Chp 1: Consumption-based model and overview

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Structure

- Basic Pricing Equation
- Stochastic Discount Rate
- Prices and Payoffs
- Classic Issues in Finance
- Discount Factors in Continuous time

Investors' Utility Function

$$U(c_t, c_{t+1}) = u(c_t) + \beta E_t\left[u(c_{t+1})\right]$$

- Utility comes from consumption and is time seperable.
- U(·)is increasing and concave. The curvature of U captures investor's aversion to risk and to intertemporal substitution.
- β is called *time preference rate* and it captures investors' impatience.

Investor's Objective

$$\underset{\xi}{Max}\, u(c_t) + E_t\left[\beta u(c_{t+1})\right]$$

$$s.t.$$

$$c_t = e_t - p_t\xi,$$

$$c_{t+1} = e_{t+1} + x_{t+1}\xi$$

- e—original consumption level.
- ξ--the amount of the asset he choose to buy.
- p_t—the price of the asset at time t.

First order condition

$$p_t u'(c_t) = E_t\left[\beta u'(c_{t+1}) x_{t+1}\right] \quad (1.1)$$

$$p_t = E_t\left[\frac{\beta u'(c_{t+1})}{u'(c_t)} x_{t+1}\right] \quad (1.2)$$

- $p_t u'(c_t)$ is the loss in utility if the investor buys another unit of the asset.
- $E_t\left[\beta u'(c_{t+1}) x_{t+1}\right]$ is the increase in (discounted, expected) utility he obtains from the extra payoff at t+1.

Stochastic discount factor

- Define the *Stochastic discount factor* m_{t+1}(1.3):

$$m_{t+1} = \beta \frac{u'(c_{t+1})}{u'(c_t)}$$

- For every investor, m is the same for every asset.
- m_{t+1} is stochastic because it's not known at t.
- m_{t+1} is also called the *marginal rate of substitution.*
- m_{t+1} is also called the *pricing kernel, change of measure* or *a state-price density.*

Basic pricing formula

- Putting (1.3) into (1.2) we can get (1.4):

$$p_t = E_t(m_{t+1}x_{t+1})$$

- Breaking (1.2) into (1.3) and (1.4) represents a deep and useful separation.
- For example, notice p=E(mx) would still be valid if we changed the utility function, but we would have a different function connecting m to data.

Prices and payoffs: Stocks

- The price p_t gives rights to a payoff x_{t+1}.
- For stocks, the one-period payoff is: $x_{t+1}=p_{t+1}+d_{t+1}$,so we get:

$$p_t = E_t\left[m_{t+1}(p_{t+1} + d_{t+1})\right]$$

Prices and payoffs: Return

- Defines gross return:

$$R_{t+1} = \frac{x_{t+1}}{p_t}$$

- We can think return as a payoff with price one.

- So we get: 1=E(mR)

Prices and payoffs: Excess return

- If you borrow a dollar at R^f and invest it in an asset with return R, you pay no money out-of-pocket today, and get the payoff R- R^f.
- You can also short-sell stock b and invest the proceeds in stock a and get an excess return Re with zero price.
- Se we get:
- $$0 = E_t\left[m_{t+1}(R_{t+1} - R^f)\right] or$$
$$0 = E_t\left[m_{t+1}(R^a_{t+1} - R^b_{t+1})\right]$$

Prices and payoffs: discount bond

- The payoff of one-period discount bond is 1.
- So we have:
- $p_t = E_t(m_{t+1})$

Prices and payoffs: options

- For an European call option, we have:

$$c_t = E_t[m_{t+1} \max(S_T - X, 0)]$$

Prices and payoffs: Real or nominal

- If prices and payoffs are nominal, we should use a nominal discount factor. For example, if p and x denote nominal values, then we can create real prices and payoffs to write:

$$\frac{p_t}{\Pi_t} = E_t[\beta \frac{u'(c_{t+1})}{u'(c_t)} \frac{x_{t+1}}{\Pi_{t+1}}]$$

- Obviously, it is the same as defining a nominal discount factor by

$$p_t = E_t[(\beta \frac{u'(c_{t+1})}{u'(c_t)} \frac{\Pi_t}{\Pi_{t+1}}) x_{t+1}]$$

Prices and payoffs: risk-free rate

Because the risk-free rate is known ,so:

$$1=E(mR^f)=R^fE(m)$$

$$R^f=1/E(m) \qquad (1.6)$$

If a risk-free rate is not traded, we can define 1/E(m)as the “shadow” risk-free rate. It is sometime called “zero-beta” rate.

Economics behind risk-free rate

- Suppose: no uncertainty, and power utility function:

$$u(c_t) = \frac{1}{1-\gamma} c_t^{1-\gamma}$$

- Then we have:

$$R^f = \frac{1}{\beta}\left(\frac{c_{t+1}}{c_t}\right)^{\gamma}$$

- Findings:

1. R^f is high when people are impatient(β is low).

2. R^f is high when consumption growth is high.

3. R^f is more sensitive to consumption growth if γ is large.

Risk-free rate under uncertainty

- Suppose c growth is lognormally distributed, Start with

$$R^f = 1/E_t\left[\beta\left(\frac{c_{t+1}}{c_t}\right)^{-\gamma}\right]$$

- Using the fact that normal X means:

$$E(e^X) = e^{E(X)+0.5\sigma^2}$$

- We have:

$$R^f = \frac{1}{\beta}\left[E_t\left(e^{-\gamma \ln\frac{c_{t+1}}{c_t}}\right)\right]^{-1} = \frac{1}{\beta}\left[e^{-\gamma E_t(\Delta \ln c_{t+1})+0.5\gamma^2\sigma^2(\Delta \ln c_{t+1})}\right]^{-1}$$

$$r^f = \delta + \gamma E_t(\Delta \ln c_{t+1}) - 0.5\gamma^2\sigma^2(\Delta \ln c_{t+1})$$

$$where: r^f = \ln R^f; \beta = e^{-\delta}, \Delta \ln c_{t+1} = \ln c_{t+1} - \ln c_t$$

Risk-free rate under uncertainty(2)

Additional finding:

- $-0.5\gamma^2\sigma^2(\Delta \ln c_{t+1})$ captures the effect of precautionary savings.
- For the power utility function, the curvature parameter γ simultaneously controls intertemporal substitution—aversion to a consumption stream that varies over time, risk aversion—aversion to a consumption stream that varies across states of nature, and precautionary savings.

The Risk-Free rate Puzzle

$$r^f = \delta + \gamma E_t(\Delta \ln c_{t+1}) - 0.5\gamma^2\sigma^2(\Delta \ln c_{t+1})$$

- 若使用$\delta = 2\%$，μ_c=2%,σ_c=2%,则

$$r^f = 0.02 + 0.02\gamma - 0.0002\gamma^2$$

rra	rf
2	5.92%
3	7.82%
4	9.68%
5	11.50%
50	52.00%
250	-7.48%

- 这就是无风险利率之谜。（see Claus Munk, Financial Asset Pricing Theory, p297-299）

Risk Corrections--price

- Using the definition of covariance cov(m,x)=E(mx)-E(m)E(x), we can write p=E(mx) as:

$$p = E(m)E(x) + \text{cov}(m, x)$$

$$p = E(x)/R^f + \text{cov}(m, x)$$

$$p = E(x)/R^f + \frac{\text{cov}\left[\beta u'(c_{t+1}), x_{t+1}\right]}{u'(c_t)}$$

- The first term is the asset's price in a risk-neutral world. The second term is a risk adjustment.
- Marginal utility u'(c) declines as c rises. Thus an asset's price is lowered if its payoff covaries positively with consumption.
- It's the covariance not the variance determines the riskiness.

$$\sigma^2(c + \xi x) = \sigma^2(c) + 2\xi\,\text{cov}(c, x) + \xi^2\sigma^2(x)$$

Risk Corrections--Returns

- From 1=E(mR^i), we have:(1.11-13)

$$1 = E(m)E(R^i) + \text{cov}(m, R^i)$$

$$E(R^i) - R^f = -R^f \text{cov}(m, R^i)$$

$$E(R^i) - R^f = -\frac{\beta \text{cov}(u'(c_{t+1}), R^i)}{u'(c_t)} \frac{u'(c_t)}{\beta E_t(u'(c_{t+1}))}$$

$$E(R^i) = R^f - \frac{\text{cov}(u'(c_{t+1}), R^i)}{E_t(u'(c_{t+1}))}$$

- Assets whose returns covary positively with consumption must promise higher expected returns to induce investors to hold them.

Idiosyncratic risk does not affect risk

- Only the component of a payoff perfectly correlated with m generates an extra return. Idiosyncratic risk, uncorrelated with m, generate no premium.
- We can decompose any payoff x into a part correlated with m and an idiosyncratic part uncorrelated with m by running a regression:
- $x = proj(x|m) + \varepsilon$
- Projection means linear regression without a constant:

$$proj(x|m) = \frac{E(mx)}{E(m^2)} m$$

$$\text{Proof}: p(proj(x|m)) = p\left(\frac{E(mx)}{E(m^2)} m\right) = E\left(\frac{E(mx)}{E(m^2)} m^2\right) = E(mx) = p(x)$$

小测

- 如果没有其他收入来源，只有证券投资的收入。那么市场组合的预期收益率是否等于无风险利率？单个证券的预期收益率取决于什么？
- 对一个房地产占资产90%的投资者来说，房地产股票的预期收益率如何？

Expected Return-Beta Representation

- From (1.12) we have (1.14):

$$E(R^i) = R^f + \left(\frac{\text{cov}(R^i, m)}{\text{var}(m)}\right)\left(-\frac{\text{var}(m)}{E(m)}\right)$$

$$E(R^i) = R^f + \beta_{i,m}\lambda_m$$

风险价格取决于m的方差

- This is a beta pricing model. The λ_m is the price of risk and is the same for all assets, and $\beta_{i,m}$ is the quantity of risk in each asset and varies from asset to asset.
- With $m=\beta(c_{t+1}/c_t)^{-\gamma}$,we can get (1.15) by taking a Taylor approximation of (1.14):

$$E(R^i) = R^f + \beta_{i,\Delta c}\lambda_{\Delta c}$$

$$Where: \lambda_{\Delta c} = \gamma \, \text{var}(\Delta c)$$

Mean-Variance Frontier

- All assets priced by m must obey(1.17):

$$\left|E(R^i)-R^f\right| \le \frac{\sigma(m)}{E(m)}\sigma(R^i)$$

- It's because:

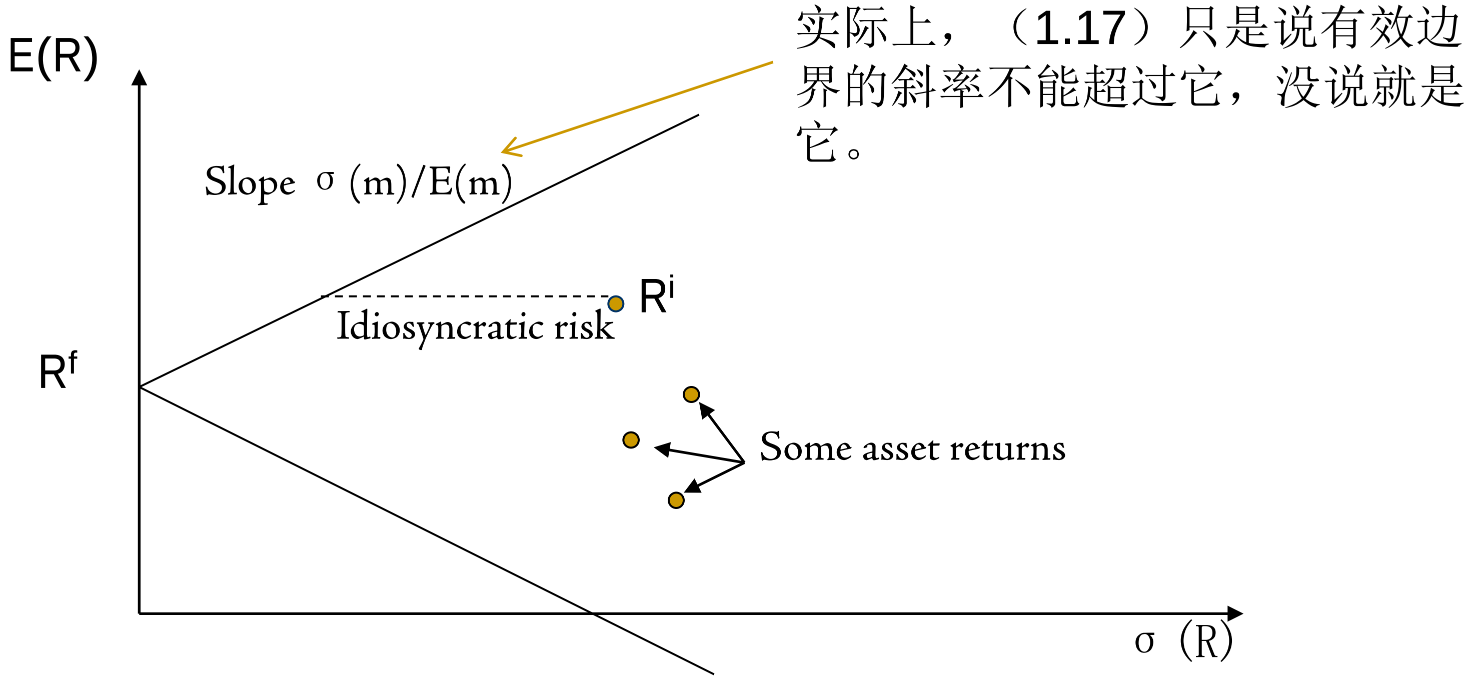
$$1=E(mR^i)=E(m)E(R^i)+\rho_{m,R^i}\sigma(R^i)\sigma(m)$$

$$E(R^i)=R^f-\rho_{m,R^i}\sigma(R^i)\frac{\sigma(m)}{E(m)}$$

$$and: \left|\rho\right| \le 1.$$

Classic Implications(1)

- Means and variances of asset return must lie in the wedge-shaped region as Fig.1.1.The boundary is called the *mean-variance frontier*.



Mean-standard deviation frontier

- The slope of the mean-standard deviation frontier is the largest available Sharpe ratio. From (1.17),the slope of the frontier is

$$\left|\frac{E(R^{mv})-R^f}{\sigma(R^{mv})}\right| \le \frac{\sigma(m)}{E(m)} = R^f\sigma(m)$$

- Under power utility function, we have

$$\left|\frac{E(R^{mv})-R^f}{\sigma(R^{mv})}\right| \le \frac{\sigma[(c_{t+1}/c_t)^{-\gamma}]}{E[(c_{t+1}/c_t)^{-\gamma}]}$$

Mean-standard deviation frontier(2)

- Suppose consumption growth is lognormal, we have

$$\sigma[(c_{t+1}/c_t)^{-\gamma}] = \sqrt{E(e^{-2\gamma\Delta\ln c_{t+1}}) - E(e^{-\gamma\Delta\ln c_{t+1}})^2}$$

$$= \sqrt{e^{-2\gamma E(\Delta\ln c_{t+1})+2\gamma^2\sigma^2(\Delta\ln c_{t+1})} - e^{-2\gamma E(\Delta\ln c_{t+1})+\gamma^2\sigma^2(\Delta\ln c_{t+1})}}$$

$$= e^{-\gamma E(\Delta\ln c_{t+1})+0.5\gamma^2\sigma^2(\Delta\ln c_{t+1})}\sqrt{e^{\gamma^2\sigma^2(\Delta\ln c_{t+1})} - 1}$$

$$E[(c_{t+1}/c_t)^{-\gamma}] = E(e^{-\gamma\Delta\ln c_{t+1}}) = e^{-\gamma E(\Delta\ln c_{t+1})+0.5\gamma^2\sigma^2(\Delta\ln c_{t+1})}$$

$$\therefore \left|\frac{E(R^{mv}) - R^f}{\sigma(R^{mv})}\right| \le \frac{\sigma[(c_{t+1}/c_t)^{-\gamma}]}{E[(c_{t+1}/c_t)^{-\gamma}]} = \sqrt{e^{\gamma^2\sigma^2(\Delta\ln c_{t+1})} - 1}$$

Mean-standard deviation frontier(3)

- Using the approximation for small x that $e^x \approx 1+x$, get(1.20)

$$\left| \frac{E(R^{mv}) - R^f}{\sigma(R^{mv})} \right| = \sqrt{e^{\gamma^2 \sigma^2 (\Delta \ln c_{t+1})} - 1} \approx \gamma \sigma(\Delta \ln c)$$

Equity premium puzzle

- *the slope of the mean-standard deviation frontier is higher if the economy is riskier – if consumption is more volatile – or if investors are more risk averse.*
- Over the last 50 years in the U.S., real stock returns have averaged 9% with a standard deviation of about 16%, while the real return on treasury bills has been about 1%. Thus, the historical annual market Sharpe ratio has been about 0.5. Aggregate consumption growth has A mean and standard deviation of about 1%. Thus, we can only reconcile these facts with (1.20) if investors have a risk aversion coefficient of 50!
- This is the "equity premium puzzle."

Equity premium puzzle(2)

- Aggregate consumption has about 0.2 correlation with the market return, If you add this fact, you need risk aversion of 250 to explain the market Sharpe ratio in the face of 1% consumption volatility!
- Clearly, either 1) people are a *lot* more risk averse than we might have thought 2) the stock returns of the last 50 years were largely good luck rather than an equilibrium compensation for risk, or 3) something is deeply wrong with the model, including the utility function and use of aggregate consumption data.
- 部分解决方案见Munk(2013，p317-370)

Random walk

- From (1.1), we have (1.21)

$$p_t u'(c_t) = E_t\left[\beta u'(c_{t+1})(p_{t+1} + d_{t+1})\right]$$

- It says that prices should follow a martingale after adjusting for dividends and scaling by marginal utility.
- Since consumption and risk aversion don't change much day to day, we might expect the random walk view to hold pretty well on a day-to-day basis.
- However, more recently, evidence has accumulated that long-horizon excess returns are quite predictable.

Time-varying Expected returns

- Writing our basic equation for expected return as:

$$E_t(R_{t+1}) - R_t^f = -\frac{\text{cov}_t(m_{t+1}, R_{t+1})}{E_t(m_{t+1})}$$

$$= -\frac{\sigma_t(m_{t+1})}{E_t(m_{t+1})}\sigma_t(R_{t+1})\rho_t(m_{t+1}, R_{t+1})$$

$$\approx -\gamma_t\sigma_t(\Delta c_{t+1})\sigma_t(R_{t+1})\rho_t(m_{t+1}, R_{t+1})$$

- It says that returns can be somewhat predictable. risk and risk aversion change over the business cycle, and this is exactly the horizon at which we see predictable excess returns.

Present-value statement(1)

- Suppose an investor can purchase a stream $\{d_{t+j}\}$ at price p_t.
- His long-term objective is: $E_t \sum_{j=0}^{\infty} \beta^j u(c_{t+j})$
- The first order condition gives (1.23):

$$p_t = E_t \sum_{j=1}^{\infty} \beta^j \frac{u'(c_{t+j})}{u'(c_t)} d_{t+j} = E_t \sum_{j=1}^{\infty} m_{t+j} d_{t+j}$$

- From $p_t=E_t[m_{t+1}(p_{t+1}+d_{t+1})]$, we can get

$$p_t = E_t \sum_{j=1}^{\infty} \beta^j \frac{u'(c_{t+j})}{u'(c_t)} d_{t+j} + \lim_{T\to\infty} E_t[\beta^T \frac{u'(c_{t+T})}{u'(c_t)} p_{t+T}]$$

- When there is no speculative bubbles, it will get (1.23).

Risk adjustment

- From (1.23) we have:

$$p_t = \sum_{j=1}^{\infty} \frac{E_t d_{t+j}}{R^f_{t,t+j}} + \sum_{j=1}^{\infty} \mathrm{cov}_t(d_{t+j}, m_{t,t+j})$$

$$Where: R^f_{t,t+j} \equiv E_t(m_{t,t+j})^{-1}$$

Discrete-time framework
另一种表达方法

- In the discrete-time multiperiod framework, a stochastic discount rate is an adapted stochastic process such that
  - $m_0 = 1$
  - $P_t = E_t\left[\sum_{s=t+1}^{T} \frac{m_s}{m_t} D_s + P_T \frac{m_T}{m_t}\right]$

Discrete-time framework

- $1 = E_t\left[\frac{m_{t+1}}{m_t}R_{t+1}\right]$
- ${R_t}^f = \left[E_t\left(\frac{m_{t+1}}{m_t}\right)\right]^{-1}$

Continuous time

- Suppose an asset's price is p_t, and it pays a dividend stream D_t.
- The Utility function is

$$U(\{c_t\}) = E_t \int_{s=t}^{\infty} e^{-\delta(s-t)} u(c_s) ds = u(c_t)dt + E_t \int_{s=0}^{\infty} e^{-\delta s} u(c_{t+s}) ds$$

$$s.t.: c_t dt = e_t dt - \xi p_t, c_{t+s} ds = e_{t+s} ds + \xi D_{t+s} ds$$

$$FOC: p_t u'(c_t) = E_t \int_{s=0}^{\infty} e^{-\delta s} u'(c_{t+s}) D_{t+s} ds$$

- Define discount factor in continuous time as: $\Lambda_{t+s} \equiv e^{-\delta s} u'(c_{t+s})$
- Then we can write the pricing equation as(1.28):

$$p_t \Lambda_t = E_t \int_{s=0}^{\infty} \Lambda_{t+s} D_{t+s} ds$$

One-period pricing equation

- From (1.28) we have:

$$p_t\Lambda_t = E_t\int_{s=0}^{\Delta}\Lambda_{t+s}D_{t+s}ds + E_t[\Lambda_{t+\Delta}p_{t+\Delta}]$$

- For small Δ we get(1.29):

$$p_t\Lambda_t \approx \Lambda_t D_t\Delta + E_t[\Lambda_{t+\Delta}p_{t+\Delta}] = \Lambda_t D_t\Delta + E_t[\Lambda_t p_t + \Lambda_{t+\Delta}p_{t+\Delta} - \Lambda_t p_t]$$

$$0 \approx \Lambda_t D_t\Delta + E_t[\Lambda_{t+\Delta}p_{t+\Delta} - \Lambda_t p_t]$$

$$As\Delta \to 0,$$

$$0 = \Lambda_t D_t dt + E_t[d(\Lambda_t p_t)]$$

- (1.29)says that after adjusting for dividends, marginal utility-weighted price should follow a martingale.

One-period pricing equation

- Using Ito lemma ,we have

$$d(\Lambda p) = p d\Lambda + \Lambda dp + dp d\Lambda$$

- The one period pricing equation become

$$0 = \frac{D}{p}dt + E_t\left(\frac{d\Lambda}{\Lambda} + \frac{dp}{p} + \frac{d\Lambda}{\Lambda}\frac{dp}{p}\right)$$

Pricing risk-free asset

- We can think of a risk-free asset as having price 1 and paying r^f_t as a dividend. i.e. $p=1, D_t=r^f_t$. Or as a asset pays no dividend but whose price climbs deterministically at a rate:
- $dp_t/p_t=r^f_t dt$
- Applying (1.29) we get (1.34):

$$0 = r_t^f dt + E_t[\frac{d\Lambda_t}{\Lambda_t}]$$

$$r_t^f dt = -E_t(\frac{d\Lambda_t}{\Lambda_t})$$

- It's equivalent to $R^f_t=1/E_t(m_{t+1})$.

More intuitive version

- Using (1.34), we can rearrange (1.33) as(1.35):
- $$E_t(\frac{dp_t}{p_t}) + \frac{D_t}{p_t}dt = r_t^f dt - E_t[\frac{d\Lambda_t}{\Lambda_t}\frac{dp_t}{p_t}]$$
- It's analogue to E(R)=R^f-R^fcov(m,R).

Relationship between asset return and consumption risk

- From $\Lambda_t \equiv e^{-\delta t}u'(c_t)$ we have:

$$d\Lambda_t = -\delta e^{-\delta t}u'(c_t)dt + e^{-\delta t}u''(c_t)dc_t + 0.5e^{-\delta t}u'''(c_t)dc_t^2$$

$$\frac{d\Lambda_t}{\Lambda_t} = -\delta dt + \frac{c_t u''(c_t)}{u'(c_t)}\frac{dc_t}{c_t} + 0.5\frac{c_t^2 u'''(c_t)}{u'(c_t)}\frac{dc_t^2}{c_t^2}$$

$$Define: \gamma_t = -\frac{c_t u''(c_t)}{u'(c_t)}, \eta_t = \frac{c_t^2 u'''(c_t)}{u'(c_t)}$$

$$\frac{d\Lambda_t}{\Lambda_t} = -\delta dt + \gamma_t\frac{dc_t}{c_t} + 0.5\eta_t\frac{dc_t^2}{c_t}$$

Relationship between asset return and consumption risk(2)

- Ignoring the orders higher than dt, we have:

$$E_t\left(\frac{d\Lambda_t}{\Lambda_t}\frac{dp_t}{p_t}\right) = \gamma_t E_t\left(\frac{dc_t}{c_t}\frac{dp_t}{p_t}\right)$$

- (1.35)becomes (1.38)

$$E_t\left(\frac{dp_t}{p_t}\right) + \frac{D_t}{p_t}dt - r_t^f dt = \gamma_t E_t\left[\frac{dc_t}{c_t}\frac{dp_t}{p_t}\right]$$

Sharp Ratio

- (1.38)can be rewritten as

$$E_t(\frac{dp_t}{p_t})+\frac{D_t}{p_t}dt-r_t^f dt=\gamma_t\rho_t\sigma_c\sigma_p$$

- Using ρ≤1 and

$$\mu_p \equiv E_t(dp_t/p_t),\sigma_p^2=E_t[(dp_t/p_t)^2],\sigma_c^2=E_t[(dc_t/c_t)^2]$$

- We have:

$$\frac{\mu_p+\frac{D_t}{p_t}dt-r_t^f dt}{\sigma_p}\leq\gamma_t\sigma_c$$

Problem 1.8

- Suppose the utility function includes leisure. Derive the pricing function. It's a multifactor model.

$$U(\{c_t, l_t\}) = E_t \int_{s=t}^{\infty} e^{-\delta(s-t)} u(c_s, l_s) ds = u(c_t, l_t) dt + E_t \int_{s=0}^{\infty} e^{-\delta s} u(c_{t+s}, l_{t+s}) ds$$

$$s.t.: c_t dt = e_t dt - \xi p_t, c_{t+s} ds = e_{t+s} ds + \xi D_{t+s} ds$$

$$FOC: p_t u_c(c_t, l_t) = E_t \int_{s=0}^{\infty} e^{-\delta s} u_c(c_{t+s}, l_{t+s}) D_{t+s} ds$$

$$Defining: \Lambda \equiv e^{-\delta t} u_c(c, l)$$

$$d\Lambda = -\delta \Lambda dt + e^{-\delta t}[u_{cc} dc + u_{cl} dl + 0.5 u_{ccc} dc^2 + 0.5 u_{cll} dl^2 + u_{ccl} dcdl]$$

$$\frac{d\Lambda}{\Lambda} = -\delta dt + [\frac{u_{cc}}{u_c} dc + \frac{u_{cl}}{u_c} dl + 0.5 \frac{u_{ccc}}{u_c} dc^2 + 0.5 \frac{u_{cll}}{u_c} dl^2 + \frac{u_{ccl}}{u_c} dcdl]$$

$$E_t(\frac{dp_t}{p_t}) + \frac{D_t}{p_t} dt - r_t^f dt = -E_t[\frac{d\Lambda_t}{\Lambda_t} \frac{dp_t}{p_t}] = -\frac{c_t u_{cc}}{u_c} E_t[\frac{dp_t}{p_t} \frac{dc_t}{c_t}] - \frac{l_t u_{cl}}{u_c} E_t[\frac{dp_t}{p_t} \frac{dl_t}{l_t}]$$

$$or, E_t(R^i) - R^f \approx -\frac{c_t u_{cc}}{u_c} \sigma_t^{i,\Delta c} - \frac{l_t u_{cl}}{u_c} \sigma_t^{i,\Delta l}$$

请提问

- Any Questions?

謝謝