Chp. 15 Tests of linear factor models

- Three approaches to the CAPM in size portfolios:Time series,cross-section and GMM/DF tests
- Monte Carlo and Bootstrap

Tested model-CAPM

- $$1 = E\left(m_{t+1} R_{t+1}\right)$$
  $$\Leftrightarrow m_{t+1} = a + bR_{t+1}^{m}$$
  $$\Leftrightarrow E\left(R^{i}\right) - \gamma = \beta_{i,R^{m}}\left[E\left(R^{m}\right) - \gamma\right]$$

$$m_{t+1} = a + bR_{t+1}^{m} \Leftrightarrow E\left(R^{e}\right) = \beta\lambda$$

$$E\left(R^{em}\right) = E\left(f\right) = \lambda$$

$$E\left(R^{e}\right) = \beta\lambda = \beta E\left(f\right) = \beta E\left(R^{em}\right)$$

Tested assets

- 10 size portfolios

Time series-Black,Jensen and Scholes(1972)

- 1. Estimate the factor risk premium
- 2.Run time-series regression for each test asset

$$R_t^{ei} = \alpha_i + \beta_i R_t^{em} + \varepsilon_t^i, i = 1, 2, \cdots, N$$

$$\hat{\beta}_i = \left(\sum_{t=1}^{T} R_t^{em2} \right)^{-1} \sum_{t=1}^{T} R_t^{ei} R_t^{em}$$

$$\widehat{E(f)} = \hat{\lambda} = \frac{1}{T} \sum_{t=1}^{T} R_t^{em}$$

OLS cross section(1)

- 1. find $\hat{\beta}$ from time-series regressions

$$R_t^{ei} = a_i + \beta_i' R_t^{em} + \varepsilon_t^i, \qquad t = 1, 2, \cdots, T$$

- Then, estimate $\hat{\lambda}$ from cross-sectional regression.

$$E_T\left(R^{ei}\right) = \beta_i' \lambda + \alpha_i, \qquad i = 1, 2, \cdots, N$$

OLS cross section(2)

$$\hat{\lambda} = (\beta'\beta)^{-1}\beta' E_T(R_t^e)$$

$$\hat{\alpha} = E_T(R_t^e) - \hat{\lambda}\beta$$

GLS cross section

$$\hat{\lambda} = \left(\beta'\Sigma^{-1}\beta\right)^{-1}\beta'\Sigma^{-1}E_T\left(R_t^e\right),$$

$$\hat{\alpha} = E_T\left(R_t^e\right) - \hat{\lambda}\beta$$

GMM(1)

- $$E\left(R_i^e\right) = \beta\lambda = \frac{\operatorname{cov}\left(R_i^e, R^{em}\right)}{\operatorname{var} R^{em}} E\left(R^{em}\right)$$

$$= \frac{E\left(R^{em}\right)}{E\left(R^{em2}\right)} E\left(R_i^e R^{em}\right) = bE\left(R_i^e R^{em}\right)$$

$$b = \frac{E\left(R^{em}\right)}{E\left(R^{em2}\right)}$$

GMM(2)

$$g_T(b) = E_T\left(R_i^e - bR_i^e R^{em}\right) = E_T\left(R_i^e\right) - E_T\left(R_i^e f\right)b$$

- The GMM estimates of b are

$$fisrstage : \hat{b}_1 = (d'd)^{-1} d' E_T\left(R_i^e\right)$$

$$\sec ondstage : \hat{b}_2 = \left(d' S^{-1} d\right)^{-1} d' S^{-1} E_T\left(R_i^e\right)$$

$$d' = \frac{\partial g_T(b)}{\partial b} = E_T\left(f R_i^e\right) = E_T\left(R^{em} R_i^e\right)$$

GMM(3)

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$$fisrstage:$$

$$\hat{b}_1 = (d'd)^{-1} d' E_T\left(R_i^e\right) = \frac{E_T\left(R_i^e\right)}{E_T\left(R^{em} R_i^e\right)}$$

$$\sec ondstage:$$

$$\hat{b}_2 = \left(d' S^{-1} d\right)^{-1} d' S^{-1} E_T\left(R_i^e\right) = \frac{E_T\left(R_i^e\right)}{E_T\left(R^{em} R_i^e\right)} ?$$

Time series and cross section(1)

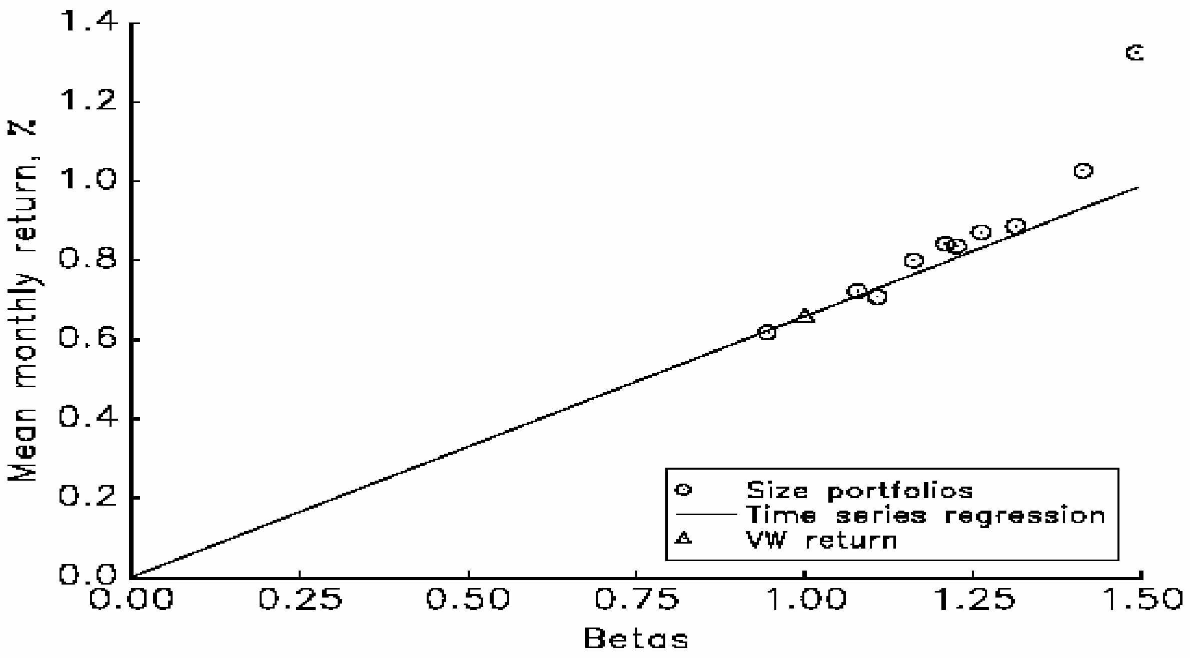

Figure 28. Average excess returns vs. betas on CRSP size portfolios, 1926-1998. The line gives the predicted average return from the time-series regression, $E(R^e) = \beta E(R^{em})$.

Time series and cross section(2)

- The time-series regression estimates the factor risk premium from the average of the factor,
- $$\widehat{E(f)} = \hat{\lambda} = \frac{1}{T}\sum_{t=1}^{T} f_t$$
- ignoring any information in the other assets.
- Thus, a time-series regression draws the expected return-beta line by making it fit precisely on two points, the market return and the riskfree rate (this can be see from the estimation of the slop)

Time series and cross section(3)

- small firm anomaly
- Why does ML ignore all the information?
- First,When we write $R^e = a + \beta f_t + \varepsilon_t$ and ε independent of f, we tell ML that Sample=factor sample+noise
- Second, we tell ML that the factor risk premium equals the mean of the factor

Time series and cross section(4)

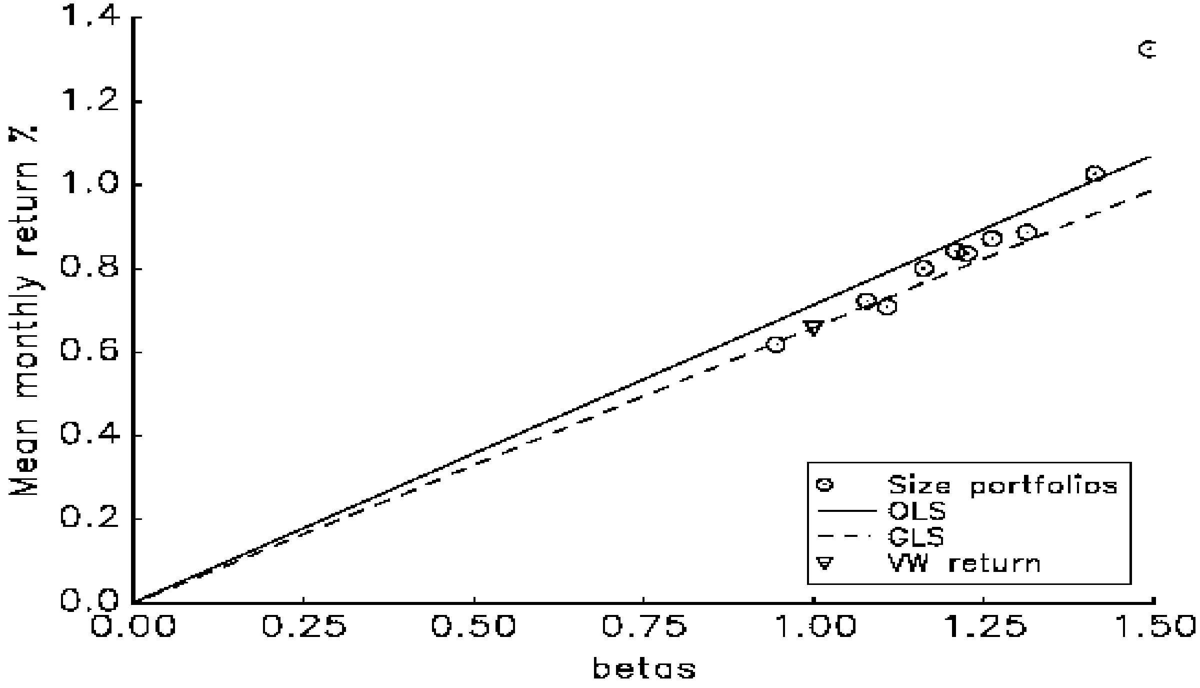

Figure 29. Average excess returns vs betas of CRSP size portfolios 1926-1998, and the fit of cross-sectional regressions.

Time series and cross section(5)

- The OLS cross -sectional regression minimizes the sum of squared pricing errors, so allows some market pricing error to fit other assets better.(real line)
- GLS is visually indistinguishable from the time series regression， because

$$\hat{\lambda} = \left(\beta' \Sigma^{-1} \beta\right)^{-1} \Sigma^{-1} E_T\left(R_t^e\right),$$

$$\hat{\alpha} = E_T\left(R_t^e\right) - \hat{\lambda}\beta$$

Time series and cross section(6)

- The GLS cross-sectional regression weights the various portfolios by the inverse of the residual covariance matrix Σ ▷
- As the residual variance of a portfolio goes to zero, the GLS regression pays more and more attention to that portfolio, until you have achieved the same result as a time-series regression.
- The size portfolios nearly span the market return, so the GLS cross-sectional regression is visually indistinguishable from the time series regression in this case.

GMM/DF first and second stage(1)

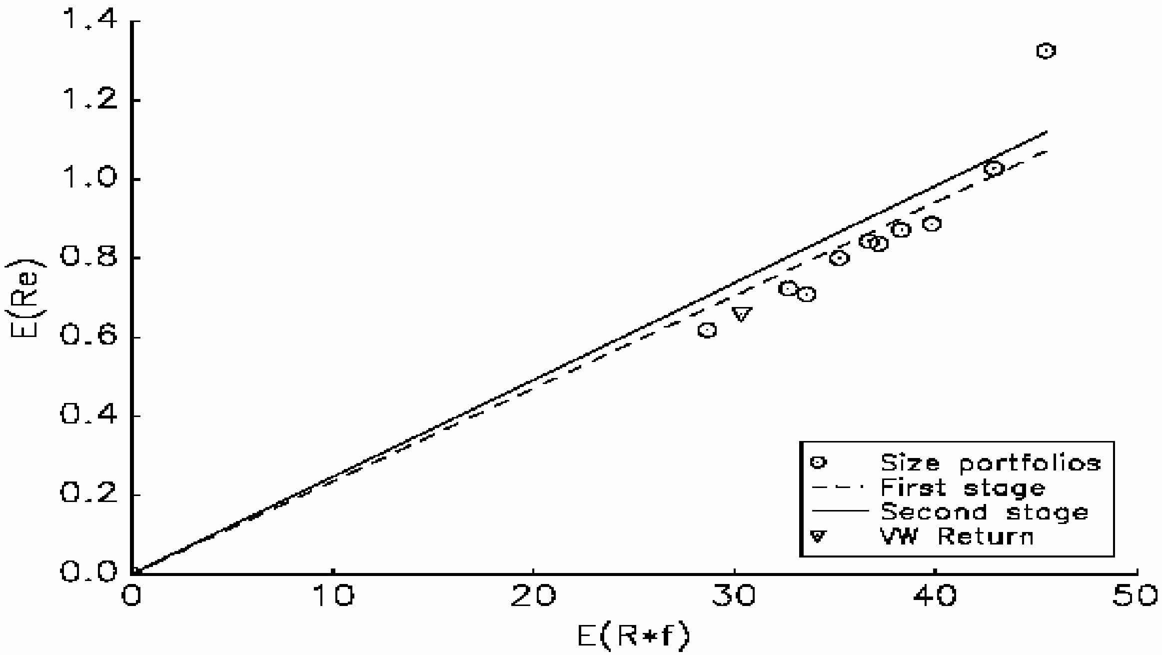

Figure 30. Average excess return vs. predicted value of 10 CRSP size portfolios, 1926-1998, based on GMM/SDF estimate. The model predicts $E(R^e) = bE(R^e R^{em})$. The second stage estimate of b uses a spectral density estimate with zero lags.

GMM/DF first and second stage(2)

- The first stage estimate is an OLS cross-sectional regression of average returns on second moments.
- The second stage estimate minimizes pricing errors weighted by the spectral density matrix.
- The slope of the line is slightly higher for the second stage estimate.

GMM/DF first and second stage(3)

- The graphs and analysis do not strongly suggest that any method is better than any other
- In particular, the size of the small firm anomaly is substantially affected by how one draws the market line.

Parameter estimates, standard errors, and tests(1)

	Beta model λ			GMM/DF b		
	Time-Series	Cross section OLS	GLS	1st stage	2nd stage *Est.*	Std. Err.
Estimate	*0.66*	*0.71*	*0.66*	2.35		
i.i.d.	0.18 (3.67)	0.20 (3.55)	0.18 (3.67)			
0 lags	0.18 (3.67)	0.19 (3.74)	0.18 (3.67)	0.63 (3.73)	*2.46*	0.61 (4.03)
3 lags, NW	0.20 (3.30)	0.21 (3.38)	0.20 (3.30)	0.69 (3.41)	*2.39*	0.64 (3.73)
24 lags	0.16 (4.13)	0.16 (4.44)	0.16 (4.13)	1.00 (2.35)	*2.15*	0.69 (3.12)

Parameter estimates, standard errors, and tests(2)

- The rows compare results with various methods of calculating the spectral density matrix.

Parameter estimates, standard errors, and tests(3)

- The big point of Table c1 is that the GMM/discount factor estimate and standard errors behave very similarly to the traditional estimates and standard errors.
- From OLS

$$b = \frac{E\left(R^{em}\right)}{E\left(R^{em2}\right)} = \frac{E\left(R^{em}\right)}{\left(E\left(R^{em}\right)^2 + \mathrm{var}\left(R^{em}\right)\right)} = 2.17$$

Parameter estimates, standard errors, and tests(4)

	Time series		Cross section		GMM/DF	
	$\chi^2_{(10)}$	% p	$\chi^2_{(9)}$	% p	$\chi^2_{(9)}$	% p
i.i.d.	8.5	58	8.5	49		
GRS F	0.8	59				
0 lags	10.5	40	10.6	31	10.5	31
3 lags NW	11.0	36	11.1	27	11.1	27
24 lags	-432	-100	7.6	57	7.7	57

Table c2. χ^2 tests that all pricing errors are jointly equal to zero.

Parameter estimates, standard errors, and tests(5)

- The big point of Table c2 is that the GMM/discount factor method gives almost exactly the same result as the cross-sectional regression.

Monte Carlo and Bootstrap

- Conduct Monte Carlos and bootstraps one each under the null that the CAPM is correct, to study size, and one each under the alternative that the CAPM is false, to study power.
- The important question is to compare the time series regression – which is ML with i.i.d. normal returns and factors – to the first and second stage GMM/DF procedure.

Monte Carlo

- The Monte Carlo experiments follow the standard ML assumption that returns and factors are i.i.d. normally distributed, and the factors and residuals are independent as well as uncorrelated.

Bootstrap

- The bootstraps check whether non-normalities, autocorrelation, heteroskedasticity, and non-independence of factors and residuals matters to the sampling distribution in this data set.

χ^2 tests(1)

	Monte Carlo						Block-Bootstrap					
	Time series			GMM/DF			Time series			GMM/DF		
Sample size:	240	876		240	876		240	876		240	876	
level (%):	5	5	1	5	5	1	5	5	1	5	5	1
i.i.d.	7.5	6.0	1.1				6.0	2.8	0.6			
0 lags	7.7	6.1	1.1	7.5	6.3	1.0	7.7	4.3	1.0	6.6	3.7	0.9
3 lags, NW	10.7	6.5	1.4	9.7	6.6	1.3	10.5	5.4	1.3	9.5	5.3	1.3
24 lags	25	39	32	25	41	31	23	38	31	24	41	32

Table 6c. Size. Probability of rejection for χ^2 statistics under the null that all pricing errors are zero

χ 2 *tests(2)*

	Monte Carlo						Block-Bootstrap					
	Time-Series			GMM/DF			Time-Series			GMM/DF		
Sample size:	240	876		240	876		240	876		240	876	
level (%):		5	1		5	1		5	1		5	1
i.i.d.	17	48	26				11	40	18			
0 lags	17	48	26	17	50	27	15	54	28	14	55	29
3 lags, NW	22	49	27	21	51	29	18	57	31	17	59	33
24 lags	29	60	53	29	66	57	27	63	56	29	68	60

Table 7c. Power. Probability of rejection for χ^2 statistics under the null that the CAPM is false, and the true means of the decile portfolio returns are equal to their sample means.

$\chi 2$ *tests(3)*

- The size and power of $\chi 2$ test statistics is nearly identical for time-series regression test and the GMM/discount factor test.

Parameter estimates and standard errors(1)

	Monte Carlo				Block-Bootstrap			
	Time series	GMM/DF			Time series	GMM/DF		
		1st stage	2nd stage $\sigma(\hat{b})$	2nd stage E (s.e.)		1st stage	2nd stage $\sigma(\hat{b})$	2nd stage E (s.e.)
T=876:								
$\sigma(\hat{\lambda}), \sigma(\hat{b})$	*0.19*	*0.64*			*0.20*	*0.69*		
i.i.d.	0.18				0.18			
0 lags	0.18	0.65	*0.61*	0.60	0.18	0.63	*0.67*	0.60
3 lags NW	0.18	0.65	*0.62*	0.59	0.19	0.67	*0.67*	0.62
24 lags	0.18	0.62	*130*	0.27	0.19	0.66	*1724*	0.24
T=240:								
$\sigma(\hat{\lambda}), \sigma(\hat{b})$	*0.35*	*1.25*			*0.37*	*1.40*		
i.i.d.	0.35				0.35			
0 lags	0.35	1.23	*1.24*	1.14	0.35	1.24	*1.45*	1.15
3 lags NW	0.35	1.22	*1.26*	1.11	0.36	1.31	*1.48*	1.14
24 lags	0.29	1.04	*191*	0.69	0.31	1.15	*893*	0.75

Parameter estimates and standard errors(2)

- The parameter distribution for the time-series regression estimate is quite similar to that from the GMM/discount factor estimate.

Parameter estimates and standard errors(3)

- A bad spectral density matrix can ruin either time-series or GMM/discount factor estimates and tests.

Parameter estimates and standard errors(4)

- There is enough serial correlation and heteroskedasticity in the data that conventional i.i.d. formulas produce test statistics with about 1/2 the correct size. If you want to do classic regression tests, you should correct the distribution theory rather than use the ML i.i.d. distributions.