Market Incompleteness and Divergences Between Forward and Futures Interest Rates

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I. Introduction

A PRESIDENTIAL ADDRESS IS a solemn ceremonial event. The words should perhaps be sung rather than recited. Not being in very good voice, I intend to give the shortest presidential address in AFA history. The presidential address is 1600 Pennsylvania Avenue, N.W. In the rest of the time allotted to me, I profess only to deliver a paper.

My paper offers some new perspectives on the logical basis for expecting divergences between yields quoted for futures contracts and parallel forward yields implicit in the term structure of interest rates. The presentation proceeds by focusing on the equilibrium *prices* of alternative ways of making two-period investments and on assumptions that *differentially* affect the ways that spot, futures, and options markets for bonds can be completed. Implications are drawn regarding observable divergences between expected, forward, and futures prices of post-dated bonds. Since we are free to interpret the first or the second period in our two-period equilibrium conditions to be as long or as short as we please, our results readily generalize to investments of any maturity.

My specific objective is to develop an efficient-market explanation of the allegedly "confusing" time-series behavior of the differential between the forward and futures interest rates on U.S. Treasury securities. Forward rates implicit in the term structure have been close to parallel yields on futures contracts traded on the Chicago Mercantile Exchange only for the Treasury bill contract closest to execution (Poole; Lang and Rasche). Differentials on more distant contracts have been consistently large and in 1977 reversed sign (Struble). Even allowing for transactions and carrying costs, most scholars find these events puzzling (Capozza and Cornell; Vignola and Dale). A few even interpret the persistent failure of futures and forward rates to converge as evidence of segmentation

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(Branch) or "inefficiency" (Rendleman and Carabini). I argue instead that, once we recognize the implicit and explicit costs of guaranteeing futures-market performance and the capital-gains tax treatment of futures-market profit and loss, divergence becomes the typical equilibrium state.

I. Prices Versus Yields

Traditional term-structure theory focuses on single-payment securities, uncomplicated by default risk or special features of any kind. The unit price, $P_{n,t}$, of a security that matures in n periods is the discounted present value of a dollar at the maturity date. We find this value by discounting the future dollar n times at $R_{n,t}$, the yield to maturity on an n-period bond:

$$P_{n,t} \equiv 1/(1 + R_{n,t})^n, \quad \text{for} \quad n = 1, 2, 3, \ldots \tag{1}$$

In this discrete-time conception, the term to maturity of any bond spans n unit maturities. As against the continuous-time conception, we can defend the notion of a "unit maturity" as a minimum period for economical investment in open-market securities. Much empirical work arbitrarily treats this interval as the calendar quarter.

Discounting may proceed in terms of either nominal or real yields. Term-structure theory seeks only to explain differential or relative yields, so that one yield (or one bond price) may be given exogenously. It is convenient to conceive of this exogenous yield as the shortest rate in the system, which we can term the "bill rate."

Traditional economic analyses root their explanations in the yield side of the pricing identity (1), asymmetrically viewing hypothetical changes in equilibrium yields as driving observable changes in bond prices. On the other hand, modern finance theory roots itself in state-preference theory, treating the equation's price side as a dog that wags an interest-rate tail. In this approach, bond prices arise as the sums of values of state-dependent claims, with yields viewed as artificial by-products (e.g., see Banz and Miller).

As the identity sign implies, apparent differences between a price and yield approach can only be matters of emphasis, not of substance (Malkiel). Nevertheless, I hope to convince you that reformulating expectational theories of the term structure of interest rates as arbitrage theories of bond *prices*, provides new insights into the ways in which futures markets function.

II. The Pure-Expectations Theory Revisited

A. *Alternative Strategies for Term Lending.* In modern contingent-claims theory (e.g., Black and Scholes), the trick is to define a hedge portfolio. Since the long and short sides of a hedge portfolio must be perfect substitutes, their value is governed by the Law of One Price. Investments maturing in exactly n periods can be made in at least seven distinct ways. In turn, these seven basic investment strategies can be broken down into two strategies for holding strips of maturities *shorter* than the planned holding period (i.e., the planned maturity of the overall investment), two strategies for *matching* the maturities to be held with the planned holding period, and three strategies for holding maturities *longer* than the planned holding period:

Strip or Unit-Maturity Strategies

1. The "naked rollover" or "uncovered rollover strategy" of buying a one-period bond (i.e., a "bill") in the current spot market and planning to let the matured funds ride in the one-period spot market for each of the next $n - 1$ periods.
2. The "covered rollover" or "futures-market strategy" of buying a bill in the current spot market and simultaneously contracting in the futures market for a sequence of bill purchases over the next $n - 1$ periods.

Matching Strategies

3. The "repurchase-agreement strategy" of buying an n-period security today and arranging to borrow against it *via* a series of one-period "buy-back" agreements.
4. The "buy-and-hold strategy" of buying an n-period bond in the current spot market and planning to hold it until it matures. This strategy is also called the "implicit forward-market strategy," to emphasize that the investor may be viewed as executing a series of implicit "forward' transactions in future one-periods bonds.

Yield-Curve Rides

5. The "uncovered roll-out strategy" of buying a bond whose maturity is longer than n periods and planning to sell it in the spot market n periods later for whatever it will bring at the time.
6. The "covered roll-out strategy" of buying a bond whose maturity is longer than n periods and simultaneously contracting in the futures market to sell it in exactly n periods.
7. The "options strategy" of buying a bond whose maturity is longer than n periods and simultaneously selling a call option and buying a put option on the bond, both exercisable n periods in the future at an identical striking price.

Metaphorically, each investment strategy maps out a set of roads along which current funds can travel to a destination n periods in the future. For any planned period of investment, each strategy constructs a point-input, point-output transaction in bond, options, and futures markets that converts an outflow of present dollars into a larger return flow of future dollars. Assuming that bond denominations are perfectly divisible, all strategies can be scaled to offer the same *expected* payoff of one dollar n periods hence.

B. *Prices in Perfect Markets With Risk Neutrality and Perfect Performance Guarantees*. We begin by assuming risk neutrality, identical expectations, perfect divisibility, costless transacting, identical tax treatment of interest and capital-gains income, *and* costless guarantees of futures-market performance. Under these assumptions, all n-period investment strategies have the same present value, since certain and expected future dollars are equally desirable. This is the situation contemplated in the pure-expectations theory of the term structure (PET), in which so-called term premia do not exist. By definition, term premia are differences between the market yield on n-period securities and the expected average yield on an n-period naked rollover in bills.

Under the PET assumptions, futures, spot, repurchase, and options markets are complete. For each holding period, all seven strategies must have the same price. To see what theorems equilibrium pricing entails, it is convenient to focus on two-period investments. Although we have not yet explained our notation, Table 1 gives the equilibrium price under PET of an expected dollar generated by each of the seven basic investment strategies.

On the left-hand side of the equals signs, $P_{2,t}$ represents the current price of a dollar receivable for sure in two periods. Superscripts designate each of the other strategies being valued. N signifies naked or uncovered positions, C covered ones. S indicates that the maturity bought in the spot market is shorter than the planned holding period, while L indicates that the maturity purchased in the spot market is longer than the planned holding period. RP and O stand for repurchase agreement and options, respectively.

On the righthand side, $E_t(P_{n,t+k})$ represents the expected value as of time t of the price of n-period bonds k periods in the future. Superscripts are used to

Table 1

Value of an Expected Dollar Receivable Two Periods Later From Each of the Seven Basic Investment Strategies, Given PET Assumptions

1. Naked Rollover:

$$P_{2,t}^{NS} = P_{1,t}E_t(P_{1,t+1}).$$

2. Covered Rollover:

$$P_{2,t}^{CS} = P_{1,t}P_1^*(t, t+1).$$

3. Implicit Forward Contract:

$$P_{2,t} = P_{1,t}P_1^F(t, t+1).$$

4. Repurchase Agreement:

$$P_{2,t}^{RP} = [P_{2,t} - P_{2,t}] + P_{1,t}P_1^{BB}(t, t+1) = P_{1,t}P_1^{BB}(t, t+1).$$

5. Naked Rollout:

$$P_{2,t}^{NL} = P_{n,t}[E_t(P_{n-2,t+2})]^{-1}, \qquad n > 2.$$

6. Covered Rollout:

$$P_{2,t}^{CL} = P_{n,t}[P_{n-2}^*(t, t+2)]^{-1}, \qquad n > 2.$$

7. Options Strategy:

$$P_{2,t}^{O} = \{P_{n,t} - P^{\text{call}}[t, t+2; E_t(P_{n-2,t+2})] + P^{\text{put}}[t, t+2; E_t(P_{n-2,t+2})]\} \cdot [E_t(P_{n-2,t+2})]^{-1}, \qquad n > 2.$$

designate explicit and implicit per-dollar prices of futures, forward, repurchase, and options contracts. In these contracts, each price has two timing dimensions. The first index represents the date at which the contract is entered. The second gives the future date at which the contract is scheduled for execution. Asterisks indicate futures prices, while *BB* denotes a "buy-back" price. The notation used for prices of call and put options includes as a third argument the "striking price" at which the option may be executed at $t + 2$. The reader may recognize that we have priced the options strategy by drawing on Hans Stoll's "put-call parity."

Finally, the forward price of $t + 1$ bills implicit in the term structure of interest rates at time t is defined as:

$$P_1^F(t, t+1) \equiv P_{2,t}/P_{1,t}. \tag{2}$$

We can clarify the mechanics of the first six strategies by interpreting the righthand side of each equation as the product of the price and quantity of the security purchased at t. The second term of each product tells us *how many* securities we must buy today to produce an expected inflow of one dollar at $t + 2$. As an example, suppose that $P_{1,t}$ and $E_t(P_{1,t+1})$ were each 0.87. Then, a naked rollover would require us to buy 0.87 of current bills, thereby putting ourselves in the position to be able to put $E_t(P_{1,t+1})$ into bills at $t + 1$. The price of two-period bonds—indeed the price of all two-period strategies—would of course be $(0.87) \cdot (0.87) = 0.76$. In passing, we may note that unless interest-rate expectations are single-valued:

$$E_t(P_{1,t+k}) = E_t[1/(1 + R_{1,t+k})] \neq 1/[1 + E_t(R_{1,t+k})],\ k \geqq 1. \tag{3}$$

To illustrate a naked roll-out, let us suppose that $P_{3,t}$ is 0.701 and $E_t(P_{1,t+2})$ is 0.926. Purchasing $(0.926)^{-1}$ or 1.08 three-period bonds at time t would produce an expected dollar's worth of bills to sell at $t + 2$. To carry out the options strategy, we must buy the same amount of three-period bonds at t and sell put and call options on an expected dollar's worth of the bills that these bonds evolve into at $t + 2$.

Since all strategies with the same expected future payoff must have the same current price, we can establish three theorems. First, the expected, futures, forward, and repurchase prices of next period's bills must all be equal:

$$E_t(P_{1,t+1}) = P_1^*(t, t+1) = P_1^F(t, t+1) = P_1^{BB}(t, t+1). \tag{4}$$

Substituting (4) into expressions for successively longer investment strategies would let us establish that futures, forward, and repurchase prices for every future bill must each also equal the contemporary forecast of the relevant bill price.

Second, the expected and futures prices of all longer bonds must also be equal:

$$E_t(P_{n-2,t+2}) = P_{n-2}^*(t, t+2), \qquad n > 2. \tag{5}$$

By increasing the length of investment period to $k > 1$, we can easily establish that:

$$E_t(P_{n-k,t+k}) = P_{n-k}^*(t, t+k), \qquad n > k > 1. \tag{5'}$$

Implicit forward and repurchase prices for bonds would also equal these same values.

Third, the options strategy constructs a *riskless* combination of bond-market and options transactions. Whatever happens to the price of $(n-2)$-period bonds at $t+2$, the investor is assured of receiving exactly one dollar. If $P_{n-2,t+2}$ turns out to exceed its expected value, the put option becomes valueless and the call would be exercised. If $P_{n-2,t+2}$ falls short of its expected value, the call becomes valueless and the investor would exercise the put. Either way, the bonds are exchanged for their expected value.

Setting $P_{2,t}^{O}$ equal to $P_{2,t}^{NL}$, we can easily establish that the PET assumptions imply that a put and call whose common striking price equals the bond's expected price have the same value. Cancelling the expected-value term, we get:

$$P_{n,t} - P_{n,t} = 0 = -P^{\text{call}}[t, t+2; E_t(P_{n-2,t+2})] + P^{\text{put}}[t, t+2; E_t(P_{n-2,t+2})]. \quad (6)$$

Replacing $t+2$ by $t+k$ (where $n > k \geqq 1$) extends the result to cover holding periods of any length.

III. Relaxing PET Assumptions

Term-structure theory is concerned with costs and benefits generated by moving funds through time. In the pure-expectations theory, risks are irrelevant and temporal "transportation" costs are zero. Hence, every conceivable n-period path of investment offers the same equilibrium expected return. All n-period strategies are equally efficient.

If we introduce differential tax rates or transactions costs, different individuals may find some n-period paths more efficient than others. Since efficient paths dominate inefficient paths, n-period securities must always be priced according to the risks and transportation costs encountered along what are the most efficient paths for the marginal investor.

A. *Introducing Risk Aversion: The Risk-Averse Pricing Theory of the Term Structure.*[1] Leaving the other PET assumptions untouched, we now remove the assumption of risk neutrality. Instead we assume that a positive price is paid for risk-bearing and that some nonarbitrageable risk exists in every future bill and every current bond. Perhaps the easiest way to justify this assumption is to postulate the absence of perfect hedges against unanticipated inflation.

These new assumptions break the equivalence between cash flows that are certain and those that are merely expected. While covered strategies remain priced as before, the risk-bearing inherent in *uncovered* strategies must now be priced. Parallel futures and implicit forward bond prices remain equal to each other, but they can no longer also equal the *expected* price. We can see this clearly in the two-period case. If the futures price *did* equal the expected price, the risk-adjusted value of the naked rollover strategy would be too high. No

[1] Observable implications developed in this section parallel those in section V of Cox, Ingersoll, and Ross. However, our approach makes it possible to finesse the specific stochastic assumptions required to implement their "general-equilibrium" continuous-time model.

lender would go unhedged. The forward and futures-market strategies would *dominate* the uncovered rollover. To complete markets again, the risk-bearing inherent in the uncovered strategy must be compensated. The price paid for *not* covering is the market exchange rate between a dollar certain and a dollar expected to be generated at $t + 1$ *by the particular rollover or rollout opportunity.* In the multiperiod CAPM (Merton), this difference would be the compensation paid for bearing undiversifiable or "systematic" risk. We denote the t-period price of $t + 1$ certainty equivalence in bills as $a_{t,t+1}$. Risk-averse pricing implies that this price be *less* than unity.[2] It is, however, an instrument-specific measure.

As compared to the risk-neutral situation, the certainty-equivalence factor scales down the equilibrium price of the naked-rollover strategy to:

$$P_{2,t}^{NS'} = P_{2,t}^{NS} a_{t,t+1} = P_{1,t}[E_t(P_{1,t+1})a_{t,t+1}]. \quad (7)$$

Referring again to our price-quantity interpretation of these pricing equations, we need to buy *fewer* bills today to acquire an uncertain opportunity whose two-period expectation is one dollar than to purchase a certain opportunity with the same expectation.

Drawing again on the Law of One Price and the concept of market completeness, riskless futures, forward, rollout, and options strategies must sell at this same risk-adjusted price. As long as performance can be costlessly guaranteed in markets for options on future bills or bonds, riskless options strategies can also be constructed. This means that put and call options on securities that are exercisable at expected prices would still have equal values.

Equation (7) has an important observable implication. Risk-averse pricing implies that forward, futures, and repurchase-agreement prices for bills and bonds should *lie below* expected prices:

$$P_1^F(t, t + k) = P_1^*(t, t + k)$$
$$= P_1^{BB}(t, t + 1) = E_t(P_{1,t+k})a_{t,t+k}, \qquad k = 1, 2, 3, \ldots \quad (8)$$

$$E_t \left[\prod_{k=1}^{n-1} P_{1,t+k}\right] a_{t,t+n}^n = \prod_{k=1}^{n-1} P_1^*(t, t + k). \quad (9)$$

In equation (9), all forecasts are conditional on the t-period information set and $a_{t,t+n}^n$ represents the certainty-equivalence conversion factor for $t + n$ cash flows in n-period bonds evaluated at time t. Substituting (8) into (9) recursively, we can show that

$$a_{t,t+n}^n = \prod_{k=1}^{n-1} a_{t,t+k}. \quad (10)$$

With each $a_{t,t+k}$ assumed to be less than unity, bond-market certainty-equivalence factors decline with maturity. Translated to yield space, this cumulative "term

[2] Risk-averse pricing is, of course, merely an hypothesis. If all interest-rate risk could be costlessly arbitraged away, $a_{t,t+k}$ would equal unity for all k. Alternatively, in the preferred-habitat theory of Modigliani and Sutch, where $a_{t,t+k}$ is interpreted as manifesting the balance of borrower and lender maturity preferences, the certainty-equivalence factors can equal or even exceed unity. Other researchers (e.g., Green; Hirshleifer; Roberts) suggest still-different interpretations and restrictions. However, empirical research on the term structure (e.g., Kessel; Nelson; Kane and Malkiel; McCulloch; Pesando) supports the hypothesis of risk-averse pricing.

price" implies conventionally positive term premia. By definition,

$$T_{n,t} \equiv [P_{1,t}E_t \prod_{k=1}^{n-1} (P_{1,t+k})a_{t,t+k}]^{-1/n} - [P_{1,t}E_t \prod_{k=1}^{n-1} (P_{1,t+k})]^{-1/n}. \qquad (11)^3$$

Moreover, term-price theory can be turned into an explicit theory of term-price *incrementation.* In particular, if the price of bill-market certainty-equivalence is anticipated to be the same for all future periods, the term price would increase (at a decreasing rate) with maturity. In yield space, this would produce term premia that increase to an asymptote at a decreasing rate. This theory is observationally equivalent to the Hicksian Liquidity-Premium Theory of the term structure. In our theory, the $a_{t,t+k}$ are marginal elements parallel to Hicksian liquidity premia, while the term price is a cumulative average on the order of a term premium.

We can illustrate by extending the numerical example we used to illustrate Table 1. If for all k ($k = 1, 2, \ldots$), $a_{t,t+k} = 0.98$ and $E(P_{1,t+k}) = P_{1,t} = 0.87$, we obtain $P_{2,t} = 0.74$ and $P_{4,t} = 0.539$, with $T_2 = 0.012$, $T_4 = 0.018$, and $\text{Lim}_{n\to\infty} T_n = 0.0235$.

Implications can also be drawn from this theory concerning the effect of interest-rate levels on term premia. For a given term price, the term premium on any maturity *increases* with the average expected yield on interim bills. (See footnote 3.) Existing writings on this issue (Kessel; Cagan; Van Horne; Nelson; Pesando) do not distinguish the possible indirect effect of interest rates through the term price from the direct effect shown here.

B. *Introducing Differential Capital-Gains Taxes.*[4] With risk-averse pricing, interest-rate futures contracts carry an *anticipated* capital gain. U.S. tax law designates futures contracts—unlike Treasury bills themselves, but like options on bonds—as capital assets (*The Bank Tax Report*). Net capital gains are taxed differently from ordinary interest income.

In principle, the deferral of capital-gains taxes until realization and scheduled revisions in the capital-gains tax structure, along with changes in the tax situation of marginal investors in interest-rate futures, go a long way toward explaining observed variations in the average differences between forward and futures prices. The major tax law revisions are:

1. For assets other than the long side of a futures contract traded in an organized market, the holding period necessary for trading profits to qualify as long-term capital gains increased from 6 months to 9 months in 1977 and to one year from 1978 forward. Profits earned on the *short side* of futures contracts are always classified as *short-term* capital gains, regardless of the holding period.
2. In 1979, taxpayers' long-term capital-gains tax rate was lowered from 50 percent of the statutory marginal tax rate on ordinary income to 40 percent of the statutory rate, producing a maximum capital-gains tax of 28 percent.
3. The optional "alternative tax," which had placed a cap of 25 percent on the

[3] Defining the expected yield on an n-period naked rollover as $h_{n,t}$, (11) simplifies to:

$$T_{n,t} = (1 + h_{n,t})[(a_{t+n}^{n})^{-1/n} - 1]. \qquad (11')$$

[4] I am grateful to Stephen Buser for suggesting a role for capital-gains tax rates.

rate for a household's first $50,000 in net long-term gains, was eliminated in 1979. For corporate taxpayers, the alternative ceiling on the long-term capital-gains tax rate was at the same time lowered from 30 to 28 percent.

4. Except for market-makers (for whom these assets are deemed stock-in-trade), net futures losses are not fully deductible from ordinary income. In any year, household taxpayers may deduct the sum of short-term losses and 50 percent of long-term losses only up to a maximum of $3,000 (up from $2,000 in 1978 and $1,000 before). However, by taking delivery rather than closing out a loss position, capital losses on T-bill futures can be converted into ordinary income.

Three important points emerge. First, except for market-makers, net *losses* on futures contracts are potentially tax-disadvantaged. Second, only net *long-term* capital gains are taxed advantageously. Third, favorable tax treatment applies only to *net* realized long-term gains on the aggregate of calendar-year transactions. Capital losses developed elsewhere in the portfolio lessen the effective tax preference afforded gains on long futures.

Recognizing these complications breaks the completeness of the hypothetical cover provided by futures-market transactions. Gains on long futures more than offset equal losses in bill markets, while losses on long futures may not offset equal gains in bill markets. Contract values must price both the tax advantage and the incompleteness. Equilibrium requires an equality of *after-tax* risk-adjusted returns on all strategies.

We can pull the risks and net potential tax advantages into a "futures factor," $f_{t,t+k}$. We hypothesize that:

$$P_1^*(t, t+1) = P_1^F(t, t+1) f_{t,t+k}, \tag{12}$$

where $f_{t,t+k} > 1$ for contracts with six months or more to run. In holding that long bill-rate futures are bid to a premium relative to implicit forward prices, we are assuming that, for the *marginal investor* in long futures, the lure of tax-favored expected capital gains outweighs the risks of unanticipated adverse price declines here and elsewhere in the investor's portfolio.[5]

C. *Prices in Perfect Markets with Risk Neutrality But with Costly Performance Guarantees.* We now restore the assumptions of equal tax rates and risk neutrality and relax the assumption that the ability to execute futures contracts and to exercise options can be guaranteed costlessly. This introduces a new probability into the pricing equations: the *probability of nonperformance.*

1. *Implicit Prices Serve to Complete Markets*

Unless the performance of futures and option contracts is costlessly guaranteed, spot, futures, and options markets are no longer complete. We need an implicit market for insurance against nonperformance to complete them again. When real rather than nominal discount rates are employed, the probability distributions of one-period prices are unbounded on both sides. Even using nominal rates, the

[5] To pursue the tax-effect issue, readers may consult Scholes. His paper analyzes tax effects on options pricing for investors in different tax brackets.

upper tail is unbounded. In either case, it is unprofitable for producers of costly guarantees to carry them to perfection. This means that some unlikely events cannot be hedged against in futures markets. If investors can acquire only *partial* performance guarantees, it is impossible for them to construct a perfectly hedged portfolio by going long in current bonds and short in an intervening series of interest-rate futures on one-period bonds or by trading in puts and calls on two-period bonds. The "fatter" the uninsured tail (or tails) of the bond-price distributions, the more severe the moral hazard.

To complete the spot and future markets, we must include the implicit prices of perfect-performance guarantees. Assuming that bonds are default-free does not imply that future commitments by individuals to buy and sell these bonds are also default-free. A would-be hedger has to recognize that futures-market or options transactors who take positions opposite to the hedger's act to maximize their own wealth position. As maximizers, they must be expected to default on their contracts whenever the benefit from reneging exceeds the penalties imposed on them for doing so. Similarly, a hedger retains the implicit option to engage in advantageous default himself. Even with universal risk neutrality, transactors in default-free long-term bonds offer the other side of the market a commitment that is valuable because it is irrevocable. Because they are executed immediately, implicit forward transactions are free from the moral hazard that besets a futures contract.

Investment strategies that take long positions in real-world futures markets are equivalent to the simultaneous issuance of *three* contracts:

1. Spot purchase of a one-period bill and a futures-market long position in a rollover portfolio of the same maturity;
2. An implicit put option allowing holders of short positions in futures markets ("shorters") to default whenever the one-period spot rate of interest becomes sufficiently *low*;
3. An implicit call option allowing long positions in futures markets (the "longers") to default on futures-market commitments whenever the spot rate becomes sufficiently *high*.

Although exercising either option constitutes nonperformance of the explicit futures contract, the true price of a long position in the futures market must have three components, one for each of the explicit contracts issued.

Introducing the possibility of nonperformance does not disturb the PET equilibrium conditions. The expected and forward prices of post-dated bills continue to equal futures prices, but each futures price now has implicit as well as explicit elements. Hence, the unobservable *true* price of futures transactions will seldom be the contract execution price quoted in the marketplace, $P_1^{ex}(t, t+1)$.

2. *Technology of Contemporary Performance Guarantees*

Without external constraints, only in the razor's edge case where $P_1^{ex}(t, t+k)$ happens to equal $P_{1,t+k}$ would maximizing individuals on *both* sides of the futures market be willing to live up to their contractual commitments.

In the contemporary United States, Chicago Mercantile Exchange (CME) rules provide futures-market transactors with *partial* performance guarantees, whose value is hidden in escrow deposits, net-worth screening, negotiated brokerage fees (usually \$60 per roundtrip million-dollar contract), and costs of margin maintenance. These guarantees narrow the effective range of potential defaults, improving the deliverability of the product but increasing explicit transactions and maintenance costs.[6]

First, brokers require most transactors to deposit \$10,000 in escrow and to put up an "initial margin" of at least \$1500 per million-dollar transaction. Since these deposits may take the form of a letter of credit or pledges of interest-bearing securities, they impose only minor costs on *wealthy* individuals. However, subsequent gains and losses in the value of the contract cumulate as cash in the margin account. Accounts are marked to market daily. Positions that fall below a "maintenance margin" of \$1200 are subject to a margin call for cash, on which the broker keeps the subsequent interest. Customers who fail to meet a margin call are promptly sold out.[7] However, because of exchange limits on daily price changes, sell-outs cannot always take place immediately. Occasionally, brokers must wait for futures markets to "catch up" with spot markets. Balances in excess of escrow and initial margin requirements generate interim cash inflows that need to be invested at interest. When the futures price is *expected* to change in a specific direction (as, for example, it is expected to *rise* under risk-averse pricing), margin accounts introduce an *asymmetry* in expected cash flows for long and short positions that the contract execution price must adjust away.

Additionally, each brokerage firm is pledged to make good all defaults by its own customers and to bring suit in civil court against the defaulting party. Finally, the Exchange operates an emergency fund, which backstops transactors against individual-broker bankruptcy, even to the extent of authorizing the Exchange to levy make-good charges on surviving members of the Exchange.

In perfect markets, the quality of Exchange guarantees would increase with their cost. Hence, they would have a conflicting dual effect on the true futures price, $P_1^*(t, t+1)$. The more costly the guarantee, the higher the probability of performance, but also the greater the implicit charge for the guarantee. In practice, performance guarantees are produced from various combinations of high-rated cosigners, letters of credit, escrow accounts, and Exchange commitments. If we assume that guarantee quality is produced at nondecreasing costs, the *optimal quality* of guarantee would *never* be perfect.

Especially if the distribution of bond-price changes should be stable Paretian (Roll, McCulloch), one-day interest-rate movements could in principle have a sharp-enough spike to exhaust the finite sum of reserves implicit in Exchange arrangements. For a given set of penalties, the "fatter" the uninsured tails of the distribution of future one-period bond prices, the greater the probability of nonperformance.

CME arrangements only approximate guarantees of perfect futures perform-

[6] I am grateful to Kurt dew for clarifying the mechanics of CME margin requirements and guarantees.

[7] The probability of nonperformance may be particularly high for repurchase agreements, because interim changes in contract value and collateral are not systematically monitored.

ance. Persons who take long and short positions in interest-rate futures are *forced* by Exchange rules implicitly to sell their brokers almost all of their unconstrained default option. They retain only a residual option exercisable at a highly unlikely striking price. Still, with risk neutrality and perfect markets, the value of the options sold and retained must enter the true price of the overall futures contract.

3. *Guarantee Costs and True Futures Prices.*

For futures-market commitments maturing in any period, three possibilities exist:

1. S_s = default by short positions;
2. S_l = default by long positions;
3. S_{ex} = execution of all maturing commitments.

The true price of a two-period futures strategy is its expected value. This is the product of $P_{1,t}$ and the net expected value of three second-period components. We let G_{t+1} equal the cost of Exchange guarantees and $\Pr(S_{ex})$ represent the conditional probability of execution, given the set of guarantees in force. If substantial inflows are expected to accrue to long-position margin accounts, G_{t+1} could well be negative. $Y_{t+1,l}$ and $Y_{t+1,s}$ denote the conditional expected values of Exchange penalties and make-good payments in the event of the indicated type of default. The alternative expected second-period cash flows become:

1. $\Pr(S_{ex})P_1^{ex}(\mathrm{t}, \mathrm{t} + 1) - G_{t+1}$
2. $-Y_{t+1,l}$
3. $Y_{t+1,s}$. (13)

The first of these expected cash flows may be interpreted as the price of a hypothetical perfect futures contract, with the indicated expected value, $P_1^{**}(t, t + 1)$. The other two elements may be interpreted as the values of the *residual* long-position and short-position options to violate the contract, $N_1^L(t, t + 1)$ and $N_1^S(t, t + 1)$. Defining the "net option value,"

$$N_1(t, t + 1) = N^L(t, t + 1) - N^S(t, t + 1), \tag{14}$$

the true price of the futures contract becomes:

$$P_1^*(t, t + 1) = P_1^{**}(t, t + 1) - N_1(t, t + 1). \tag{15}$$

4. *Relation Between Futures Contract Prices and Forward Prices*

Risk neutrality requires that forward prices and true future prices be the same. But equation (15) implies that futures-contract execution prices may differ in either direction from both of these.

If the net option value just happens to equal zero, contract execution prices would probably *exceed* forward prices. But when price appreciation is expected on the futures contract (as it would be under risk-averse pricing), the reverse could be true. The differential depends on both the quality and the net cost of performance guarantees. Assuming that guarantees of given quality are cheap to produce for near-term contracts but progressively more costly to produce as the delivery date becomes more distant in time would let us explain the pattern of differentials on T-bill futures observed since mid-1977. The gap between forward and futures prices has been negligible for contracts close to delivery and increased with the futurity of the contract.

To explain the pre-1977 pattern without either appealing to tax effects or relaxing the assumption of risk neutrality, we must argue either that in 1976 and early 1977 the mechanics of CME guarantees promised large interim net cash *in*flows to long positions or that the net option value was positive and increased with maturity.

D. *Prices in Perfect Markets with Risk-Averse Pricing and Costly Guarantees.* Retaining costly guarantees *and* introducing a positive price for risk-bearing breaks the equality between expected and forward rates. For futures and options markets, the major implication is that nonperformance *risk* must be priced, too. In the two-period model, equilibrium requires:

$$E_t(P_{1,t+1})a_{t,t+1} = P_1^F(t, t+1) = P_1^{**}(t, t+1)a_{t,t+1}^{**} - Y_{t+1,l}a_{t,t+1}^L + Y_{t+1,s}a_{t,t+1}^S. \quad (16)$$

In this more realistic model, variations in the systematic risk of post-dated bills and each of the three elements of the futures contracts combine with changes in their expected values to explain divergences between forward prices, expected prices, and contract prices for interest-rate futures. The equilibrium condition makes it clear that expected and forward bill prices can depart substantially, not only from each other, but especially from the execution prices of futures-market contracts. Reintroducing capital-gains tax differentials would further increase opportunities for divergence.

IV. Conclusions and Agenda for Future Research

Recognizing that it is costly to guarantee futures-market performance is sufficient to destroy any presumed identity between futures and forward interest rates. Introducing a positive price for risk-bearing services, applicable to whatever nonarbitrageable risk inheres in every risky opportunity, suffices both to make forward interest rates differ from expected rates and, especially when capital gains are taxed preferentially, to make the representation of futures rates very complicated indeed. When PET assumptions are relaxed, expected, forward, and futures yields are free to travel different roads. Movements in comparable expected, forward, and futures yields *constrain* each other, but not to the point of equality.

Although the various parameters identified in this paper may be estimated in principle, data on the true prices of different contracts and options and on the cost functions for performance guarantees will be difficult to align.

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