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Journal of Finance, Volume 34, Issue 4 (Sep., 1979), 895-914.

THE JOURNAL OF FINANCE • VOL. XXXIV, NO. 4 • SEPTEMBER 1979

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RICHARD J. RENDLEMAN, JR. and CHRISTOPHER E. CARABINI*

On January 6, 1976, the International Monetary Market of the Chicago Mercantile Exchange began trading the Treasury bill futures contract. Compared with other financial instruments, the Treasury bill futures contract is relatively simple to price. As we will show, the equilibrium price of a contract can be determined or closely approximated from observable spot Treasury bill prices. Given the ease of pricing the contract, one would expect the market for Treasury bill futures to be highly efficient. The purpose of this paper is to test the efficiency of the Treasury bill futures market.

The specifications of the contract call for delivery of a $1,000,000 par value, 90-day U.S. Treasury bill, although 91 or 92 day bills are substitutable. The delivery months are March, June, September, and December, and eight contract maturities are currently traded. Contracts are deliverable on the second day following the Federal Reserve 3-month (13 week) Treasury bill auction of the third week of the delivery month. This generally falls on the third Thursday of the month.

Recently, several studies (Branch [2], Capozza and Cornell [3], Emery and Scott [5], Lang and Rasche [6], Oldfield [7], Poole [8], and Puglisi [9]) of the Treasury bill futures market have attempted either to test the efficiency of the market or to test the empirical validity of the Expectations Hypothesis of the term structure of interest rates. Both types of tests focused on the existence of arbitrage opportunities between the futures and spot markets. Although the non-existence of arbitrage opportunities does not prove the Expectations Hypothesis, the studies that intended to test the Hypothesis can be viewed as indirect tests of market efficiency.[1]

These studies find conflicting evidence regarding the efficiency of the futures market. These conflicting results can be partially explained by differences in sample sizes and sample periods. However, it is possible that the same data may not have been employed in a consistent manner in all studies. For example, only in the Branch, Poole, and Lang and Rasche papers is there any recognition of the fact that the delivery vehicle for contracts, other than the nearby contract, has never existed. Therefore, it is not clear how this problem was resolved in the development of the data to be tested. In addition, several studies recognize the

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[1] See Cox, Ingersoll and Ross [4] for an excellent description of the confusion surrounding the Expectations Hypothesis and other theories of the term structure of interest rates.

existence of transaction costs, either objectively or subjectively, but in some cases the treatment of transaction costs is inconsistent.

Emery and Scott, Poole, and Oldfield found that differences between futures prices and forward prices implied in spot bills were not of the magnitude to permit profitable arbitrage. Oldfield's tests showed a tendency for the market to become more efficient during the first year of trading. Poole focused only on the futures contract nearest to maturity.

Using weekly data for the first eighteen months of trading, Capozza and Cornell concluded that the nearest term contract was priced efficiently. The longer term contracts tended to be under-priced and the extent of under-pricing was directly related to the time remaining until the futures matured. However, none of these discrepancies could have been directly arbitraged due to the cost of shorting the spot bill necessary to establish the appropriate position.

Branch, Lang and Rasche, and Puglisi found the futures market to be inefficient. Branch's sample of only eighteen dates casts considerable doubt on any general conclusions about efficiency that can be inferred from his study. Much of the evidence in the Lang and Rasche study in support of inefficiency is provided by futures contracts with a year or more to maturity. The delivery vehicles for these contracts do not exist; therefore, Lang and Rasche used the yields of coupon securities to compute theoretical futures yields. Given the well-known problems associated with yield to maturity as a return measure which result from coupon and tax effects, it is unlikely that one can infer theoretical futures prices from the yields of such securities. As shown later, some of the empirical results obtained by Puglisi contrast sharply with the results of the present study. The conflicting findings of the above studies suggest that the question of market efficiency remains an unresolved issue.

I. Pricing the Futures Contract

The Equilibrium Futures Price

The equilibrium price of the futures contract can be determined on the basis of arbitrage relationships between the futures contract and spot bills. If the futures and spot markets are in equilibrium, there can be no pure arbitrage or quasi arbitrage opportunities available between the two markets.[2]

Consider the following graphical representation of the time dimensions of a typical Treasury bill futures contract.

Time 0 ————————	m ————————	n
Current Date	Maturity Date of Futures Contract	Maturity Date of Delivery Vehicle for Futures Contract

[2] Pure arbitrage refers to shorting a security or portfolio to fund a position in an economically equivalent security or portfolio at a lower price. Quasi arbitrage refers to selling securities from an existing portfolio to fund an economically equivalent position at a lower price.

The futures contract allows the investor to contract to either buy or sell a Treasury bill that will mature at time n for a fixed commitment price at time m.

Let P_m and P_n represent the spot prices per \$100 of par value for T-bills maturing at time m and n, respectively. In addition, let FP represent the futures price per \$100 of par. Consider a situation in which the bill maturing at time n is purchased for P_n at the present time and its time m selling price is locked in at FP through a futures contract. If this transaction offers a different return than could be obtained by simply purchasing a bill for P_m(FP/100) with a time m maturity value of FP, market pressures should eventually bring the returns into parity. Similarly, it might be possible to purchase a bill for P_m(FP/100) with a time m maturity value of FP and use the maturity proceeds of the bill to fund a position in the futures contract that would provide a higher return than could be obtained by paying P_n for the bill maturing at time n. Again, one would expect market pressures to close this gap until the prices of the two equivalent portfolios were the same. In the absence of transaction costs, both of these situations imply that equilibrium will be attained in the futures market when

$$P_m\,(\mathrm{FP}/100) = P_n \tag{1A}$$

or

$$\mathrm{FP} = 100\,P_n/P_m, \tag{2A}$$

where FP is the theoretical no-arbitrage futures price.[3]

[3] This derivation treats the futures contract as if it is a forward contract. Futures contracts differ from forward contracts, however, in that day-to-day changes in the futures prices are either debited of credited to the customer's account with any deficits having to be replenished with cash. Thus, to be technically correct, any futures pricing model should take these day-to-day changes into account.

Cox, Ingersoll and Ross (4) have shown that (2A) will not hold if interest rates are uncertain. They develop a closed form solution for pricing unit discount bonds (Treasury bills) and futures contracts under the assumption that the instantaneous interest rate (which might be viewed as the Federal Funds rate in practice) follows a mean reverting square root diffusion process. The inputs to the unit discount bond model are the current instantaneous interest rate, the natural rate, the variance of percentage changes in the interest rate, the mean reverting diffusion process speed of adjustment coefficient, the covariance of changes in interest rates with percentage changes in optimally invested wealth, and the time to maturity of the bond. The inputs to the futures pricing model are the same except that one must specify the time to maturity for both the futures contract and the bond that serves as the contract's delivery vehicle.

Using a variety of inputs, we have computed futures prices using the Cox, Ingersoll and Ross model. For input parameters that give rise to "reasonable" unit discount bond prices, the difference between theoretical forward and futures IMM Index values (which we define later) is generally less than 3–4 basis points for contracts with 270 days to maturity. The difference is generally less for shorter maturities. An example of a "reasonable" unit discount bond price would be \$.92 per \$1.00 of par for a one year bond if the instantaneous (Federal Funds) rate were 6%. We would not consider a \$.75 price per \$1.00 of par to be reasonable, however, given a 6% instantaneous rate. At the "unreasonable" prices, we do find wide discrepancies.

Using numerical methods, we have also computed futures and forward prices using an alternative model that assumes that the Expectations Hypothesis of the term structure of interest rates holds and that the instantaneous interest rate follows a lognormal distribution. The inputs to this model are the current instantaneous rate, the expected drift and variance of the rate, and the time parameters associated with the maturity of the various securities. Using a wide range of input parameters, we were unable to find any differences between futures and forward prices that we could view as significant (all differences were generally less than one basis point). Based on these calculations, we feel that it is unlikely that our tests, which price the futures contract as if it was a forward contract, are significantly biased.

Even if the actual and theoretical futures prices are not the same, the transaction costs involved with using futures to improve a portfolio's yield may eliminate much of the potential gain. Prior to negotiated rates,[4] round-trip commissions on the futures contract were \$60.00 or \$.006 per \$100 of par. Unlike commissions on stocks, these brokerage fees are paid when the position in the futures contract is reversed. In addition to this cost, the buying and selling prices of Treasury bills are not the same due to the dealer's bid-asked spread. These transaction costs imply that a range of futures prices will exist over which arbitrage between the futures and spot markets will not be possible.

To determine the lower bound of this range, consider the situation in which the time m bill and the futures contract are purchased as a substitute for the bill maturing at time n. With transaction costs, the bill that matures at time m must be purchased to fund both the futures contract and the commission on the contract. In addition, the bill must be purchased at the dealer's asking price.

Let P_m^A represent the asking price of the bill maturing at time m per \$100 of par. The amount paid to fund the long position in the bill maturing at time m for FP + \$.006 would be $P_m^A\left(\frac{FP + .006}{100}\right)$. This long position in the time m bill in conjunction with the long position in the futures contract will ensure a return of \$100 at time n. If the dealer's bid price, P_n^B, for the bill maturing at time n for \$100 is greater than $P_m^A\left(\frac{\text{FP} + .006}{100}\right)$, arbitrage would be possible. Thus, in an efficient market, one would expect the futures price to be set so that this type of arbitrage opportunity would not be present,

or
$$P_m^A\left(\frac{\text{FP} + .006}{100}\right) \geq P_n^B. \tag{1B}$$

This implies that the equilibrium futures price will meet the following condition:

$$\text{FP} \geq 100\, P_n^B/P_m^A - .006. \tag{2B}$$

In a similar manner, it can be shown that the upper bound of the equilibrium futures price is given by:

$$\text{FP} \leq 100\, P_n^A/P_m^B + .006. \tag{2C}$$

Determining the Equilibrium Value of the IMM Index

The International Monetary Market of the Chicgo Mercantile Exchange has adopted the IMM Index for pricing the Treasury bill futures contract. This index is quite similar to the bankers' discount method of pricing Treasury bills. The index value is simply the difference between the par value of the bill (on a \$100 basis) and its annualized discount from par, assuming 360 days to the year. For example, an IMM Index value of \$92 corresponds to an actual contract price of \$100 − \$(100–92) (90/360) = \$98 for a futures contract on a 90-day bill. The futures contract is actually written in terms of \$1,000,000 or par rather than \$100.

[4] Fixed rates were phased out completely in March, 1978, the final month of our sample period.

Therefore, a one basis point move in the index represents a $\$.01 \left[\frac{\$1{,}000{,}000}{\$100}\right] \times \frac{90}{360} = \$25$ gain or loss in the actual price of the contract.[5]

Subtracting the futures prices in (2B) and (2C) from \$100 and multiplying by 360/91, the annualized discounts from par of the futures prices can be determined. These discounts can then be subtracted from \$100 to obtain the following range of equilibrium IMM Index values:

$$100 - 395.6\,[1 - P_n^B/P_m^A] - .0237 \leq \text{IMM} \leq 100 - 395.6\,[1 - P_n^A/P_m^B] + .0237. \quad (3)$$

If P_m and P_n are expanded in terms of the bankers' discount pricing convention, and one basis point is subtracted from the lower bound of the IMM Index and added to the upper bound to allow for the market maker's spread in the futures contract, the equilibrium range for the index can be restated in terms of the percentage discount yields (Y_m and Y_n) and days to maturity (D_m and D_n) of the two bills.[6]

$$100 - 395.6\left[\frac{Y_n^B D_n/360 - Y_m^A D_m/360}{100 - Y_m^A D_m/360}\right] - .0337 \leq \text{IMM} \leq 100 - 395.6\left[\frac{Y_n^A D_n/360 - Y_m^B D_m/360}{100 - Y_m^B D_m/360}\right] + .0337 \quad (4)$$

In the absence of all transaction costs, the equilibrium IMM Index value reduces to:

$$\text{IMM} = 100 - 395.6\left[\frac{Y_n D_n/360 - Y_m D_m/360}{100 - Y_m D_m/360}\right], \quad (5)$$

which is the IMM Index value corresponding to the futures price given in (2A).

II. Analysis

If the spot and futures markets are in equilibrium, observed IMM Index values should fall within the bounds of (4). Otherwise, it would be possible to improve the yield of an existing portfolio by trading futures. As we explain later, one might also expect to observe a tendency for prices to converge to the no-transaction cost price of (5), even in the presence of transaction costs. Thus, to test the

[5] If a position in a futures contract is reversed prior to maturity, the investor's gain or loss is \$25 per basis point difference in the IMM Index values at which the contract is bought and sold. Therefore, prior to maturity, settlement in the contract is based on a 90-day maturity, even if 90 days will not be the maturity of the delivery vehicle. During the period of this study, a 91-day bill has always served as the delivery vehicle for the futures contract. If one either makes or takes delivery in a 91-day bill, the settlement price is adjusted to reflect the extra day. Thus, each basis point move in the IMM Index represents a $\$25 \times 91/90$ or \$25.28 gain or loss in a contract in which delivery takes place. In the remaining analysis, we compute all IMM Index values on a 91 day basis.

[6] According to the research department of the Chicago Mercantile Exchange, the market maker's bid-asked spread is typically two basis points.

efficiency of these markets, we examine the relationship between actual IMM Index values and theoretical values given by (4) and (5).

Description of the Data

Price data for the futures contracts were provided by the Chicago Mercantile Exchange. These data contained the price and time of trade of every transaction from the commencement of trading on January 6, 1976, through March 31, 1978. From this data source, the prices of the last trades and trading times for each contract included in our analysis were collected on a daily basis for this study. Since the maximum maturity of a Treasury bill is one year, a more complicated pricing model would have to be developed to price futures beyond the first three maturities. Therefore, we limited our analysis to the first three contract months trading at any given date. For those days in which a trade did not take place in a given contract, the contract in question was omitted from the analysis for that day.

Bid and asked bankers' discount yields of Treasury bills maturing on the maturity date of each contract as well as the bill maturing 91 days thereafter were collected on a daily basis from the Federal Reserve Bank of New York's "Composite Closing Quotations for U.S. Government Securities." In many cases, the actual bill that would serve as the delivery vehicle for a given contract or the bill that would mature on the contract's maturity date did not exist.[7] For these contracts, we recorded the bid and asked quotes of the Treasury bills whose maturity dates most closely surrounded the date in question. In a limited number of cases, the longest term bill matured prior to the maturity date of the delivery vehicle for the third contract month. In these cases, we omitted the contract from the analysis for the day. Finally, only those days in which both the futures and Treasury bill markets were open were included in the analysis. The resulting data contained 1606 observations.

Measurement Problems

To compute the range of no-arbitrage IMM Index values, one must know the bid and asked bankers' discount yields of Treasury bills maturing on the futures' maturity dates and 91 days therafter. In many cases, these Treasury bills had not been auctioned, making it necessary to estimate what the yields would have been had these maturities been available.

Our method of estimating such yields is based on the assumption that the yield curve can be well approximated by a linear function between any two dates that are "reasonably" close.[8] Let Y^- and Y^+ denote the bankers' discount yields of

[7] As a general rule, the actual delivery vehicle for each contract is auctioned 93 days prior to the contract's maturity date. Except for the period 1/06/76 through 3/18/76, the maturity date of the delivery vehicle for any contract month corresponded to the maturity date of the futures contract for the next contract month.

[8] Technically, the "yield curve" describes the relationship between annual rates of return and maturity. We actually assume a linear relationship between bankers' discount yields and maturity. Although the bankers' discount yield approximates the true rate of return, these two measures are not the same. It is easy to show, however, that the yield estimates that we obtain would not be significantly different had we assumed a linear relationship between rates of return, rather than discounts.

bills maturing just before and just after the maturity date of the bill for which the yield estimate is being made. Similarly, let D^*, D^- and D^+ represent the days to maturity of the bill in question and the days to maturity of the bills maturing just prior and just after this bill, respectively. A linear yield curve between days D^- and D^+ implies the following yield estimate:

$$Y^* = \frac{Y^-(D^+ - D^*) + Y^+(D^* - D^-)}{D^+ - D^-}. \tag{6}$$

When estimating either bid or asked yields to determine the range of equilibrium IMM Index values, the appropriate bid or asked yields are input into (6). When estimating yields to determine the equilibrium IMM Index without transaction costs, we use the means of the bid and asked yields as inputs into (6).

Comparison of Actual and Theoretical IMM Index Values (Transaction Costs not Considered)

The existence of transaction costs gives rise to a range of values for the IMM Index over which arbitrage opportunities are not available. Assuming that buyers and sellers of futures contracts face transaction costs of the same order of magnitude, neither group would be expected to dominate the market. Therefore, one should not expect a preponderance of trades to take place at either the high or low end of the no-arbitrage price range. Instead, the typical or average trade is likely to take place near the no-transaction cost value of the index, which will also approximate the mid-point of the no-arbitrage range. If the actual prices deviate from the no-arbitrage values, market inefficiency would not be implied due to the existence of trading costs. However, an average deviation of zero between the actual and theoretical IMM Index values of equation (5) would at least suggest a long-run tendency toward equilibrium.

In Table I, we present summary statistics for the difference between actual and theoretical index values. A positive difference indicates that the futures is over-priced. Summary statistics are cross-tabulated for each of the three contract months over each of the three nine-month periods of the sample. In addition, summary statistics are shown for the entire sample.

In Table I, the term "first contract month" refers to the futures contract with the least time to maturity at any given date. These contracts will always have 1–91 days until maturity. The second and third contract months refer to those contracts which mature on the two subsequent maturity dates.

These statistics suggest that futures for the nearest contract month have tended to be over-priced, while the longer-term futures have been under-priced. However, average deviations of actual IMM values from their no-transaction cost theoretical values do not appear to be consistent over time. For example, during the first eighteen months of the same period, the average deviation for the first contract month was approximately 6 basis points. The average difference increased to 15 basis points, however, during the last nine month period of the sample. During the second nine months, the second contract was under-priced by an average of 4 basis points, but during the next nine months, the relationship reversed; the contract was over-priced on average by approximately 14 basis points. Although the average deviation for the longest term contract was consist-

Table I

Summary Statistics for Basis Points Differential Between Actual and No-Transaction Cost Values of IMM Index

Trading Period		1st Contract Month	2nd Contract Month	3rd Contract Month	All Contracts
1st 9 Months	μ	5.599	.322	−26.788	−6.260
1/06/76–9/30/76	$\|\mu\|$	6.670	17.468	35.219	19.245
	σ	7.113	21.393	28.811	25.008
	σ_ϵ	4.940	8.004	8.945	N/A
	ϕ_1, ϕ_2	.633, .124	.752, .179	.855, .083	N/A
	N	187	187	168	542
	t	3.766[a]	.038	−2.407[a]	N/A
	$Q(21, N)$	19.746	23.911	40.328[b]	N/A
2nd 9 Months	μ	6.472	−4.349	−30.884	−8.700
10/1/76–6/30/77	$\|\mu\|$	8.089	11.344	32.545	16.688
	σ	7.880	14.055	24.857	22.689
	σ_ϵ	4.693	8.119	10.627	N/A
	ϕ_1, ϕ_2	.709, .108	.743, .087	.883, .017	N/A
	N	185	186	163	534
	t	3.433[a]	−1.236	−.242	N/A
	$Q(21, N)$	16.494	30.216	21.266	N/A
3rd 9 Months	μ	15.421	13.811	−10.140	7.236
7/1/77–3/31/78	$\|\mu\|$	15.681	17.691	15.471	16.324
	σ	11.182	15.943	17.200	18.694
	σ_ϵ	5.745	7.443	5.940	N/A
	ϕ_1, ϕ_2	.716, .163	.665, .236	.939, −.002	N/A
	N	186	186	158	530
	t	4.430[a]	2.505[a]	−1.430	N/A
	$Q(21, N)$	28.999	17.132	21.467	N/A
Entire sample period	μ	9.162	3.256	−22.774	−2.670
	$\|\mu\|$	10.144	15.504	27.947	17.431
	σ	9.934	19.016	25.759	23.362
	σ_ϵ	5.159	7.887	8.794	N/A
	ϕ_1, ϕ_2	.715, .160	.743, .176	.879, .059	N/A
	N	558	559	489	1606
	t	5.244[a]	.701	3.528[a]	N/A
	$Q(21, N)$	17.957	26.060	35.312[b]	N/A

Legend:

μ = sample mean

$|\mu|$ = sample mean of absolute value

σ = sample standard deviation

σ_ϵ = standard error of estimate of second order autoregressive process

ϕ_1, ϕ_2 = first and second order autocorrelation coefficients

N = sample size

$t = \mu(1 - \phi_1 - \phi_2)/(\sigma_\epsilon/\sqrt{N})$

$Q(21, N)$ = Box-Pierce Q statistic using 21 residual autocorrelations with sample size N.

[a] Significantly different from zero at 5% level.

[b] Null hypothesis that residuals follow a white noise process is rejected at the 5% level.

ently negative, the average deviation varied by approximately 21 basis points from the second nine months to the third. Four of nine trading period-contract month cells of Table I show an average deviation that is not significantly different from zero.[9] The relatively low t values in the remaining cells together with an average basis point differential of only −2.6 for the entire sample suggests that one could not expect to have made large excess profits in the Treasury bill futures market, even in the absence of transaction costs.

An average basis point differential of zero does not necessarily imply market efficiency, since deviations of opposite sign will tend to offset when a simple average is taken. Therefore, in Table I, the mean of the absolute value of the basis point differential is also presented.

The average absolute differential over all contracts in the entire sample is 17.431 basis points. The means of the differentials in the first and third contract are roughly equivalent to the means of the absolute values of the differentials. This implies that the first contract was generally over-priced and the third contract was generally under-priced over the sample period. Except for the third nine month period, the average absolute differential increased with contract maturity.

Although the average absolute basis point differential has declined by approximately three basis points from the first nine months to the third, there does not appear to be significant evidence to suggest that the Treasury bill futures market has become more efficient. A reduction in the mean of the absolute basis point differential in the third contract from 35.219 to 15.471 from the first to the third nine months suggests that the contract has become more efficiently priced over time. In contrast, the mean of the absolute basis point differential in the first contract month increased from 6.670 to 15.681, indicating a tendency to become less efficiently priced.

The standard deviations of the basis point differential increased monotonically across contract maturities, indicating less predictability for the prices of the longer term contracts. It is interesting to note that the standard deviation across

[9] t statistics were adjusted to reflect autocorrelated basis point differentials. Let Δ_τ represent the differential observed at time τ. The following autoregressive model was fit to the data presented in both Tables 1 and 2:

$$\Delta_\tau = \delta + \phi_1\Delta_{\tau-1} + \phi_2\Delta_{\tau-2} + \epsilon_\tau.$$

The coefficients ϕ_1 and ϕ_2, and the standard error of estimate, σ_ϵ, are reported in the tables. In addition, the Box-Pierce (1970) Q statistic is presented. This statistic, which follows a chi-square distribution, can be used to test the residuals for significant departures from white noise. Except for a few cases which are noted in the tables, one cannot reject the hypothesis of zero autocorrelation in the residuals at the 5% level.

Letting μ and N represent the sample mean and sample size, respectively, the t statistic for testing the hypothesis of a zero mean basis point differential is:

$$t = \frac{\mu(1 - \phi_1 - \phi_2)}{(\sigma_\epsilon/\sqrt{N})}$$

With zero first and second order autocorrelation (i.e., $\phi_1 = \phi_2 = 0$), the standard error of estimate equals the estimate of the population standard deviation, and we obtain the usual t statistic.

all maturities has decreased over time. This suggests that prices have become more predictable, but not necessarily more efficient.

In Figure 1, we present the basis point differential plotted against the number of days remaining on the contract. In this figure, as well as Figures 2 and 3, ten percent of the data points are plotted on a random basis.[10]

The same types of relationships discussed above can be seen in Figure 1. In addition, we can see that very few contracts have been under-priced during the last 120 days to maturity. Only during the last two weeks of the contract does there appear to be a convergence to the equilibrium price. The fact that the longer term contracts have been generally under-priced while the shorter-term contracts have been over-priced for all subperiods, suggests that one could expect to earn excess returns beyond the original basis point differential by purchasing a long-term contract in an arbitrage arrangement and reversing the position 30 to 120 days prior to the contract's maturity.

The data presented thus far, as well as results obtained by Branch and Capozza and Cornell, suggest that futures contracts with shorter maturities appear to be more efficiently priced. These results can be misleading, however, if one does not recognize the fact that the arbitrage profits in the shorter term contracts can be earned in a shorter time period than those in the longer term contracts. With frequent trading in near term contracts, it might be possible to earn a higher return in the long run than could be earned in a long-term contract, even though the basis point differential in the longer term contract is higher. To adjust the basis point differential to reflect the time period over which the arbitrage profit is earned, the differential can be multiplied by $(91/360) \times (365/D_m)$. The first term converts the differential from IMM Index units to dollar units. The second term annualizes the return by multiplying the dollar return by the number of times per year it can be earned. In this adjustment it is assumed that the futures and spot markets will converge to equilibrium on the maturity date of the contract and that similar investment opportunities will be available in the future.

Table II presents summary statistics for the basis point differential on an annualized basis. Figure 2 presents a graphical representation of the annualized basis point differential and days to maturity.

It should be noted that these data contrast sharply with those of Puglisi. Puglisi found an average of 127 basis points difference between annualized yields in the spot market and those of equivalent positions in the September 1976 futures contract. In contrast, we find that no single data point in our sample provides a basis point differential of this magnitude. Moreover, in three of nine contract month-subperiod cells, the mean annualized basis point differential is not significantly different from zero.

In contrast to the non-annualized data, these exhibits illustrate that the arbitrage return potential has been the highest in the shortest term contracts. The dispersion in annualized arbitrage profit as measured by σ and σ_ϵ has been highest in the nearest term contracts and has tended to increase over time. The average absolute basis point differential has also increased in the shortest term contracts through time. On the other hand, the absolute deviation has decreased

[10] Separate graphs of each subperiod showing all data points will be furnished by the authors upon request.

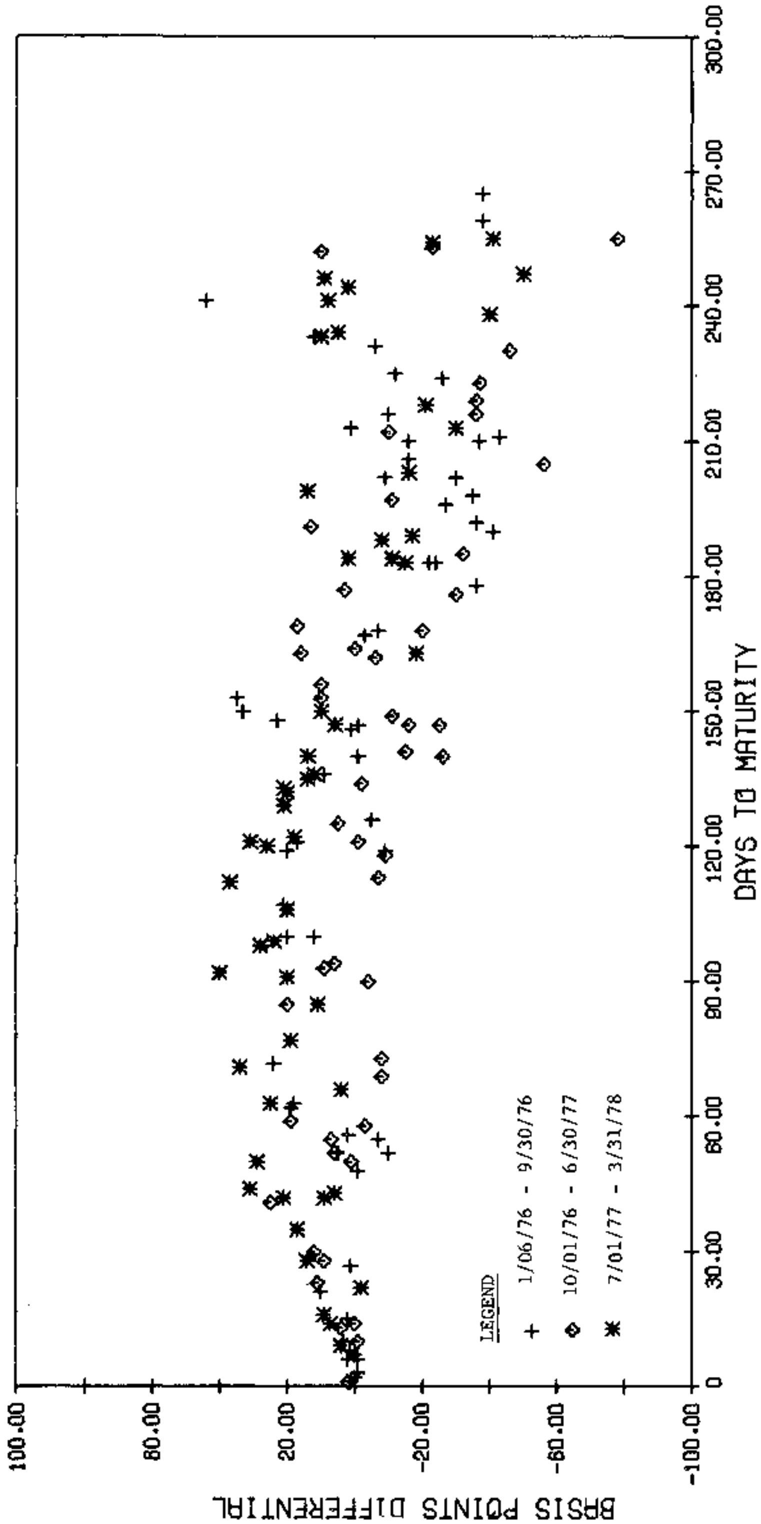

Figure 1. Non-annualized basis points differential vs. days to maturity

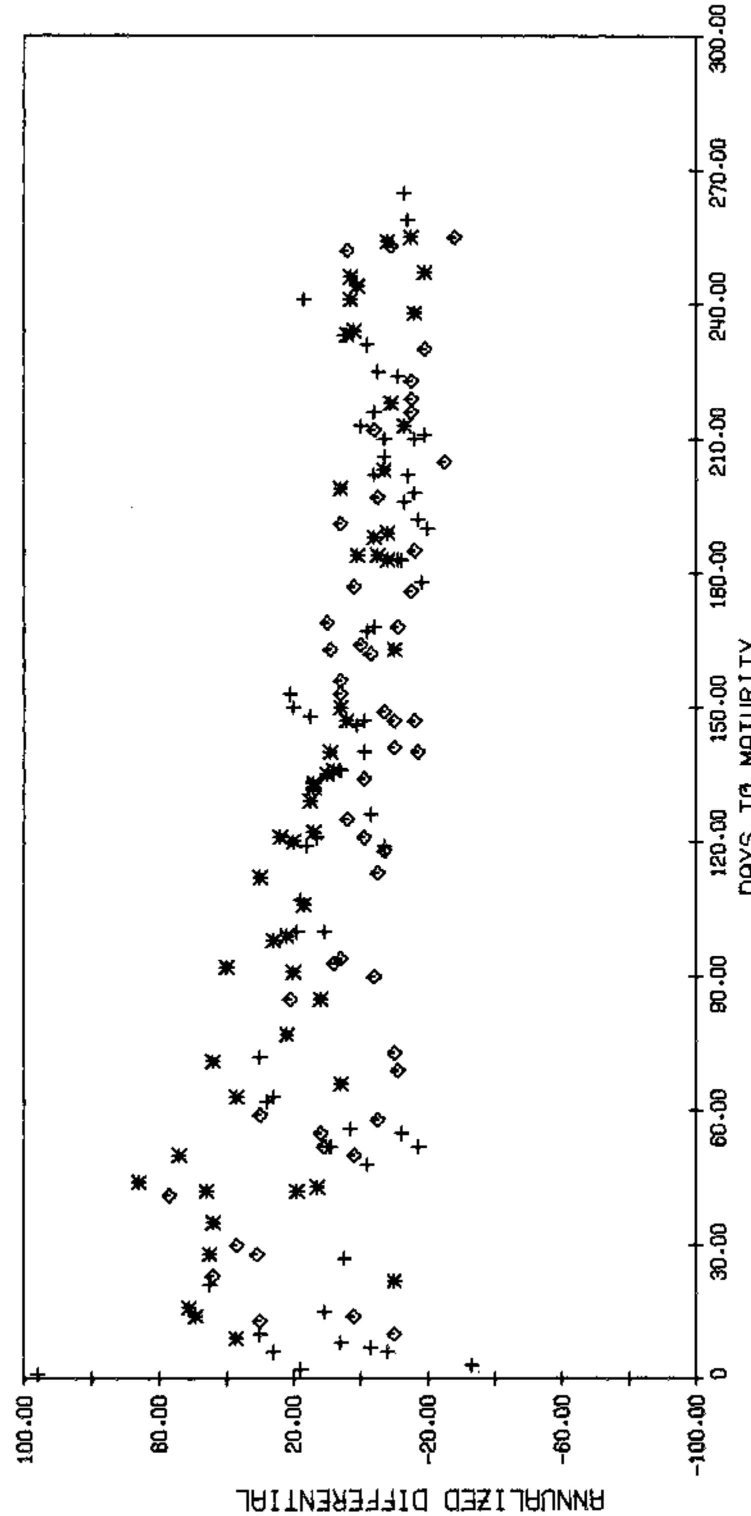

Figure 2. Annualized basis points differential vs. days to maturity

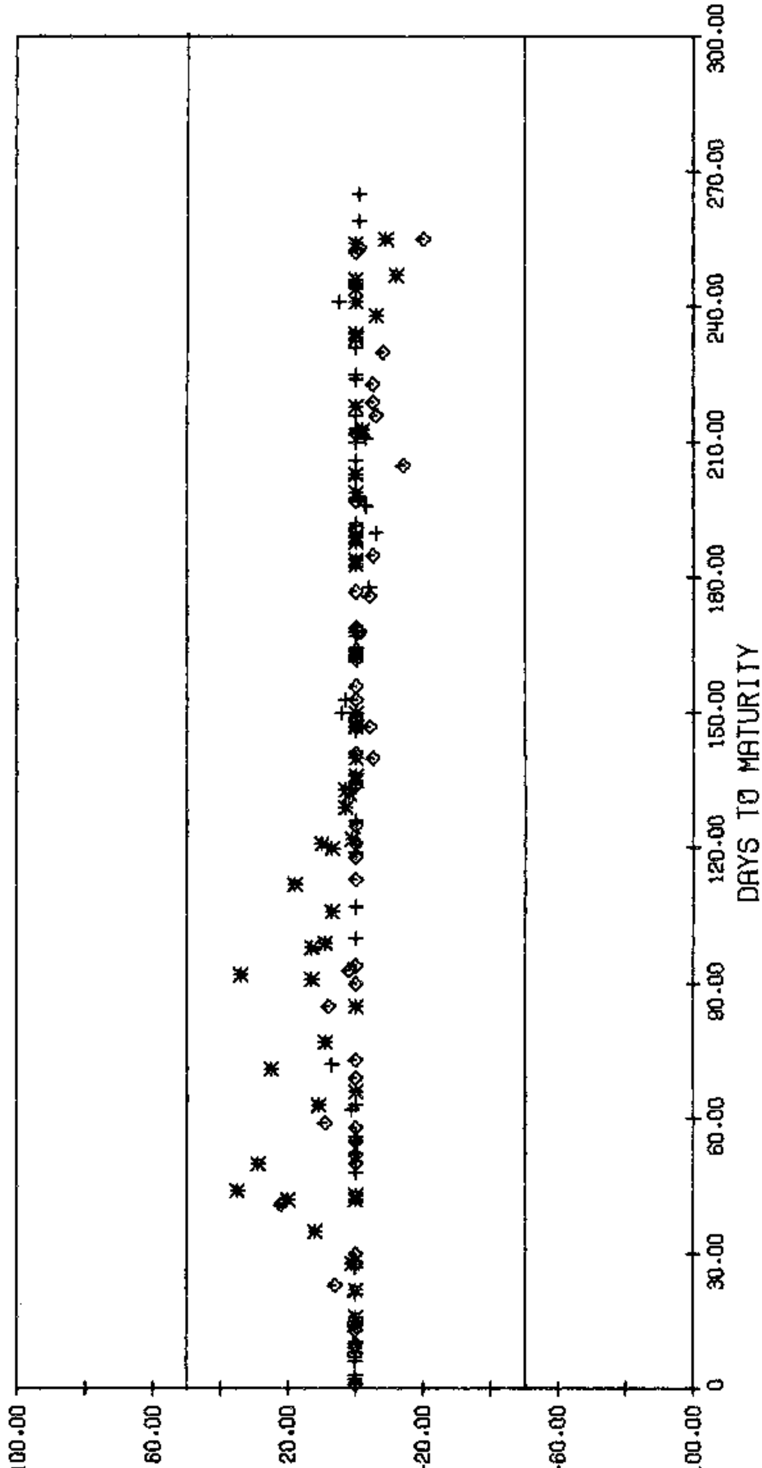

Figure 3. Annualized basis points differential vs. days to maturity considering transaction costs

Table II

Summary Statistics for Annualized Basis Points Differential Between Actual and No-Transaction Cost Values of IMM Index

Trading Period		1st Contract Month	2nd Contract Month	3rd Contract Month	All Contracts
1st 9 Months	μ	14.406	1.195	−10.991	1.976
1/06/76–9/30/76	$\|\mu\|$	17.971	11.497	14.135	14.548
	σ	23.370	13.651	11.023	19.879
	σ_ϵ	21.661	5.350	3.665	N/A
	ϕ_1, ϕ_2	.292, .184	.739, .189	.836, .095	N/A
	N	187	187	168	542
	t	4.766[a]	.220	−2.682[a]	N/A
	$Q(21, N)$	15.560	21.365	40.655[b]	N/A
2nd 9 Months	μ	17.694	−2.314	−12.429	1.530
10/1/76–6/30/77	$\|\mu\|$	21.080	7.563	13.182	13.961
	σ	26.144	9.218	9.827	21.219
	σ_ϵ	23.132	5.507	4.084	N/A
	ϕ_1, ϕ_2	.400, .143	.670, .125	.878, .028	N/A
	N	185	186	163	534
	t	4.755[a]	−1.152	−3.652[a]	N/A
	$Q(21, N)$	14.587	28.208	23.673	N/A
3rd 9 Months	μ	31.651	11.409	−3.940	13.937
7/1/77–3/31/78	$\|\mu\|$	35.671	13.537	6.273	19.140
	σ	28.096	12.635	6.854	23.534
	σ_ϵ	24.871	5.615	2.504	N/A
	ϕ_1, ϕ_2	.197, .378	.766, .140	.924, .008	N/A
	N	186	186	158	530
	t	7.376[a]	2.605[a]	−1.345	N/A
	$Q(21, N)$	25.577	21.427	21.346	N/A
Entire sample period	μ	21.244	3.426	−9.192	5.775
	$\|\mu\|$	24.902	10.867	11.277	15.868
	σ	26.952	13.308	10.114	22.319
	σ_ϵ	23.460	5.498	3.516	N/A
	ϕ_1, ϕ_2	.308, .280	.755, .167	.863, .074	N/A
	N	558	559	489	1606
	t	8.813	1.149	−3.642[a]	N/A
	$Q(21, N)$	25.031	24.072	35.341[b]	N/A

Legend:

μ = sample mean

$|\mu|$ = sample mean of absolute value

σ = sample standard deviation

σ_ϵ = standard error of estimate of second order autoregressive process

ϕ_1, ϕ_2 = first and second order autocorrelation coefficients

N = sample size

$t = \mu(1 - \phi_1 - \phi_2)/(\sigma_\epsilon/\sqrt{N})$

$Q(21, N)$ = Box-Pierce Q statistic using 21 residual autocorrelations with sample size N.

[a] Significantly different from zero at 5% level.

[b] Null hypothesis that residuals follow a white noise process is rejected at the 5% level.

for the longest term contracts. This evidence supports the earlier conclusion that the efficiency in the pricing of the longest term contract has improved, while the pricing in the shortest term contract has become less efficient. Therefore, none of the evidence presented thus far suggests that the market as a whole has become more efficient since trading began. However, before reaching any firm conclusions about the degree of efficiency in the Treasury bill futures market, the impact of trading costs must be considered.

Comparison of Actual and Theoretical IMM Index Values in the Presence of Transaction Costs

Table III presents the means of the non-annualized and annualized basis point differential net of transaction costs in cross tabular form. For those contracts for which the actual IMM Index falls below the theoretical lower bound of equation (4), the basis point differential is the difference between the actual index value and the lower bound. Similarly, for those contracts for which the index falls above the upper bound, the basis point differential is the difference between the actual value and the upper bound. For those contracts for which the index falls within the two bounds, the basis point differential is the difference between the actual index value and the no-transaction cost value of equation (5).

Figure 3 presents a plot of the annualized basis point differential and days to maturity after considering transaction costs. The values plotted are the annualized differences between the actual index values and the appropriate bounds. For those contracts for which the actual prices fall within the bounds, a value of zero is plotted.

Table III shows that 66% of the observations could not have been arbitraged, given the existence of transaction costs. As observed earlier, the average annualized arbitrage returns decreased with the length of time to maturity. Of course, the returns that actually could have been earned on an annualized basis would have been less than those indicated in the table, because many contracts did not provide recurring profitable arbitrage opportunities.

In contrast to Oldfield's observation that the market became more efficient during the first year of trading, this analysis suggests that the market became less efficient during the first twenty-seven months of trading. The percentage of observations that fell between the two no-arbitrage price bounds decreased from 72% to 58% from the first to the third nine month subperiod. Most of this decline was in the first and second contract months in which the percentage of non-arbitrageable observations fell from 89% to 45% and from 78% to 55%, respectively. However, during this same period, the pricing of the third contract became more efficient as evidenced by an increase from 43% to 77% in the number of contracts that could not be arbitraged profitably.[11]

These relationships can be seen visually in Figure 3 in which very few contracts with less than 130 days until maturity provided arbitrage opportunities in the first nine months of trading, while a large number of near term contracts would have provided arbitrage profits in the last nine months. It is also apparent from

[11] Given the autocorrelation in the data reported earlier, caution should be used in interpreting these results.

Table III

Summary Statistics for Basis Points Differential when Transaction Costs are Considered

		1st Contract Month				2nd Contract Month				3rd Contract Month				All Contracts			
Trading Period		N	%	μ	μ_a	N	%	μ	μ_a	N	%	μ	μ_a	N	%	μ	μ_a
1/06/76–9/30/76	B	0	0	0.000	0.000	18	10	−8.076	−4.994	77	46	−17.433	−6.674	95	18	−15.660	−6.335
	W	167	89	4.241	13.458	146	78	−0.020	1.045	80	48	−15.511	−7.034	393	72	−1.348	4.675
	A	20	11	3.445	4.113	23	12	7.535	5.053	11	6	10.031	3.724	54	10	6.529	4.430
		187	100			187	100			168	100			542	100		
10/1/76–6/30/77	B	1	1	−0.574	−0.582	30	16	−8.645	−4.890	98	60	−23.006	−9.103	129	24	−19.492	−8.057
	W	140	76	3.261	14.295	148	80	−0.730	0.410	65	40	−6.625	−2.762	353	66	−0.233	4.989
	A	44	23	4.417	7.166	8	4	6.486	6.309	0	0	0.000	0.000	52	10	4.735	7.034
		185	100			186	100			163	100			534	100		
7/1/77–3/31/78	B	0	0	0.000	0.000	3	1	−7.957	−4.074	37	23	−13.657	−5.159	40	7	−13.230	−5.113
	W	84	45	5.258	17.332	102	55	3.461	2.508	121	77	−2.544	−1.025	307	58	1.586	5.172
	A	102	55	10.771	17.950	81	44	13.566	11.537	0	0	0.000	0.000	183	35	12.009	15.112
		186	100			186	100			158	100			530	100		
Entire sample period	B	1	0	−0.574	−0.582	51	9	−8.404	−4.879	212	43	−19.350	−7.539	264	16	−17.164	−6.999
	W	391	70	4.108	14.590	396	71	0.626	0.878	266	54	−7.441	3.257	1053	66	−0.119	4.925
	A	166	30	8.204	13.425	112	22	11.822	9.830	11	3	10.031	3.724	289	18	9.676	11.662
		558	100			559	100			489	100			1606	100		

Legend:
- N = Number of observations
- % = % of total observatic ns within cell
- μ = Sample mean, not annualized
- μ_a = Sample mean, annualized
- B = Below lower index bound (Means are differences between actual IMM Index values and lower Index bound)
- W = Within index bounds (Means are differences between actual and no-transaction cost Index values)
- A = Above upper index bound (Means are differences between actual IMM Index values and upper Index bound)

Table IV

Average Time of Last Futures Trade

Trading Period		1st Contract Month			2nd Contract Month			3rd Contract Month			All Contracts		
		N	%	$\bar{T}$	N	%	$\bar{T}$	N	%	$\bar{T}$	N	%	$\bar{T}$
1/06/76–9/30/76	B	0	0	N/A	18	10	14.536	77	46	14.124	95	18	14.202
	W	167	89	14.125	146	78	14.437	80	48	14.142	393	72	14.244
	A	20	11	14.539	23	12	14.196	11	6	11.851	54	10	13.845
		187	100		187	100		168	100		542	100	
10/1/76–6/30/77	B	1	1	14.594	30	16	14.578	98	60	14.554	129	24	14.559
	W	140	76	14.098	148	80	14.554	65	40	14.569	353	66	14.376
	A	44	23	14.441	8	4	14.542	0	0	N/A	52	10	14.456
		185	100		186	100		163	100		534	100	
7/1/77–3/31/78	B	0	0	N/A	3	1	14.536	37	23	14.510	40	7	14.512
	W	84	45	14.234	102	55	14.562	121	77	14.448	307	58	14.427
	A	102	55	14.413	81	44	14.550	0	0	N/A	183	35	14.473
		186	100		186	100		158	100		530	100	
Entire sample period	B	1	0	14.594	51	9	14.561	212	43	14.390	264	16	14.424
	W	391	70	14.139	396	71	14.513	266	54	14.386	1053	66	14.342
	A	166	30	14.436	112	20	14.476	11	3	11.851	289	18	14.353
		558	100		509	100		489	100		1606	100	

Legend:
N = number of observations
% = % of total observations within cell
$\bar{T}$ = average time of last trade expressed in hours and fractions thereof after midnight EST
B = mean of basis point differential falls below lower index bound
W = mean of basis point differential falls within index bounds
A = mean of basis point differential falls above upper index bound

these graphs that the pricing of the longest term contracts became more efficient. One can observe that contracts with 130 to 210 days to maturity were the most efficiently priced during the sample period.

It is possible that timing differences in the reporting of prices in the futures and spot markets could create the appearance of arbitrage opportunities when, in fact, such opportunities do not exist.[12] Trading in futures halts at 2:35 p.m. EST, whereas trading in the underlying Treasury bills continues until 3:30 p.m. EST. Of course, it is possible that the last trade in each market may occur prior to the close.

If apparent arbitrage opportunities can be explained by timing differences, one would expect to observe the greatest difference in the timing of trades in those contracts that appear to sell for disequilibrium prices. Our futures data provide the time of last trade for each futures contract. According to the Federal Reserve Bank of New York's trading room, the spot quotations reflect "representative" closing bids and offers of the various dealers in the U.S. Government Securities Market. Therefore, the greatest timing differences should occur in futures contracts with the earliest time of last trade.

In Table IV, the average time of last trade is presented for the futures contract classifications in Table 3. Surprisingly, the average trading time for non-arbitrageable contracts for the entire sample is earlier than that of contracts which fall outside the no arbitrage bounds. Only in one trading period-contract month classification (3rd contract month for the period 1/06/76–9/30/76) does there appear to be any support of the hypothesis that arbitrage opportunities can be explained by timing differences in the trading of futures and spot T-bills.[13]

The potential arbitrage profits observed thus far can be viewed as quasi-arbitrage profits that could have been earned by using futures contracts to improve the return of an existing portfolio. To form a pure arbitrage position, however, one must short a spot Treasury bill. Capozza and Cornell have pointed out that the cost of borrowing a spot bill is approximately 50 basis points per year. This cost is shown in Figure 3 by the parallel lines at the +50 and −50 annualized basis point differentials. This 50 annualized basis point cost can be offset if the absolute value of the annualized return potential of the arbitrage position exceeds 50 basis points. Consistent with Capozza and Cornell's results, no observations in Figure 3 fall outside the bounds. A plot of all data points would show the same result. This implies that no pure arbitrage opportunities were available in the market during the sample period, assuming that all positions would have been maintained to maturity. Therefore, the Treasury bill futures market appears to have been highly efficient with respect to pure arbitrage opportunities. However, it appears that the futures could have been used to improve the return of a Treasury bill portfolio.

[12] It may also be possible for the opposite to occur; disequilibrium prices may appear to be equilibrium prices due to timing differences in the reporting of prices.

[13] The upper bound on the time of trade casts doubt on the appropriateness of conventional statistical tests of significance based on normally distributed trading times. Moreover, in many cases the average trading time was later for the arbitrageable contracts. Thus, we believe that tests to determine whether average trading times in the arbitrageable contracts are significantly lower than the trading times in the non-arbitrageable contracts would provide little additional insight.

III. Concluding Remarks

The evidence in this study suggests that many quasi-arbitrage opportunities have existed in the Treasury bill futures market. To determine the extent of inefficiency in the market, it is necessary to assess the significance of the quasi-arbitrage returns that could have been earned.

From the cell of Table 3 that describes the potential arbitrage returns for all contracts during the entire sample period, a potential annualized return of 7–12 basis points can be observed. Considering that profitable arbitrage opportunities were not continuously available, the true annualized return would have been lower.

It is doubtful that these potential arbitrage returns would have been worth exploiting, given the indirect costs of educating traders and policymakers within financial institutions, the costs of monitoring the futures market, the inability (in some cases) to cover a futures obligation with the exact Treasury bill required, and the reluctance by many financial institutions to alter the present maturity structure of their short term portfolios. Moreover, if a portfolio manager were to trade futures more frequently than is implied by the hold-to-maturity strategy underlying our analysis, trading costs would be significantly higher than those considered here. Finally, if a portfolio manager desired to improve the yield of an existing short term portfolio, financial instruments such as high grade commercial paper and negotiable certificates of deposit are likely to provide higher returns and greater acceptance within financial institutions than futures contracts.[14] Thus, the inefficiencies in the Treasury bill futures market do not appear to be significant enough to offer attractive investment alternatives to the short term portfolio manager.

To the extent that quasi-arbitrage opportunities have existed in the market, there appears to have been a tendency for the market to become less efficient over time. The pricing of the near term contract has become less efficient while the pricing of the third contract has become more efficient. However, it is doubtful that these inefficiencies have been large enough to induce portfolio managers to alter their investment policies.

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[14] Some financial institutions may view the futures contract as containing an element of default risk. Although the clearing members of the Chicago Mercantile Exchange guarantee the contract, the recent default on the Maine potato contract of the New York Mercantile Exchange may cast doubt on the Exchange members' guarantee. Thus, a portion of the apparent quasi-arbitrage opportunities that we observe may actually reflect a premium for default risk.

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