by the bank is zero and, if the counterparty goes bankrupt when $U < 0$, the contract is sold to a new zero-risk counterparty.

Suppose that payment dates are at times $t_1, t_2, \ldots, t_n$, and that time zero is denoted by t_0. Under the assumptions made, the probability, q_i, of the counterparty going bankrupt between times t_{i-1} and t_i is given by

$$q_i = \exp\left(-\lambda t_{i-1}\right) - \exp\left(-\lambda t_i\right) .$$

If $U(t_i) > 0$, a bankruptcy between times t_{i-1} and t_i leads to a default at time t_i.[7] Equation (1), therefore, becomes

$$W = \sum_{i=1}^{n} q_i C\left(t_i\right) . \tag{5}$$

Immediately prior to a payment date

$$B_F = A_F\left(1 + z_F\right) , \tag{6}$$

$$B_D = A_D\left(1 + z_D\right) , \tag{7}$$

[7] This assumes that the bankrupt company is able to wait until time t_i before deciding whether to default.

where A_F is the face value of the foreign bond measured in the foreign currency, A_D is the face value of the domestic bond measured in the domestic currency, and z_F and z_D are the coupon payments in the two currencies. From the assumptions that have been made

$$z_F = 2\left[\exp\left(0.5 r_F\right) - 1\right] ,$$

and

$$z_D = 2\left[\exp\left(0.5 r_D\right) - 1\right] ,$$

where r_F and r_D are the continuously compounded foreign and domestic risk-free rates of interest.

From Equation (4),

$$\max\ (U,0) = B_F \max\ \left(S - B_D / B_F, 0\right) .$$

It follows from Equations (6) and (7) that $C(t_i)$ is the value of $A_F(1 + z_F)$ European call options on a foreign currency with exercise price, X, equal to $[A_D(1 + z_D)]/[A_F(1 + z_F)]$. If we assume that S follows geometric Brownian motion with constant volatility, σ, the price of each of these options is shown by Biger and Hull (1983), Garman and Kohlhagen (1983), and Grabbe (1983) to be

$$(8) \qquad S \exp\left(-r_F t_i\right) N\left(d_1\right) - X \exp\left(-r_D t_i\right) N\left(d_2\right) ,$$

$$\text{where } d_1 = \frac{\mathrm{Ln}(S/X) + \left(r_D - r_F + \sigma^2/2\right) t_i}{\sigma \sqrt{t_i}} , \text{ and}$$

$$d_2 = d_1 - \sigma \sqrt{t_i} .$$

In Table 1, Equation (5) has been used to calculate the value of the default option when $\lambda = 0.01$, $\sigma = 0.06$, $A_D = 100$, $B_F = 100$, and the initial value of S is 1.0. As might be expected, the value of the default option increases with both σ and the life of the swap. Note that it is significantly higher when $r_F < r_D$ than when $r_F > r_D$. This is because, when $r_F < r_D$, the foreign currency is expected to appreciate over time and the bank's swap has a greater chance of becoming in-the-money than out-of-the-money.[8] Similarly, when $r_F > r_D$, the swap has a greater chance of becoming out-of-the-money than in-the-money.

When Formulation 2 is used, the default option can be valued using the binomial lattice approach of Cox, Ross, and Rubinstein (1979). In a small time interval $(t, t + \Delta t)$, S is assumed to move up to Su with probability p and down to Sd with probability $1 - p$

where $u = \exp(\sigma \sqrt{\Delta t})$,

[8] An alternative way of understanding this is as follows. When $r_F < r_D$, the bank can expect a net outflow on the early payment dates and a significant inflow when principals are exchanged at the end of the life of the swap. If bankruptcy occurs, the final exchange of principals never takes place.

TABLE 1

Value of Default Option on a Currency Swap using Formulation 1 when the Principals, A_D and A_F, in the Two Currencies are Both 100, the Initial Exchange Rate, S, is 1.0, the Domestic Interest Rate, r_D, is 6 percent per Annum, and the Parameter, λ, in the Poisson Process Generating Defaults is 0.01.

Foreign Risk-Free Interest Rate, r_f (% p.a.)	Volatility of Exchange Rate, σ (% p.a.)	Life of Swap (Years)		
		5	10	15
2	5	0.42	1.42	2.75
	10	0.52	1.59	2.96
	15	0.64	1.84	3.31
4	5	0.25	0.75	1.35
	10	0.37	1.01	1.72
	15	0.50	1.30	2.15
6	5	0.13	0.30	0.45
	10	0.26	0.59	0.89
	15	0.39	0.89	1.33
8	5	0.06	0.09	0.11
	10	0.18	0.33	0.43
	15	0.30	0.59	0.80
10	5	0.03	0.03	0.03
	10	0.12	0.18	0.20
	15	0.23	0.39	0.47

$$d = 1/u,$$
$$p = (a-d)/(u-d), \text{ and}$$
$$a = \exp[(r_D - r_F)\Delta t].$$

The probability of default during the time interval is

$$\exp(-\lambda t) - \exp(-\lambda(t+\Delta t)) .$$

Denoting this by $q(t)$,

$$V(s,t) = e^{-r\Delta t}\left[pX_u + (1-p)X_d\right] + Q(S,t) , \tag{9}$$

$$\text{where } X_u = V(Su,t+\Delta t) - q(t)[U(Su,t+\Delta t) + \max(-V(Su,t+\Delta t),0)],$$
$$X_d = V(Sd,t+\Delta t) - q(t)[U(Sd,t+\Delta t) + \max(-V(Sd,t+\Delta t),0)],$$

and $Q(S,t)$ is the financial institution's cash flow, if any, from the swap between times t and $t+\Delta t$, discounted to time t. The value of the swap immediately prior to time T is known to be $S_T A_F(1+z_F) - A_D(1+z_D)$, where S_T is the exchange rate at time T. Equation (9) can be used repeatedly to work back from time T to time 0.

The results of using this approach for the same parameter values as before are shown in Table 2.[9] As might be expected, the values of the default option in

[9] To increase accuracy, the lattice was used to calculate the differences between the value of the default option under Formulation 2 and the value of the default option under Formulation 1. The number of time intervals used for each result was 60.

Table 2 are very close to, but slightly higher than, the corresponding values in Table 1.

TABLE 2

Value of Default Option on a Currency Swap using Formulation 2 when the Principals, A_D and A_F, in the Two Currencies are Both 100, the Initial Exchange Rate, S, is 1.0, the Domestic Interest Rate, r_D, is 6 percent per Annum, and the Parameter, λ, in the Poisson Process Generating Defaults is 0.01.

Foreign Risk-Free Interest Rate, r_f (% p.a.)	Volatility of Exchange Rate, σ (% p.a.)	Life of Swap (Years) 5	10	15
	5	0.42	1.43	2.78
2	10	0.52	1.60	2.99
	15	0.64	1.86	3.35
	5	0.25	0.75	1.36
4	10	0.38	1.02	1.74
	15	0.51	1.31	2.18
	5	0.13	0.30	0.45
6	10	0.26	0.60	0.91
	15	0.40	0.90	1.35
	5	0.06	0.09	0.11
8	10	0.18	0.33	0.43
	15	0.30	0.59	0.81
	5	0.03	0.03	0.03
10	10	0.12	0.18	0.20
	15	0.23	0.39	0.48

The analysis in this section can be used for plain vanilla interest rate swaps. A plain vanilla interest rate swap is an agreement to exchange fixed-rate interest payments on a certain notional principal for floating-rate interest payments on the same notional principal. Consider a bank that is receiving fixed and paying floating. Define

A = notional principal in the swap,

B_1 = value of a bond with principal A, and coupons equal to the fixed payments underlying the swap, and

B_2 = value of a bond with principal A, and coupons equal to the floating payments underlying the swap.

In this case,

$$U = B_1 - B_2 .$$

Immediately prior to payout date, t_i,

$$B_2 = A + y_i ,$$

where y_i is the floating-rate payment required at time t_i. Hence,

$$\max\ (U, 0) \ = \ \max\ \left(B_1 - A - y_i, 0\right) .$$

The value of the default option in Formulation 1 is, therefore, given by Equation (5), with $C(t_i)$ being the value of a security paying

$$\max\ \left(B_1 - A - y_i, 0\right)$$

at time t_i. The security is a call option on a bond with exercise price $A + y_i$. Since y_i $(i > 1)$ is unknown at time zero, this option cannot be valued using standard analytic models. However, Monte Carlo simulation can be used.

IV. Assumptions to Justify Credit Risk Weighting

In July 1988, the Bank for International Settlements published a risk weighting scheme that had been approved by the Group of Ten central-bank governors. This scheme involves assigning to each asset and off-balance sheet contract a weight reflecting its risk per dollar of principal. Floating-rate loans to corporations are chosen as a benchmark and assigned a risk weight of 1.0. The total risk-weighted exposure is calculated as the sum of the products of the risk weights and principals for all assets and off-balance sheet contracts. Capital adequacy is monitored by setting minimum levels for the ratio of bank capital total risk-weighted exposure.[10] Risk weights for off-balance sheet contracts are calculated using specified "credit conversion factors." A credit conversion factor is the ratio of the risk weight for an off-balance sheet contract to the risk weight for a floating-rate loan with the same maturity, counterparty, and principal as the off-balance sheet contract.

Kim and Santomero (1988) discuss risk weighting schemes in the context of the current fixed-rate deposit insurance system. In this section, we take a more general approach and consider the assumptions necessary to justify credit risk weighting. In Section V, these assumptions will be critically reviewed.

Consider a bank supervisory authority that is setting a bank's capital requirements at time t. Following Santomero and Watson (1977), we assume that its objective is to ensure that the probability of the bank's capital dropping to zero between times t and $t + s$ is less than some level π for some s. We assume that no new capital issues are made between times t and $t + s$ and that, at time $t + s$, new capital requirements are specified by the supervisory authority.

We will show that credit risk weighting, as outlined above, can be justified when the following assumptions are made:

1. Banks have very large portfolios of commercial loans and off-balance sheet contracts.
2. The process generating bankruptcies is the same for all counterparties—both those with loans contracts and those with off-balance sheet contracts.

[10] Under the agreed international standard, two definitions for bank capital are used. The first is "equity less goodwill"; the second includes other items such as subordinated term debt. The standards require that, by 1992, the ratio of bank capital to total risk-weighted exposure be greater than 0.04 when the first definition is used, *and* greater than 0.08 when the second definition is used.

3. Average recoveries made in the event of a default are the same for all contracts.
4. The exposures on two different off-balance sheet contracts at any given time are independent.
5. The exposure on each off-balance sheet contract is independent of the variables underlying the process generating bankruptcies.

Consider one particular bank. The proportion of counterparties that go bankrupt between times t and $t+s$ is uncertain. However, assumptions 1 and 2 imply that it can, to a reasonable approximation, be assumed to be the same for both commercial loans and off-balance sheet contracts. Suppose that this proportion is b. The exposure on each defaulting loan can be assumed to be a random sample from the distribution of loan principals. Suppose that the number of loans is n and L_i is the principal amount of the ith loan ($1 \leq i \leq n$). Since n is large, the Central Limit Theorem can be used to show that the total exposure on all defaulting loans is approximately $b\Sigma_i L_i$, and the loss on the loan portfolio is approximately

$$b(1-y)\sum_i L_i \ ,$$

where y is the average proportion of a defaulting loan that is recovered.[11]

Suppose next that there are m off-balance contracts. The bank's exposure on the jth off-balance sheet contract is $\max(U_j, 0)$, where $U_j (1 \leq j \leq m)$ is the value of the contract to the bank, assuming no default possibilities. With Assumptions 4 and 5, the Central Limit Theorem can be used to show that the total loss on the off-balance sheet portfolio between times t and $t+s$ is approximately

$$b(1-y)\sum_j E\Big(\max \left(U_j, 0\right)\Big) \ ,$$

where E denotes expectations taken over all possible values of U_j, and all times between t and $t+s$.

It follows from the above that the bank's total loss between times t and $t+s$ is approximately

$$b(1-y)\left[\sum_i L_i + \sum_j E\Big(\max \left(U_j, 0\right) \right] \ .$$

Let b^* be the value of b that has a probability π of being exceeded. It follows that the required capital, H, should be

$$(10) \qquad H \ = \ b^*(1-y)\left[\sum_i L_i + \sum_j E \max \left(U_j, 0\right) \right] \ .$$

[11] The time value of money is assumed to affect the losses on all contracts equally.

This equation shows that the capital requirement of loan i should be $b^*(1-y)L_i$, and the capital requirement of off-balance sheet contract j should be $b^*(1-y)E[\max(U_j,0)]$. It implies that a credit risk weighting scheme is appropriate with the credit conversion factor, α, for the jth off-balance sheet contract being

$$\alpha = \frac{E\left[\max\left(U_j,0\right)\right]}{A_j}, \tag{11}$$

where A_j is the principal amount of the contract. This can be calculated using similar approaches to those discussed in Sections II and III.

The above argument shows that the assumptions in 1 to 5 above are sufficient for a risk weighting scheme to be applicable. If any of the assumptions are relaxed, it is possible to provide an example of a situation where risk weighting is inappropriate. We can, therefore, conclude that the conditions are both necessary and sufficient.

The credit conversion factor, under the assumptions made, is the ratio of the expected payoff from the default option on the off-balance sheet item to the expected payoff from the default option on the loan. As will be evident from the discussion in Section II, this is only equal to the ratio of the values of the two options when the world is assumed to be risk-neutral and the assumptions in Formulation 1 are made.

It is interesting to note that as s approaches zero, Equation (11) becomes

$$\alpha = \frac{\max\left(U_j,0\right)}{A_j}. \tag{12}$$

The credit conversion factor is then simply equal to the ratio of the current exposure on the contract to the principal amount of the contract.[12] If the contract is out-of-the-money, the credit conversion factor is zero.

V. Limitations of Credit Risk Weighting

A number of the assumptions made in Section IV are questionable. First, the bank's portfolio has to be sufficiently large that b and y are the only variables affecting the total losses experienced on all contracts. As the size of the portfolio decreases, the tails of the distribution of losses become fatter, and the capital requirement given by Equation (10) becomes too low. Supervisory authorities have, in the past, recognized this and have set the capital:assets ratio higher for small banks than for large banks. In the case of many off-balance sheet contracts, there is not only uncertainty as to whether a default will take place, but also

[12] For swaps and similar contracts, the new international standard gives bank supervisory authorities a choice between two calculations. Under the first calculation, contracts are marked to market frequently and α is set slightly higher than its value in (12). This corresponds to using a very small value of s. Under the second calculation, α remains the same throughout the life of the contract. This corresponds to using a value of s equal to the life of the contract.

uncertainty as to the size of the bank's exposure at the time of the default. It, therefore, is likely that as the size of a bank decreases, the "fat tails" effect causes the capital requirements to increase proportionately more for off-balance sheet items than for on-balance sheet items. This means that the credit conversion factors given by Equation (11) will tend to be too low for small banks.

The second questionable assumption is that the exposures on two different off-balance sheet items at any future time are independent. In many situations, this is likely to be untrue, and a large movement in interest rates or exchange rates may have a significant effect on the average exposure per contract in a large portfolio of off-balance sheet contracts. The effect of a dependence between off-balance sheet items is to make the tails of the distribution of losses from them fatter and the credit conversion factors given by Equation (11) too low. The credit risk weighting scheme is no longer valid because the incremental effect of a new contract on the loss experience depends on the other contracts in the portfolio.

A final questionable assumption is that the exposure on each off-balance sheet contract is independent of the process generating bankruptcies. In general, bankruptcies can be expected to depend on macroeconomic variables such as the level of interest rates and exchange rates. These are the very variables that determine the exposures on many contracts. If it is argued that bankruptcies become more likely when interest rates are high, there is some comfort for banks as far as their interest rate swap portfolios are concerned. In a matched pair of plain vanilla interest rate swaps, the counterparty with the higher credit risk tends to be the one paying fixed and receiving floating. The bank has a positive exposure as far as this counterparty is concerned only when rates are low.

Even if, from a macroeconomic perspective, there is no relationship between interest rates and bankruptcy risk, there may be some relationship for any given counterparty. Consider an interest rate swap with a counterparty that is paying fixed and receiving floating. If the counterparty is a speculator, the probability of bankruptcy will, as a result of the swap, increase as rates decrease. If the counterparty is perfectly hedged, there should be no relationship between bankruptcy and interest rates. If the counterparty is using the swap as a partial hedge, the probability of bankruptcy will increase as rates increase.[13]

VI. Summary

This paper has presented a model for valuing financial contracts where there is credit risk. In some circumstances, the model can be evaluated analytically. In other cases, an extension of the Cox, Ross, and Rubinstein lattice approach is appropriate.

The paper has also considered the credit risk weighting schemes now favored by bank supervisory authorities. It has presented a set of assumptions necessary to justify such schemes. The key assumptions are that every bank has a very large portfolio of loans and off-balance sheet contracts, that the future exposures on two different contracts are independent, and that the exposure on any given contract is independent of the probability of bankruptcy. The paper has

[13] This point was made by Belton (1987).

argued that, in practice, these assumptions are questionable. The appropriate risk weight for an off-balance sheet contract is likely to depend on the size of the bank, the other contracts in the bank's portfolio, and the objectives of the counterparty when it entered into the contract.

References

Arak, M.; L. S. Goodman; and A. Rones. "Defining Credit Exposure for Risk Management Products." *Review of Research in Banking and Finance*, 3 (Winter 1987), 60–72.

Bank for International Settlements. "Proposals for International Convergence of Capital Measurement and Capital Standards" (July 1988).

Belton, T. M. "Credit Risk in Interest-Rate Swaps." Working Paper, Board of Governors of the Federal Reserve System (April 1987).

Biger, N., and J. Hull. "Valuation of Currency Options." *Financial Management*, 12 (Spring 1983), 24–28.

Cooper, D. F., and I. R. Watson. "How to Assess Credit Risks in Swaps." *The Banker*, 137 (Feb. 1987), 28–31.

Cox, J. C.; J. E. Ingersoll; and S. A. Ross. "An Intertemporal General Equilibrium Model of Asset Prices." *Econometrica*, 53 (March 1985), 363–384.

Cox, J. C., and S. A. Ross. "The Valuation of Options for Alternative Stochastic Processes." *Journal of Financial Economics*, 3 (Jan./March 1976), 145–166.

Cox, J. C.; S. A. Ross; and M. Rubinstein. "Option Pricing: A Simplified Approach." *Journal of Financial Economics*, 7 (Sept. 1979), 229–263.

Federal Reserve Board and Bank of England. "Agreed Proposal of the United States Federal Banking Supervisory Authorities and the Bank of England on Primary Capital and Capital Adequacy Assessment" (January 1987).

Garman, M. B., and S. W. Kohlhagen. "Foreign Currency Option Values." *Journal of International Money and Finance*, 2 (Dec. 1983), 231–237.

Grabbe, J. O. "The Pricing of Call and Put Options on Foreign Exchange." *Journal of International Money and Finance*, 2 (Dec. 1983), 239–253.

Hull, J. *Options, Futures and Other Derivative Securities*. Englewood Cliffs, NJ: Prentice Hall (1989).

Johnson, H., and R. Stulz. "The Pricing of Options under Default Risk." *Journal of Finance*, 42 (June 1987), 267–280.

Kane, E. J. "Market Incompleteness and Divergences between Forward and Future Interest Rates." *Journal of Finance*, 35 (May 1980), 221–234.

Kim, D., and A. M. Santomero. "Risk in Banking and Capital Regulation." *Journal of Finance*, 43 (Dec. 1988), 1219–1233.

Santomero, A. M., and R. D. Watson. "Determining an Optimal Capital Standard for the Banking Industry." *Journal of Finance*, 32 (Sept. 1977), 1267–1282.

Wall, L. D., and K-W Fung. "Evaluating the Credit Exposure of Interest Rate Swap Portfolios." Working Paper 87–8, Federal Reserve Bank of Atlanta (Dec. 1987).

Whittaker, J. G. "Pricing Interest Rate Swaps in an Options Pricing Framework." Working Paper, Federal Reserve Bank of Kansas City and Northern Trust Company, Chicago (May 1987).