THE JOURNAL OF FINANCE • VOL. XXXIV, NO. 5 • DECEMBER 1979

Put-Call Parity and Market Efficiency

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I. Introduction

OPTIONS HAVE CONSTITUTED THE most dynamic segment of the securities markets since the inception of the Chicago Board Options Exchange in April 1973, which was followed shortly thereafter by four other registered option exchanges. Until June 1977, only call options were traded on these registered option exchanges. At that time, each of the exchanges was permitted to commence trading in put options on five underlying securities. The initial success of the put options promises to provide the next major thrust to the registered options markets when they can be traded on all of the underlying securities listed on the exchanges.

The advent of put trading on the registered exchanges is important because a deterministic relationship should exist between put and call prices, irrespective of investor demands, if both options are written on the same underlying security and have the same exercise (striking) price and expiration date.[1] This relationship exists because the put, call, and the underlying stock form an interrelated securities complex, in which any two of the three instruments can be combined in such a manner to yield the profit and loss opportunities of the third instrument. Because of conversion possibilities, theoretical put-call parity models can be developed to determine a put (call) price given a corresponding call (put) price and other relevant information. If the actual put or call price should deviate substantially from the parity price, an opportunity exists for investors to set up a riskless arbitrage position and earn more than the risk-free rate of return.

The original put-call parity model was developed by Stoll [10] and later extended and modified by Merton [5]. These models were used by Stoll [10] and and Gould and Galai [3] to empirically investigate parity among over-the-counter (OTC) put and call options. While these studies basically supported the put-call parity theory, some inefficiencies in the relationship were also found to exist. Today's options market is vastly different from the OTC market, however. Major changes made by the registered options exchanges, including the standardization of contractual terms such as the exercise price and expiration date, and the creation of a central clearing corporation, have transformed the options market place from a thin, negotiated market to a competitive, auction market.[2] Another

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[1] The relationship is deterministic in that put and call prices are established relative to one another in a risk-return format and not in the sense that an absolute option price can be determined. For option valuation models that determine absolute prices, see the seminal artical by Black and Scholes [1], and modifications or extensions by Rubinstein [8], Parkinson [6], Roll [7], and Geske [2]. An excellent review of option pricing may be found in Smith [9].

[2] The dramatic growth of this market can be measured by the volume of options traded on the five registered exchanges versus the over-the-counter market. The highest volume year in the over-the-

major difference between the OTC and registered markets concerns the payout protection of the option. An OTC option is dividend payout protected because its exercise price is adjusted downward by the amount of the cash dividend on the ex-dividend date. However, no such adjustment is made in the exercise price of the registered option, thus it is dividend payout unprotected.[3]

These marked differences between over-the-counter options and registered options warrant further investigation of the relationship between simultaneous put and call transactions. Section II summarizes the development of the theoretical parity model. Section III reviews previous empirical research and provides the methodology and empirical results of this study. Section IV offers the conclusions of the study.

II. The Relationship Between Put and Call Prices

A. The Basic Framework

It was pointed out earlier that a put, call, and the underlying common stock form an interrelated securities complex, in which any two of the three instruments can be combined in such a manner to yield the profit and loss opportunities of the third instrument. This statement does not hold in the most strict sense unless the assumption is made that neither the put nor call can be exercised prior to their expiration date. These type options are referred to as "European" in contrast to "American" options which can be exercised at any time before their expiration. Additional assumptions are that the options are either dividend payout protected, or the stock does not pay any dividends during the option period, that there are no transaction costs, and that riskless borrowing and lending can take place at the risk-free rate.[4]

For European options, on a stock that does not pay dividends during the option period, it can be shown that a call option can be purchased directly, or as a combination of borrowing, a long position in the underlying stock and the purchase of a put option. Likewise, a put option can be purchased directly, or as a combination of lending, a short position in the underlying stock and the purchase of a call option. That these positions are equivalent is shown in Table I using the following symbols:

K = the exercise (striking) price of the option,
S = the current market price of the underlying common stock,
S^* = the market price of the underlying stock on the expiration date of the option,

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counter market was 1968 when contracts representing approximately 30.3 million shares were traded. This compares with contracts representing 3,950 million shares traded in 1977 on the exchanges, a more than 130 fold increase. The registered markets have provided increased liquidity, continuous public reporting of transactions, and reduced transaction costs.

[3] Merton [4, footnote 23] demonstrates that OTC options are only partially payout protected, and the accompanying text indicates the adjustment necessary for American registered options to be fully payout protected.

[4] Stoll [10] has indicated a call (put) writer long (short) the stock need not deposit margin in addition to that required for the long (short) position. In addition, Gould and Galai [3] have demonstrated that parity relationships hold when stock margin requirements and taxes are considered.

Table I

The Interrelationships of European Options and Non-Dividend Paying Stock

Strategy	Cashflow at Time 0	Cashflow at Expiration $S^* \le K$	$S^* > K$
A Buy call	$-C$	0	$S^* - K$
B Buy stock	$-S$	S^*	S^*
Buy put	$-P$	$K - S^*$	0
Borrow	$K/(1 + i)$	$-K$	$-K$
Total	$-P - S + K/(1 + i)$	0	$S^* - K$
C Buy put	$-P$	$K - S^*$	0
D Short stock	S	$-S^*$	$-S^*$
Buy call	$-C$	0	$S^* - K$
Lend	$-K/(1 + i)$	K	K
Total	$S - C - K/(1 + i)$	$K - S^*$	0

C = the current market price of a European call option,
P = the current market price of a European put option, and
i = the risk-free (and constant) rate of return covering the life of the option.

From the table it is evident that strategies A and B yield identical cash flows at expiration, as do strategies C and D. It follows that there also must be a relationship between the cash flows at time period zero for strategies A and B, and C and D, to prevent the call or the put from being a dominated security.[5] The arbitrage mechanism that ensures these relationships, or put-call parity, is referred to as conversion, whereby a put can be converted into a call, or reverse conversion, whereby a call can be converted into a put, with no risk to the converter.

If call prices are too high relative to put prices, an arbitrageur can set up a profitable long hedge by writing a call and achieving a "buy a call" position indirectly, using strategy B. Since there is no risk involved, given the previous assumptions, the converter should be able to borrow the amount $K/(1 + i)$ at the risk-free rate.[6] The present value of the profit, M, from this long hedge is:

$$C - P - S + K/(1 + i) = M. \tag{1}$$

If put prices are deemed too high relative to call prices, a profitable short hedge, or reverse conversion, can be set up by the arbitrageur. Under this scenario, a put option would be written directly and a "buy a put" position taken indirectly via strategy D.[7] Since the arbitrageur desires a riskless position, he will

[5] The relationships established in this subsection for time period zero hold when the underlying stock pays cash dividends, if the European options are dividend payout protected.

[6] S may be $>$, $<$, or $=$ to $K/(1 + i)$. If $S \ge K/(1 + i)$ it is implicitly assumed that the borrowed funds are used to finance all or a portion of the long position in the stock. If $S < K/(1 + i)$ it is assumed that $K/(1 + i)$ is used to purchase the stock and finance a portion of the put, since arbitrage assures that $S + P \ge K/(1 + i)$. If this relationship did not hold, $K/(1 + i)$ could be borrowed to make a purchase $S + P$, yielding a sure profit. This result is implied by Merton [4, Theorem 1, pg. 144 and Theorem 12, pg. 157].

[7] Implicit in this argument is that the investor has the use of the short proceeds, S, and that C +

lend the amount $K/(1+i)$ at the risk-free rate. In similar fashion to the long hedge, the present value of the profit, N, from the short hedge can be represented as:

$$P - C + S - K/(1+i) = N. \tag{2}$$

Equations (1) and (2) are the result of Stoll's [10] pioneering theoretical work on put-call parity.[8] He notes that in competitive markets, arbitrage will reduce profits from these conversions to $M = N = 0$. Therefore, from either equation (1) or (2) it is evident that

$$C - P = S - K/(1+i). \tag{3}$$

Stoll argues that in rational markets neither the put nor the call should be exercised early, since doing so would needlessly sacrifice market value in excess of the exercise value. Consequently equation (3) holds as an equilibrium relationship. However, Stoll did not differentiate between European and American options and in essence was implying that the equilibrium relationships are identical regardless of the type of options used in constructing the hedges. Moreover, he was implicitly stating that there should not be a difference in the option premiums between an American put (call) and a European put (call).

Merton [5], in a comment on Stoll's paper agrees that under the assumptions stated, if the options are dividend payout protected or the stock does not pay any dividends, an American call should not be exercised prematurely, and will thus have the same value as its European counterpart,[9] but that the same does not hold true for puts. Denoting P_a as the price of an American input, it follows from the arbitrage process that

$$P_a \geq \text{MAX}[0, K - S]. \tag{4}$$

Since implicit in Stoll's paper is $P_a = P$, from equation (3) $P_a = C - S + K/(1+i) < K - S$, if $C < iK/(1+i)$, which is possible for small enough S. This however violates arbitrage condition (4) and it would be to the put holder's advantage to exercise the put immediately. Consequently equation (3) holds only for European options and $P_a - P > 0$ represents the premium for the right of early exercise.

Without a formal put valuation theory, Merton [5] claims that the best that can be done for American options, given the above assumptions, are the bounding inequalities:

$$S - K \leq C - P_a \leq S - K/(1+i), \tag{5}$$

$K/(1+i) \geq S$. While Merton [4, footnote 30] argues that arbitrage can not be used to establish the relationship expressed by equation (2), since current regulations prohibit short proceeds from being reinvested, this assumption is standard in the literature of this sort (see e.g. Stoll [10] and Gould and Galai [3]). Further, while institutional restrictions prohibit individuals from reinvesting short proceeds, such activity is entirely possible for a brokerage house. In addition, individual investors can achieve the same result by selling stock held in an existing portfolio and reinvesting the proceeds. That $C + K/(1+i) \geq S$ follows directly from noting that $C \geq \text{MAX}[0, S - K/(1+i)]$, see Merton, [4, Theorem 1, pg. 144]. It also follows from reasoning similar to that expressed in footnote 6.

[8] Actually equations (1) and (2) are generalizations of Stoll's equations (3) and (4) since Stoll only considers the case where $S = K$.

[9] See Merton [4, Theorems 1 and 2, pg. 144] for proof.

where the right-hand side inequality establishes the long hedge boundary and the left-hand side inequality that of the reverse conversion with the possibility of immediate exercise of the put against the arbitrageur.[10] Note the arbitrageur is the holder of the put in the long hedge, and he may exercise the put early if it is to his advantage to do so. In this event, the arbitrageur is in receipt of the certain amount K at an earlier date than maturity. The boundary expression, nevertheless, remains intact. Stoll [11], in reply to Merton, concedes the point, but notes the stock price decline necessary for $C < iK(1 + i)$ is highly unlikely to occur in reality.

B. Registered American Options

It was mentioned previously that American registered options are completely dividend payout unprotected. Because of this, the parity models as specified by equation (3) and boundary expression (5) are inadequate for specifying the parity relationships for these type options. Therefore, before it is possible to gain an insight into how efficiently put and call prices are determined simultaneously on the registered options exchanges, parity relationships need to be established that incorporate the payout characteristics of these options.

For the purpose of this development, new variables are defined:

S_x^* = the ex-dividend market price of the underlying stock on the expiration date of the option,
D = the known nonstochastic dividend to be paid during the conversion period,
α = the known decline in the stock price on the ex-dividend date, expressed as a proportion of the dividend,
C_r = the current market price of an American registered call,
P_r = the current market price of an American registered put,
δ = the fraction of the conversion (option) period remaining from the dividend payment date, and
γ = the fraction of the conversion (option) period remaining from the ex-dividend date.

So that a basis for analysis might be established, let it initially be assumed that it is optimal to hold to maturity both the put and call options that make up a hedge. Let it further be assumed that the arbitrageur in the long hedge would reinvest the known dividend from the time of receipt, and that compensation for the dividend accruing to the party providing the shorted stock, in the short hedge, is settled at maturity. Since the arbitrageur is attempting to establish a perfect hedge, the rate of return demanded for the dividend received would be the risk-free rate. Likewise, since there is no risk to the party providing the shorted stock, if a perfect hedge is established, this party would demand compensation at the risk-free rate for non-immediate payment of the dividend receipt. Under these

[10] Merton [5] notes that long conversion of American payout protected options is not riskless since $P_a - P > 0$; thus the necessary opportunity loss for a converter who purchases an American payout protected put with the knowledge that he may never exercise it is $(P_a - P)(1 + i)$. It remains, however, that the converter has established a perfect hedge and that, under the assumptions stated, his interim or terminal outflows can never exceed his inflows, and in this context conversion is riskless.

assumptions,[11] it is evident from Table II that the following relationships must hold to prevent either option from being a dominated security:[12]

$$C_r \leq P_r + S - [K + D(1 + i)^{\delta}]/(1 + i), \tag{6}$$

$$P_r \leq C_r - S + [K + D(1 + i)^{\delta}]/(1 + i). \tag{7}$$

By parallel reasoning, the multiple dividend equivalents of inequalities (6) and (7) are:

$$C_r \leq P_r + S - [K + \textstyle\sum_{j=1}^{n} D_j(1 + i)^{\delta_j}]/(1 + i), \tag{8}$$

$$P_r \leq C_r - S + [K + \textstyle\sum_{j=1}^{n} D_j(1 + i)^{\delta_j}]/(1 + i), \tag{9}$$

where n is the number of dividend payments and $1 > \delta_k > \delta_{k+1}$.

Unlike an American payout protected call, Merton [4] has demonstrated that it may be optimal to prematurely exercise a registered call just before an ex-dividend date. In effect, he shows that a probability exists for premature exercise if it can not be demonstrated, for all finite stock prices, that the exercise value is less than the market value immediately after the stock goes ex-dividend. From an empirical standpoint, however, early exercise of the call is seldom optimal. A sufficient condition for no premature exercising, as developed by Merton [4] via a dynamic-programming-like technique, and modified by Roll [7] to encompass nonconcurrent ex-dividend and payment dates, is:

$$\textstyle\sum_{j=1}^{n} \alpha_j D_j/(1 + i)^{(1-\gamma_j)} < iK/(1 + i), \tag{10}$$

where $1 \geq \gamma_k > \gamma_{k+1}$.

This result also obtains directly from inequality (8) and the arbitrage process that ensures $C_r \geq \text{MAX}[0, S - K]$. From (8) and the arbitrage condition,

$$P_r + S - K + [iK - \textstyle\sum_{j=1}^{n} D_j(1 + i)^{\delta_j}]/(1 + i) > S - K \tag{11a}$$

if

$$P_r + [iK - \textstyle\sum_{j=1}^{n} D_j(1 + i)^{\delta_j}]/(1 + i) > 0. \tag{11b}$$

However, since P_r may approach zero, which is possible for large enough S,

$$\textstyle\sum_{j=1}^{n} D_j/(1 + i)^{(1-\delta_j)} < iK/(1 + i) \tag{12}$$

is sufficient (but not necessary) for (11a) to hold. Hence, the call will definitely be a non-dominated security if the present value of the dividends foregone is less than the present value of the return that could be earned from investing the exercise price at the risk-free rate, implying no premature exercise of the call. Note that inequalities (10) and (12) are identical if $\alpha_j = 1/(1 + i)^{(\gamma_j - \delta_j)}$, i.e., if α_j

[11] It is also necessary to assume that the firm's investment policy and capital structure remain fixed during the life of the option. Hence considerations such as liquidating dividends are avoided.

[12] That $S + P_r \geq [K + D(1 + i)^{\delta}]/(1 + i)$ and $C_r + [K + D(1 + i)^{\delta}]/(1 + i) \geq S$ follow from reasoning similar to that expressed in footnote 6.

Table II

The Interrelationships of Registered Options and Dividend Paying Stock

	Strategy	Cashflow at Time 0	Cashflow at Expiration $S_\tau^* \le K$	Cashflow at Expiration $S_\tau^* > K$
A	Buy call	$-C_r$	0	$S_\tau^* - K$
B	Buy stock	$-S$	$S_\tau^* + D(1+i)^\delta$	$S_\tau^* + D(1+i)^\delta$
	Buy put	$-P_r$	$K - S_\tau^*$	0
	Borrow	$[K + D(1+i)^\delta]/(1+i)$	$-K - D(1+i)^\delta$	$-K - D(1+i)^\delta$
	Total	$-P_r - S + \frac{[K + D(1+i)^\delta]}{(1+i)}$	0	$S_\tau^* - K$
C	Buy Put	$-P_r$	$K - S_\tau^*$	0
D	Short stock	S	$-S_\tau^* - D(1+i)^\delta$	$-S_\tau^* - D(1+i)^\delta$
	Buy call	$-C_r$	0	$S_\tau^* - K$
	Lend	$-[K + D(1+i)^\delta]/(1+i)$	$K + D(1+i)^\delta$	$K + D(1+i)^\delta$
	Total	$S - C_r - \frac{[K + D(1+i)^\delta]}{(1+i)}$	$K - S_\tau^*$	0

represents the discount factor applicable betwen the jth ex-dividend date and the jth dividend payment date, as it should under the perfect capital market assumptions previously stated.

Since the long hedger is the put owner, and the variables of either (10) or (12) are known at inception, the long hedger can determine at that time whether a rational call owner might ever prematurely exercise the call and terminate the hedge position.[13] If (10) holds at inception, inequality (8) is the upper boundary for a registered call.

The previous subsection detailed the sufficient condition for rational premature exercise of an American payout protected put. A similar condition also exists for registered puts. Since an American registered put can be exercised immediately, its value at inception must satisfy the arbitrage condition $P_r \ge \text{MAX}[0, K - S]$. From inequality (9), however,

$$C_r - S + [K + \textstyle\sum_{j=1}^{n} D_j(1+i)^{\delta_j}]/(1+i) < K - S \tag{13}$$

if

$$C_r < [iK - \textstyle\sum_{j=1}^{n} D_j(1+i)^{\delta_j}]/(1+i), \tag{14}$$

which is possible for small enough S. If condition (14) holds, the arbitrage condition has been violated, and it would be to the put holder's advantage to exercise the put immediately, since the exercise value is greater than the corresponding portfolio value.

Since C_r is not a constant value throughout the term to maturity, it is possible that condition (14) may fail to hold at inception, but may later hold when recalculated using an interim call price, risk-free rate, and interim number of

[13] If any portion of a hedge is terminated prematurely, it is logical to assume the hedger liquidates his remaining position immediately, since his riskless position has been destroyed.

remaining dividends. Thus in contrast to a call, possible rational premature exercise of a registered put may not be determinable at inception. And for this reason, inequality (9) will be the upper boundary for a registered put, only if condition (14) failed to hold at all interim points. On the other hand, since the sufficient condition for premature exercise of a registered put, on a stock with dividends to be paid during the option period, is more stringent than the one for a payout protected put, the probability is less, before the last ex-dividend rate, that it will be exercised early. As a final remark on premature exercise, comparison of conditions (12) and (14) indicates that if condition (12) fails to hold, the put should not be exercised at inception. In the next section, the relationships established in this subsection will be empirically tested.

III. Empirical Testing of Put-Call Parity

A. Previous Work

Past empirical work on put-call parity has been done by Stoll [10] and Gould and Galai [3]. Essentially Stoll uses regression models formed from equation (3), where $K = S$ and $i/(1+i)$ was approximately by an annualized i. The resulting models were

$$C = \Gamma_0 + \Gamma_1 iS + \Gamma_2 P + \eta, \tag{15a}$$

$$C/S = \Gamma_0' + \Gamma_1 i + \Gamma_2(P/S) + \epsilon. \tag{15b}$$

The data Stoll examined were the submissions by the Put and Call Dealers Association (PCDA) to the Securities and Exchange Commission. Each week the PCDA submitted nominal OTC put and call price quotations on 15 "Regular" companies, which remained unchanged from week to week, and 10 "New Business" companies, that varied weekly, and presumably represented those stocks with the greatest amount of trading during the week. In most of Stoll's regressions, the coefficients appeared significantly different from their theoretical values. In particular, the constant terms tended to be positive and significantly different from zero at the .05 level. In general, given the quality of the data, Stoll concluded that put-call parity theory could not be rejected, but that the "New Business" data, which represented a more active market, appeared to indicate that whenever relative option prices are high (above average) the difference between relative call and put prices is greater than when relative option prices are low (below average).

Gould and Galai, due to the Merton objection of Stoll's paper, conducted empirical tests only on the long hedge boundary portion of expression (5). When $K = S$, this boundary constraint can be expressed as

$$C - P_a - \frac{iS}{(1+i)} \leq 0. \tag{16}$$

Verbally, inequality (16) states that the present value of the gross profit from a conversion constructed from American payout protected options should be less than or equal to zero in an efficient options market. If this inequality should fail to hold to the extent that economic profits, i.e. profits in excess of transaction costs, could be realized, then the options market is inefficient and put-call parity is weak.

The major data available to Gould and Galai were the 10 "New Business" put and call price equations, supplied to them by Stoll. They also had actual transaction data, rather than price quotations, made available to them by Black and Scholes. For each long hedge constructed from the data, Gould and Galai calculated a value for the left-hand side (LHS) of inequality (16), assuming a round lot. Their results indicated that profitable conversions existed in virtually all stock price ranges if, and only if, the converter was a member firm. Gould and Galai claim that the reason this inefficiency exists is that calls are overpriced relative to puts, especially in the low stock price ranges. They also suggest that this is the reason for the statistically significant positive constant terms found by Stoll in his regressions.[14]

B. The Present Study-Methodology

The empirical work in this paper is based on inequalities (8) and (9). These inequalities can be expressed, respectively, as

$$(C_r - P_r - S)(1 + i) + K + \textstyle\sum_{j=1}^{n} D_j(1 + i)^{\delta_j} \leq 0, \tag{17}$$

$$(S + P_r - C_r)(1 + i) - K - \textstyle\sum_{j=1}^{n} D_j(1 + i)^{\delta_j} \leq 0. \tag{18}$$

Verbally inequality (17) (inequality (18)) states that the gross terminal profit from engaging in a long (short) hedge constructed from American registered options should be less than or equal to zero if the options are not prematurely exercised. Put-call parity theory would be rejected if many observed cases violated these inequalities in excess of transaction costs. Note, nevertheless, that (17) and (18) are the mirror images of one another. Hence, in situations where neither option would be exercised prematurely, a profitable position is always possible unless the LHS's of both inequalities are indentically equal to zero.

Recall from the last section, however, that at inception, it is only possible to determine whether the call might rationally be exercised prematurely, and not the put. In this regard, if sufficient condition (10) holds at inception, the profit calculated according to the LHS of inequality (17) can be deemed a certainty. If (10) fails to hold the profit is subject to risk. For a short hedge, condition (14) must fail to hold throughout the duration of the option period for the profit calculated according to the LHS of inequality (18) to be realized. Consequently, if (14) fails to hold at inception, the profit calculated at that time is only *conditional.*

An equal number of long and short hedges were constructed to conduct the empirical tests. It was next determined if sufficient conditions (10) and (14) held at inception. If condition (10) failed to hold, the applicable long hedge was

[14] Because Stoll and Gould and Galai were using OTC data, their models should have included a dividend term, since OTC options are only partially payout protected. For OTC options on stock that will pay a single dividend during the option period, and will not be prematurely exercised, the upper boundary constraints can be shown to be: $C_{otc} \leq P_{otc} + S - [K + D((1 + i)^{\delta} - 1)]/(1 + i)$, $P_{otc} \leq C_{otc} - S + [K + D((1 + i)^{\delta} - 1)]/(1 + i)$.

eliminated from further analysis. For the remaining long hedges, the LHS of inequality (17) was calculated, and the hedge was categorized as profitable or unprofitable. If condition (14) held at inception, the corresponding short hedge was eliminated from further analysis. The remaining short hedges were then categorized as conditionally profitable or unprofitable according to inequality (18).[15]

If the relationships developed are adequate statements of put-call parity, one would expect to find proportionately more violations of inequality (18) and in larger dollar amounts than of inequality (17), since there is risk in realizing the profit of a short hedge. One would also expect to find relatively few long hedges in which profit in excess of transaction costs could be earned with certainty, if the options markets are efficient.

C. The Data

The hedges were constructed for each of the fifteen companies having both puts and calls listed on the CBOE, the American and the Philadelphia Stock Exchanges. One day each month was selected for the twelve month period beginning in July 1977 and ending in June 1978. The put, call, and stock transactions data utilized in constructing a hedge were obtained from Francis Emory Fitch, Inc., which provides the size, price, and time (rounded to the minute) of each transaction on the stock and options exchanges. This data makes it possible to construct a nearly simultaneously priced hedge. The criterion of this study required that the put, call and underlying stock all had to trade within one minute of each other. Only one hedge was constructed per company on each date selected for a given expiration date and striking price, although in many instances it was possible to construct numerous hedges. In total, 606 long and short hedges were constructed with the total number of observations per company varying between 16 and 68. Maturities of the hedges varied from three to a maximum of 39 weeks.

As an estimate of the risk-free rate, the bid yield quotation on Treasury Bills was used. For each hedge constructed, the annualized bid yield was obtained for the T-bill issue that on the inception date of the hedge had a maturity that most closely matched the expiration of the options. This annualized yield was then adjusted downward to reflect the rate applicable to the maturity length of the hedge. The adjusted rate was then assumed to be constant throughout the term of the hedge.

The theoretical development in Section II.B. assumed future dividends were known with certainty. This is not the case in reality, although for the companies having both puts and calls traded on the registered exchanges, dividends have been more frequently increased than decreased. And, an unexpected dividend increase will increase the profit of a long hedge. Thus a suitable conservative estimate of each of the future quarterly dividends (a maximum of three quarters)

[15] In conducting the empirical tests, some observations had the nth dividend payment date later than the maturity date of the conversion. In those cases, the nth dividend was discounted at the appropriate risk-free rate to the expiration date of the options. The α_i's were approximated as unity; this is not a gross violation since a typical figure might be .9955.

was deemed to be the last quarterly dividend paid. If a stock typically paid an "extra" dividend in a particular quarter, the prior year's amount was included as part of the expected future quarterly dividend for the present period. The form of the put call parity tests can thus be viewed as ex ante in nature.

D. Results

Sixty-six of the 606 long hedges constructed were eliminated from further analysis because sufficient condition (10) failed to hold. Of the remaining 540 long hedges, 306 appeared unprofitable and 234 profitable ex ante. Fourteen of the 606 short hedges were eliminated from further analysis because condition (14) held at inception. Of the remaining 592 short hedges, 268 were unprofitable and 324 indicated conditional ex ante profit according to the LHS of inequality (18).

The fact that slightly more than forty percent of the long hedges appeared profitable but that more than half of the short hedges appeared conditionally profitable ex ante is consistent with the hypothesis that the converter bears risk in undertaking a short conversion but none in a long conversion. Table III presents an overview of these ex ante profitable hedges (per round lot) by stock price range, the sum of the terminal value of the dividends, and the number of weeks to expiration. The figures in Table III are based on zero transaction costs.

If $20. is the transactions cost for a member firm to obtain a position in a hedge,[16] it can be seen from Table III that only 147 of the 540 acceptable observations, or twenty-seven per cent, are profitable. On the basis of this result, the data appear consistent with put-call parity theory and options market efficiency. However, 62 of the 147 profitable long hedge observations, or 42 percent, exceeding the $20. level occurred in the first three months of put trading on the registered exchanges. Thus there appears to have been a slight decrease in any inefficiency that may have existed when put trading initially began. Only 38 long hedges, or seven percent, of the 540 provided an ex ante profit in excess of $60., which represents the minimum amount needed by a nonmember investor to construct a hedge.

Table III demonstrates that the largest profits for long hedges occurred in the low to middle stock price ranges. The median profits were largest in the $60.–$80. and the $20.–$40. ranges. These results are consistent with Gould and Galai's [5] findings, as discussed earlier in this section. For the short conversions, the opposite is true; the median profit was positively related to the stock price range. The fact that the profits from the short hedges, on average, tended to be greater than the profits from the long hedges is consistent with the hypothesis stated earlier.

The sum of the terminal value of the dividends (per round lot) appears to have only a slight impact on long hedge profitability. The proportion of long hedges with profits exceeding $20. was lowest in the $0. dividend class, but fairly consistent over the other dividend classes. This was not true for the short hedges. The proportion of short hedges in each dividend class with profits in excess of

[16] Based on conversions with staff members of the CBOE and the SEC, $20. is a reasonable and conservative etimate of round-trip transaction costs to market makers for constructing a hedge position consisting of multiple option contracts and round lots.

Table III

Summary of ExAnte Profitable Hedges

Factor	0 - $20.		$20. - $40.		$40.-$60.		$60. - $80		$80. — $100.		Over $100.		Total		
	LH[a]	SH[b]	LH	SH	LH	SH	LH	SH	LH	SH	LH	SH	LH +	SH =	TO[c]
A. *Stock Price Range*															
$0. - $20.	23	6	19	7	8	1	3	3	1	0	0	0	54	17	71
$20. - $40.	25	40	34	29	23	15	11	7	6	2	5	1	104	94	198
$40. - $60.	33	34	12	31	7	24	6	4	3	7	0	14	61	124	185
$60- $80.	1	2	0	6	3	3	2	5	0	4	0	10	6	30	36
$80.- $200.	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Over $200.	5	2	1	3	2	5	1	8	0	4	0	37	9	59	68
Total	87	84	66	76	43	48	23	37	10	17	5	62	234	324	558
B. *Sum of Terminal Value of Dividends*															
$0.	19	29	15	18	2	5	1	2	0	2	0	1	37	57	94
$0. - $20.	24	14	17	7	13	1	3	4	0	0	1	0	58	26	84
$20. -$40.	10	12	13	14	6	4	2	1	1	0	1	0	33	31	64
$40. - $60.	11	9	11	13	4	7	8	3	3	2	0	4	37	38	75
$60. - $80.	4	8	1	4	4	10	2	6	1	3	1	1	13	32	45
$80. - $100.	7	6	7	3	6	5	1	5	3	3	1	2	25	24	49
Over $100	12	6	2	17	8	16	6	16	2	7	1	54	31	116	147
Total	87	84	66	76	43	48	23	37	10	17	5	62	234	324	558
C. *Weeks to Expiration*															
1 - 5	6	10	3	8	0	1	1	1	0	2	0	1	10	23	33
6 - 10	11	16	9	13	3	4	0	1	0	0	0	0	23	34	57
11 - 15	12	9	11	14	2	10	1	3	1	2	0	8	27	46	73
16 - 20	10	12	10	9	3	7	3	8	2	3	2	5	30	44	74
21 - 25	20	20	15	13	12	16	8	8	1	5	0	19	56	81	137
26 - 30	11	4	8	10	8	2	6	9	3	4	2	8	38	37	75
Over 30	17	13	10	9	15	8	4	7	3	1	1	21	50	59	109
Total	87	84	66	76	43	48	23	37	10	17	5	62	234	324	558

[a] LH represents the number of profitable long hedges.
[b] SH represents the number of profitable short hedges.
[c] TO represents the total number of observations per class.

\$20. was highest in the over \$100. class, lowest in the \$0. to \$40. classes, and constant over the three remaining classes.

Table III also shows that with the exception of the hedges having expirations of 10 weeks or less, there appears to be no relationship between weeks to expiration and proportion of hedges having profits in excess of \$20. This is true for the long hedges as well as the short hedges.

A regression model was formed from inequality (8) to determine what was responsible for making a long hedge profitable. It is:

$$C_r/S = \beta_0 + \beta_1(P_r/S) + \beta_2(1 - (K/S)/(1 + i)) + \beta_3 \left(\sum_{j=1}^{n} D_j(1 + i)^{\delta_j}/S(1 + i)\right) + \epsilon. \quad (19)$$

The individual variables were scaled by the stock price to help eliminate any heteroscedastic disturbance that might otherwise occur in a cross-sectional regression.

If inequality (8) is a correctly formulated statement for describing the relative value of a call, the resulting constant term from the regression should be either insignificantly different from zero or significantly negative. If the constant term is significantly positive, the call can be viewed as overpriced. The β_1 and β_2 coefficients would not be expected to be significantly different from unity and the β_3 coefficient should not be significantly different from negative unity if the data is consistent with put-call parity theory, regardless of the value of the constant term.

Three separate regressions were performed: one included only the 234 profitable long hedges; a second the 306 unprofitable long hedges; and the last one utilized all 540 long hedges. The results, as presented in Table IV, for the 540 case regression show simultaneous put and call prices to be thoroughly consistent with put-call parity theory; neither the constant term nor any of the three coefficients were significantly different from their hypothesized values at the .05 significance level. The regression for the 234 profitable long hedges portrays different results. The constant term is significantly positive and the dividends coefficient, β_3, is significantly different from negative unity in the positive direction. These results indicate that it is an overpriced call premium that is the most important factor in making a long hedge profitable. Dividends, while contributing to profitability, do not alone account for this profitability. Had β_3 been significantly less than negative unity, it could be asserted that dividends were relatively important in making a long hedge profitable, but the results reject this.[17] The regression for the 306 unprofitable long hedges shows the constant term to be insignificantly different from zero. The β_3 coefficient however, was significantly less than negative unity. This proves consistent with the previous assertion that dividends do not have a major impact on determining profitability, since dividends had a larger relative weight in the unprofitable conversions. The put coefficient, β_1, was significantly less than unity in the unprofitable regression. The interpretation of

[17] It is important to bear in mind that dividends contribute to the profit of a long hedge, as can be seen from inequality (17), but that the dividends coefficient, β_3, in the regression has a hypothetical negative one value since the regression model was formulated from inequality (8). Thus the smaller the coefficient value in relation to negative one, the more impact dividends have on profitability, and vice-versa.

Table IV

Long Hedge Regression Results

Analysis Group	β_0	β_1	β_2	β_3	Dependent Variable	$\bar{R}^{2b}$	DW[c]
234 Profitable							
Coefficient	.007*	1.026	.996	−.755*		.951	1.733
t value[a]	(3.245)	(.995)	−(.228)	(3.813)			
Mean		.082	.002	.013	0.83		
Std. Deviation		.055	.087	.011	.046		
306 Unprofitable							
Coefficient	−.001	.917*	.946*	−1.158*			
t value[a]	−(.836)	−(4.811)	−(4.688)	−(3.396)		.963	1.481
Mean		.068	.017	.013	.062		
Std. Deviation		.049	.074	.010	.040		
540 total							
Coefficient	−.000	1.004	.976	−1.019			
t value[a]	−(.215)	(.151)	−(1.570)	−(.301)		.896	.725
Mean		.074	.010	.013	.071		
Std. Deviation		.052	.080	.011	.044		

[a] t statistics for constant terms are computed from zero and from unity and negative unity, respectively, for the coefficients. * denotes t statistic significance at the .05 level for $(n - 4)$ degrees of freedom.

[b] $\bar{R}^2$ denotes the coefficient of determination adjusted for degrees of freedom.

[c] DW denotes Durbin-Watson statistic; the .01 significance points for $n = 100$, $k^- = 3$ are: $d_L = 1.48$, $d_U = 1.60$.

this result is that when the put premium is overpriced, it detracts from the profit of a long hedge.

The β_2 coefficient variable is a stock price weighted value of $S - K/(1 + i)$, which can be viewed as a rough measure of how much a call is "in-the-money" or "out-of-the-money". From Table IV it can be seen that the mean price weighted "in-the-money" value is .002 for the profitable long hedges and .017 for the unprofitable. In addition it can be seen that the mean stock price weighted call-put spread $(C/S - P/S)$ is .001 and −.006 for the profitable and unprofitable groups, respectively. The data suggest that the less "in-the-money" or more "out-of-the-money" a call is, the more likely it is overvalued relative to the put premium, and the more likely it is that the long hedge will be profitable.

IV. Conclusions

The purpose of this study was to review the past theoretical and empirical work on put-call parity and develop a testable models applicable to registered options. The empirical results of the models tested are consistent with put-call parity theory and thus support this aspect of efficiency for the registered options markets. The small degree of inefficiency detected appears to be the result of overpriced calls.

In addition, the study has shown that many reverse conversions prove conditionally profitable at inception and in large dollar amounts. Whether this profit will be realized is uncertain and dependent upon the premature excercise of the put. Further research should be conducted to determine the degree to which premature exercise occurs for the profitable short hedges.

It should be pointed out that the $20. profitability level may not be adequate compensation, given that the arbitrageur now operates in markets where the options are not dividend payout protected and option prices and stock prices change continuously. Further research should test the profitability of these hedges in an ex post manner by constructing the hedges using actual prices minutes after initially identifying the hedge as adequately profitable and using actual dividends over the life of the hedge in place of projected dividends. This ex post analysis may provide even stronger support for put-call parity and market efficiency.

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