



## Chapter 2: Applying the Basic Model



# Main Points

## 2.1 Assumptions and Applicability

## 2.2 General Equilibrium

## 2.3 Consumption-Based Model in Practice

## 2.4 Alternative Asset Pricing Models: Overview

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## 2.1 Assumptions and Applicability

- In writing

$$p = E(mx) \quad \text{or} \quad pu'(c_t) = E_t[\beta u'(c_{t+1})x_{t+1}]$$

- We have **not** made most of these assumptions:
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## 2.1.1 We have **not** assumed complete markets or a representative investor

- These equations apply to **each individual** investor, for **each asset** to which he has access, independently of the presence or absence of other investors or other assets.
  - Complete markets/representative agent assumptions are used if one wants to use aggregate consumption data in  $u'(c_t)$ , or other specializations and simplifications of the model.
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## 2.1.2 We have *not* said anything about payoff or return distributions

- In particular, we have not assumed that returns are normally distributed or that utility is quadratic.
  - The basic pricing equation should hold for *any* asset, stock, bond, option, real investment opportunity, etc., and *any* **monotone and concave utility function**.
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## 2.1.3 This is *not* a “two-period model”

- The fundamental pricing equation holds for any two periods of a multi-period model, as we have seen. Really, everything involves *conditional* moments, so we have not assumed i.i.d. returns over time. (see Claus Munk, Financial Asset Pricing Theory, p211-213)
  - State- or time-nonseparable utility (habit persistence, durability) complicates the relation between the discount factor and real variables, but does not change  $p = E(mx)$  or any of the basic structure.. (see Claus Munk, Financial Asset Pricing Theory, p199-201,p180-182)
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## 2.1.4 We do **not** assume that investors have no non-marketable human capital, or no outside sources of income.

- The first order conditions for purchase of an asset relative to consumption hold no matter what else is in the budget constraint.
  - By contrast, the portfolio approach to asset pricing as in the CAPM and ICAPM relies heavily on the assumption that the investor has no non-asset income.
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**2.1.5 We don't really need the assumption that the market is "in equilibrium," that investor has bought all of the asset that he wants to, or that he can buy the asset at all.**

- We can interpret  $p = E(mx)$  as giving us the value, or willingness to pay for, a small amount of a payoff  $x_{t+1}$  that the investor does not yet have.
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## Here is why:

- If the investor had a little  $\xi$  more of the payoff  $x_{t+1}$  at time  $t + 1$ , his utility  $u(c_t) + \beta E_t u(c_{t+1})$  would increase by

$$\begin{aligned} & \beta E_t [u(c_{t+1} + \xi x_{t+1}) - u(c_{t+1})] \\ &= \beta E_t \left[ u'(c_{t+1}) x_{t+1} \xi + \frac{1}{2} u''(c_{t+1}) (x_{t+1} \xi)^2 + \dots \right] \end{aligned}$$

- If  $\xi$  is small, only the first term on the right matters.
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## Here is why(2)

- If the investor has to give up a small amount of money  $v_t \xi$  at time  $t$ , that loss lowers his utility by

$$u(c_t - v_t \xi) - u(c_t) = -u'(c_t)v_t \xi + \frac{1}{2}u''(c_t)(v_t \xi)^2 + \dots$$

Again, for small  $\xi$ , only the first term matters.

- Therefore, in order to receive the small extra payoff  $\xi x_{t+1}$ , the investor is willing to pay the small amount  $v_t \xi$  where

$$v_t = E_t \left[ \beta \frac{u'(c_{t+1})}{u'(c_t)} x_{t+1} \right]$$

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# Private valuation and market value



- If this private valuation is higher than the market value  $p_t$ , and if the investor can buy some more of the asset, he will do so until the value to the investor has declined to equal the market value.
  - Thus, *after* an investor has reached his optimal portfolio, the *market value* should obey the basic pricing equation as well, using post-trade or equilibrium consumption.
  - But the formula can also be applied to generate the marginal *private* valuation, using pre-trade consumption, or to value a *potential*, not yet traded security.
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## Some other issues

- We *have* calculated the value of a “small” or marginal portfolio change for the investor.
  - For some investment projects, an investor cannot take a small (“diversified”) position.
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## Example

- For example, a venture capitalist or entrepreneur must usually take all or nothing of a project with payoff stream  $\{x_t\}$ . Then the value of a project not already taken,

$$E_t \sum_j \beta^j \left[ u(c_{t+j} + x_{t+j}) - u(c_{t+j}) \right],$$

might be substantially different from its marginal counterpart

$$E_t \sum_j \beta^j u'(c_{t+j}) x_{t+j}.$$

once the project is taken of course,  $c_{t+j} + x_{t+j}$  becomes  $c_{t+j}$ , so the marginal valuation still applies to the ex-post consumption stream.

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## 2.2 General Equilibrium

$$p = E(mx)$$

- So far, we have not said where the joint statistical properties of the payoff  $x_{t+1}$  and marginal utility  $m_{t+1}$  or consumption  $c_{t+1}$  come from.
  - We have also not said anything about the fundamental exogenous shocks that drive the economy.
  - The basic pricing equation  $p = E(mx)$  tells us only what the price should be, *given* the joint distribution of consumption (marginal utility, discount factor) and the asset payoff.
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## General Equilibrium (2)

$$u'(c_t) = E_t \left[ \beta u'(c_{t+1}) x_{t+1} / p_t \right]$$

- We can think of this equation as determining today's *consumption* given asset prices and payoffs, rather than determining today's *asset price* in terms of consumption and payoffs.
  - Thinking about the basic first order condition in this way gives the permanent income model of consumption.
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## 2.2.1 which is the chicken and which is the egg?

- Which variable is exogenous and which is endogenous?  
The answer is, neither, and for many purposes, it doesn't matter.
  - The first order conditions characterize any equilibrium; if you happen to know  $E(mx)$ , you can use them to determine  $p$ ; if you happen to know  $p$ , you can use them to determine consumption and savings decisions.
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## Linear production technologies

- Suppose: the production technologies are linear: the real, physical rate of return (the rate of intertemporal *transformation*) is not affected by how much is invested.
  - Consumption must adjust to these technologically given rates of return.
  - This is, implicitly, how the permanent income model works. This is how many finance theories such as the CAPM and ICAPM and the CIR model of the term structure work as well.
  - These models specify the return process, and then solve the consumer's portfolio and consumption rules.
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# Figure 2.1 Consumption adjusts when the rate of return is determined by a linear technology

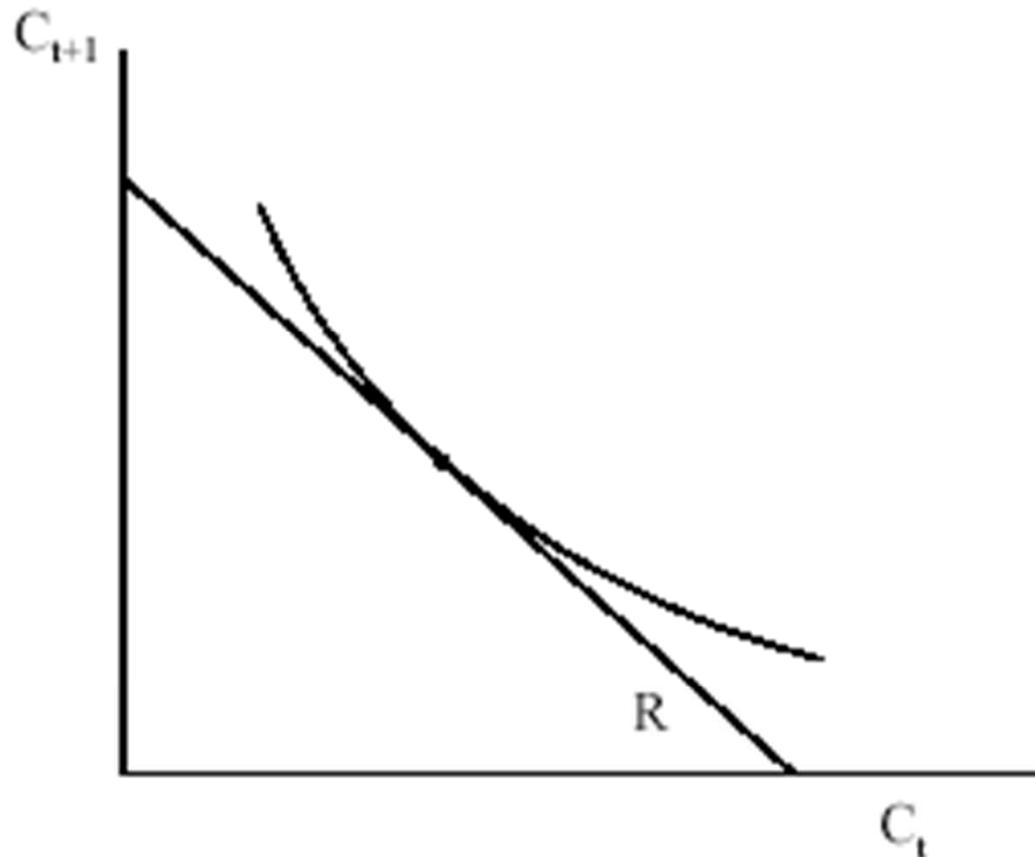


Figure 2. Consumption adjusts when the rate of return is determined by a linear technology.



# Endowment economy

- Nondurable consumption appears (or is produced by labor) every period. There is nothing anyone can do to save, store, invest or otherwise transform consumption goods this period to consumption goods next period. Hence, asset prices must adjust until people are just happy consuming the endowment process.
  - **In this case consumption is exogenous and asset prices adjust.**
  - Lucas (1978) and Mehra and Prescott (1985) are two very famous applications of this sort of “endowment economy.”
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# Figure 2.2 Asset prices adjust to consumption in an endowment economy

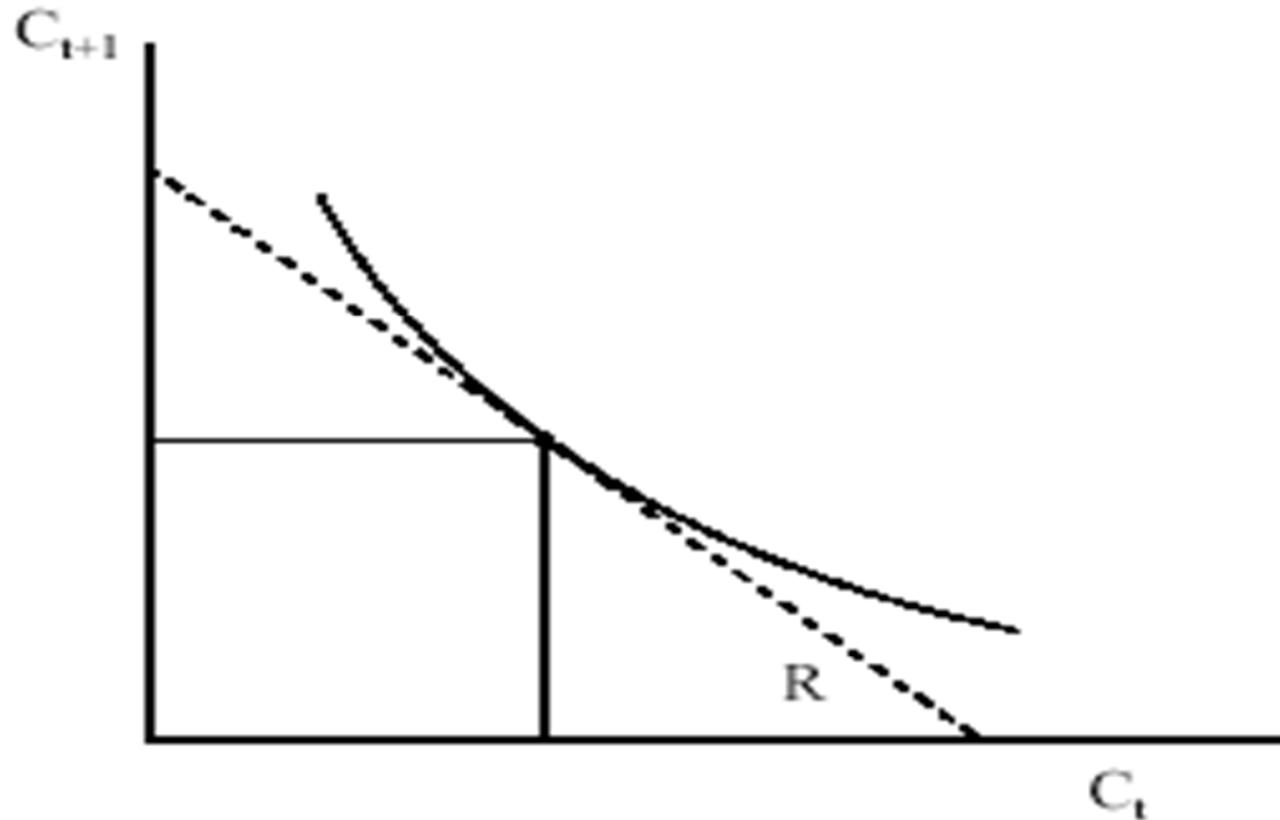


Figure 3. Asset prices adjust to consumption in an endowment economy.

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## Which of these possibilities is correct?

- Which of these possibilities is correct?  
Well, neither, of course.
  - The real economy and all serious general equilibrium models look something like figure 2.3:  
one can save or transform consumption from one date to the next, but at a decreasing rate. As investment increases, rates of return decline.
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## Figure 2.3 General equilibrium

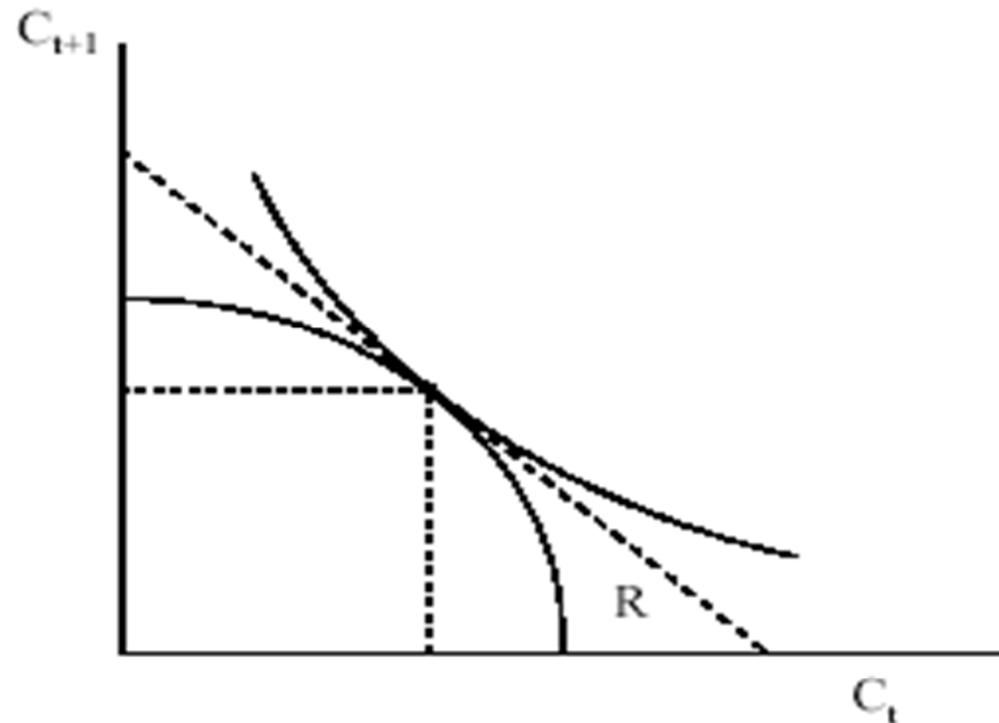


Figure 4. General equilibrium. The solid lines represent the indifference curve and production possibility set. The dashed straight line represents the equilibrium rate of return. The dashed box represents an endowment economy that predicts the same consumption-asset return process.

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# Does this observation invalidate any modeling we do with the linear technology model, or the endowment economy model? No.



- Suppose we model this economy as a linear technology, but we happen to choose for **the rate of return** on the linear technologies exactly the same stochastic process for returns that emerges from the general equilibrium. The resulting joint consumption, asset return process is exactly the same as in the original general equilibrium!
  - Similarly, suppose we model this economy as an endowment economy, but we happen to choose for **the endowment process** exactly the stochastic process for consumption that emerges from the equilibrium with a concave technology. Again, the joint consumption-asset return process is exactly the same.
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# There is nothing wrong in adopting one of the following strategies for empirical work



- Form a statistical model of bond and stock returns, solve the optimal consumption-portfolio decision. Use the equilibrium consumption values in  $p = E(mx)$ .
  - Form a statistical model of the consumption process, calculate asset prices and returns directly from the basic pricing equation  $p = E(mx)$ .
  - Form a completely correct general equilibrium model, including the production technology, utility function and specification of the market structure. Derive the equilibrium consumption and asset price process, including  $p = E(mx)$  as one of the equilibrium conditions.
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# Conclusion

- If the statistical models for consumption and/or asset returns are right, i.e. if they coincide with the equilibrium consumption or return process generated by the true economy, either of the first two approaches will give correct predictions for the joint consumption-asset return process.
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## 2.3 Consumption-Based Model in Practice

- In principle, the consumption-based model can be applied to any security, or to any uncertain cash flow, but works poorly in practice.
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## 2.3.1 Excess returns

- To be specific, consider the standard power utility function

$$u'(c) = c^{-\gamma}$$

- Then, excess returns should obey

$$0 = E_t \left[ \beta \left( \frac{c_{t+1}}{c_t} \right)^{-\gamma} R_{t+1}^e \right]$$

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## Excess returns(2)

- Taking unconditional expectations and applying the covariance decomposition, expected excess returns should follow

$$E(R_{t+1}^e) = -R^f \text{cov} \left[ \beta \left( \frac{c_{t+1}}{c_t} \right)^{-\gamma}, R_{t+1}^e \right] \quad (2.3)$$

- Given a value for  $\gamma$ , and data on consumption and returns, one can easily estimate the mean and covariance on the right hand side, and check whether actual expected returns are, in fact, in accordance with the formula.
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## 2.3.2 Present-value formula

- Similarly, the present value formula is

$$p_t = E_t \sum_{j=1}^{\infty} \beta^j \left( \frac{c_{t+1}}{c_t} \right)^{-\gamma} d_{t+j}$$

- Given data on consumption and dividends or another stream of payoffs, we can estimate the right hand side and check it against prices on the left.
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# Bonds

- An N-period default-free nominal discount bond(a U.S. Treasury strip) is a claim to one dollar at time  $t+N$ . Its price should be:

$$p_t = E_t \left( \beta^N \left( \frac{c_{t+N}}{c_t} \right)^{-\gamma} \frac{\Pi_t}{\Pi_{t+N}} 1 \right)$$

where  $\Pi = \text{price level } (\$/\text{good})$

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# option

- A European option is a claim to the payoff  $\max(S_{t+T}-K,0)$ , where  $S_{t+T}$ =stock price at time  $t+T$ ,  $K$ =strike price. The option price should be:

$$p_t = E_t \left( \beta^T \left( \frac{c_{t+T}}{c_t} \right)^{-\gamma} \max(S_{t+T} - K, 0) \right)$$

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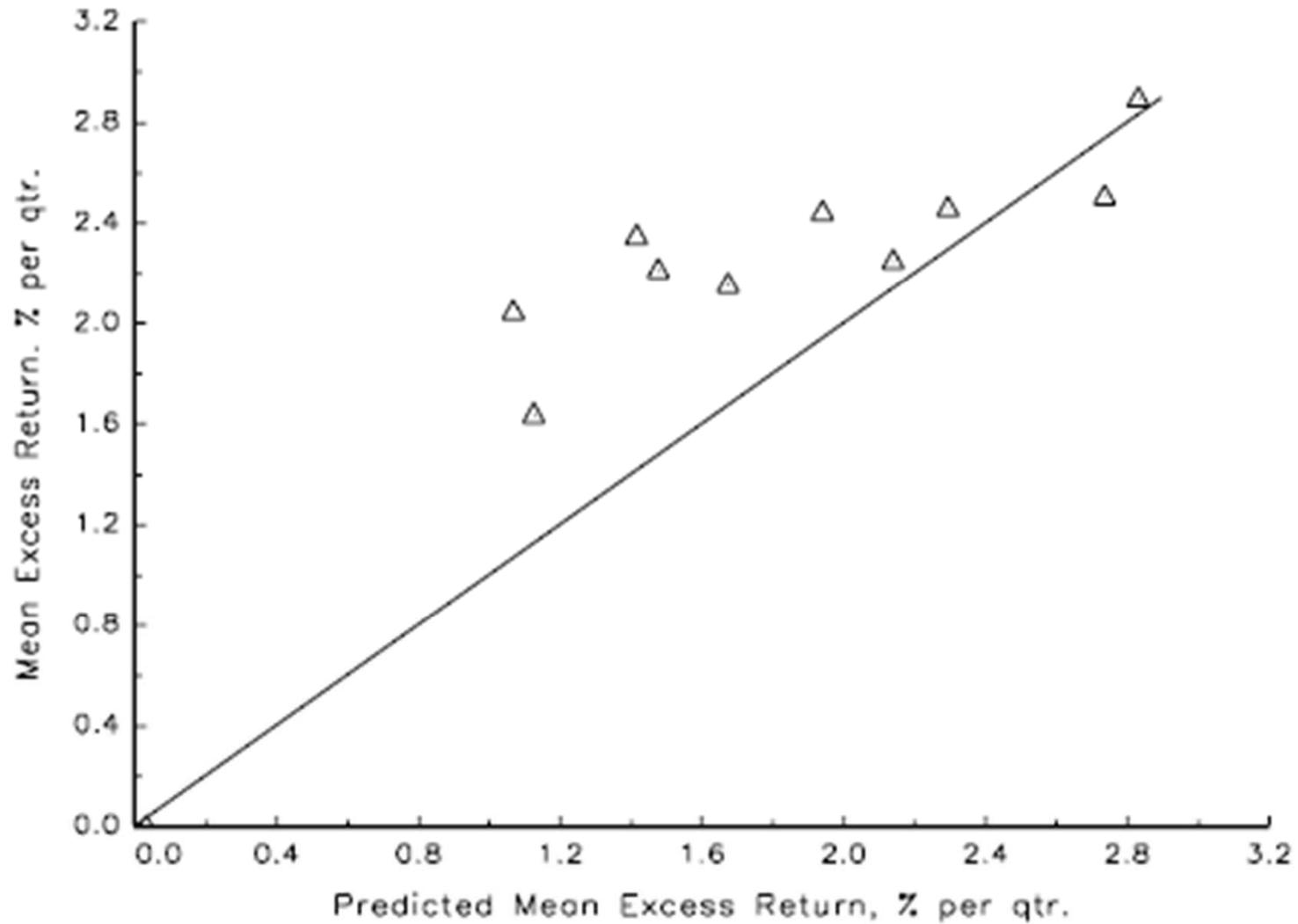
# Unfortunately, the above specification of the consumption-based model does not work very well



- Figure 2.4 presents the mean excess returns on the ten size-ranked portfolios of NYSE stocks vs. the predictions – the right hand side of (2.3) – of the consumption-based model.
  - As you can see, the model isn't hopeless—there is some correlation between sample average returns and the consumption-based model predictions.
  - But the model does not do very well. The pricing error (actual expected return - predicted expected return) for each portfolio is of the same order of magnitude as the spread in expected returns across the portfolios.
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# Figure 2.4





## 2.4 Alternative Asset Pricing Models

- The poor empirical performance of the consumption-based model motivates a search for alternative asset pricing models – alternative functions  $m = f(\text{data})$ .
  - *All* asset pricing models amount to different functions for  $m$ .
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## 2.4.1 Different utility functions

- Perhaps the problem with the consumption-based model is simply the functional form we chose for utility. The natural response is to try different utility functions.
  - Which variables determine marginal utility is a far more important question than the functional form.
  - Perhaps the stock of durable goods influences the marginal utility of nondurable goods; perhaps leisure or yesterday's consumption affect today's marginal utility.
  - These possibilities are all instances of *nonseparabilities*.
  - One can also try to use micro data on individual consumption of stockholders rather than aggregate consumption.
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## 2.4.2 General equilibrium models

- Perhaps the problem is simply with the consumption *data*.
  - General equilibrium models deliver equilibrium decision rules linking consumption to other variables, such as income, investment, etc.
  - Substituting the decision rules  $c_t = f(y_t, i_t, \dots)$  in the consumption-based model, we can link asset prices to other, hopefully better-measured macroeconomic aggregates.
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## 2.4.3 Factor pricing models

- Another sensible response to bad consumption data is to model marginal utility in terms of other variables directly.
- Factor pricing models follow this approach.
- They just specify that the discount factor is a linear function of a set of proxies,

$$m_{t+1} = a + b_A f_{t+1}^A + b_B f_{t+1}^B + \dots$$

- where  $f_i$  are factors and  $a, b_i$  are parameters.
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## 2.4.4 Arbitrage or near-arbitrage pricing

- The mere existence of a representation  $p = E(mx)$  and the fact that marginal utility is positive  $m \geq 0$  (these facts are discussed in the next chapter) can often be used to deduce prices of one payoff in terms of the prices of other payoffs.
  - The Black-Scholes option pricing model is the paradigm of this approach: Since the option payoff can be replicated by a portfolio of stock and bond, any  $m$  that prices the stock and bond gives the price for the option.
  - Recently, there have been several suggestions on how to use this idea in more general circumstances by using very weak further restrictions on  $m$  (Chapter 17).
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**The End**

Thank You!

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